



Strategy, Task Performance, and Behavioral Themes from Students Solving 2-D and 3-D Force Equilibrium Problems

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Abstract

Sophomore engineering students display cognitive strategies while solving Statics problems that yield insight into our understanding of their native levels of knowledge. Within this stage of their academic careers, initial engineering courses are being taken, laying the foundations for future engineering success. Engineering Statics – the first class in the engineering mechanics series as well as one of the first engineering courses offered to many engineering students – presents a prime environment to understand fundamental issues regarding students strategies and misconceptions in a problem solving process. Gaining an understanding of these students' approaches to Statics problems, and the possible accompanying misconceptions, is motivated by their direct correlation and impacts on future engineering coursework and success.

This study aims to discover cognitive strategies and misconceptions exhibited by engineering students as they are introduced to 2-D and 3-D force equilibrium concepts. Qualitative initial, axial, and selective coding methods, following a constant comparative analysis technique imbedded in grounded theory, were used to analyze the responses of students as they solve 2-D and 3-D force equilibrium problems recorded through a transcribed Think-Aloud protocol. An expanded pilot study – where the initial group of students solved traditional equilibrium problems and a follow-on group of students solved segmented equilibrium problems – is discussed in this paper. The study aimed to identify mental models for problem solving that can be used to frame interventions, as well as areas of need where such interventions would help students solving Statics problems. Procedural and conceptual aspects of students' strategies and misconceptions are discussed individually and interactively. Results will foster future research, refine the qualitative methods applied, and direct pedagogical descriptions of the Statics problem-solving process.

Introduction

As one of the first courses to be offered to many undergraduate engineering students, Statics serves as an introduction to the field of Engineering, and often serves as a gatekeeper for multiple fields, including mechanical, aerospace, civil, environmental, and biological engineering. Statics, essentially, is the mechanical science of things that are stationary or move at a constant rate, thus eliminating acceleration from analysis. In basic mechanics, a Statics curriculum focuses on forces exerted on a rigid-body held in equilibrium, thus preventing any translational or rotational acceleration or deformation. Failure to succeed in Statics signals an abrupt end to most educational paths in engineering, and the failure to comprehend Statics curricula creates a barrier to success in later courses in mechanics (such as Dynamics or Strength of Materials). In the interest of encouraging success by students and retaining them in the field of study¹, it becomes important to understand the concepts foundational to Statics curricula and use that understanding to improve Statics instruction. Steif and Dantzler² have pursued significant work in this direction by developing a Statics Concept Inventory that provides a detailed overview of those concepts for Statics. In doing so, Steif and Dantzler focused on the class of

questions that were purely for Statics. Of the five classes they list, three are particularly relevant to the 2 and 3-d equilibrium topics of this study: “*Free body diagrams*” (FBDs), “*Static equivalence of combinations of forces and couples*”, and “*Equilibrium conditions*”. They then present statistical validity for the problems selected for the inventory. This study will target 2-D and 3-D force equilibrium problems (i.e. no couples or moments) – in a deeper problem solving approach designed to reveal critical material from math and physics curricula that prove to be problematic for Statics students. Additionally, it looks to the behavior and strategies the students use in their problem solving approaches. Thus, the research will target math and physics concepts as well as statics. This work continues previous work developed by Goodridge, Villanueva, Call, Valladares, Wan, and Green³. Similarities and differences between this study’s findings and the work of Steif and others will be included in the Discussion section below.

Litzinger, Lattuca, Hadgraft, and Newstetter⁴ call for a deeper development of expertise within engineering students, and encourage new and improved approaches for instruction and the experiences of engineering students in hopes of improving conceptual understanding. In the spirit of that call for improvement, as well as the recommendation for discussing the relationship between educational theory and methodology by Case and Light⁵, a new approach is used in this study. Creswell⁶ recommends following quantitative measures –such as those typically used in the assessment of engineering students for their formative and summative implications – with qualitative research on the relationships within theories and models that provide insight into their mechanisms. This study adopts this model. A grounded theory approach was determined to be appropriate for identifying the cognitive strategies and misconceptions held by Statics students in 2-D and 3-D equilibrium problems in order to keep the analysis open to new findings from multiple students’ experiences – including their approaches to problem solving and the struggles that arise therein.

This paper is restricted to the subject matter complexity of a sophomore-level Statics course. The primary focus is on cataloging strategies and misconceptions in solving 2-D and 3-D equilibrium problems. The questions of what and why subjects are taught will be limited to their impact on proficiency in Statics.

Method

A grounded theory approach was utilized in this study. Through personal interaction with the students, grounding findings in their comments, behavior, and task performance, critical issues holding them back from deep conceptual understanding have been identified. The basic observations were identified by common code and grouped under shared themes which are presented in the Results section. As is the case with grounded theory, validity in this qualitative study is established through saturation. That is, when continued data analysis and reflection do not bring forth any new facets or insight, the effort is confirmed to be complete. The data collection occurred in two different sections – one in the spring and one in the fall – where similar problems were given to both groups, but an alteration of scope for the fall group was intended to provide more focus in the coding process.

A quick grading process - focused on getting “correct answers” on the problems - was used to place participants in below average, average, and above average groupings based only on problem-solving performance during the study. This enabled us to categorize via an extreme groups split⁷. Since a focus of the study is on strategies and misconceptions - which are assumed to vary between the above average and below average groups - the records of participants who performed above average and below average on the problems were given more attention during coding than those in the average grouping during the pursuit of saturation. Table 1 provides the breakdown of score ranges observed and the grouping into below average, average, and above average.

	Below Average 24-45%	Average 46-70%	Above Average 71-85%
Spring Study	5	3	3
Fall Study	2	2	3
Total %	38.9%	27.8%	33.3%

Table 1. Performance Groupings within the Study Problem Set

Data Collection

Data was collected in a laboratory setting. To facilitate this study, students were video-recorded as they took part in the study. Selected participants also participated in video-recorded open-ended post-interviews at the conclusion of the study. During the first semester of the study, participants were asked to solve a number of Statics problems. Changes were made to the format of the Statics portion of the study during the study’s evolution over two semesters, resulting in a rough division between two groups of students. The initial group of students were asked to follow a think-aloud protocol. They solved complete 2-D and 3-D force equilibrium problems. The latter group of students were given the opportunity to solve the problems in silence (this appeared to lower their stress levels), and then provide verbal reflection in an interview following completion of the problems. The second group solved similar problems, but the problems were segmented into smaller, component pieces thus allowing strategic targeting of areas of difficulty observed in the first group. For example, given the graphic shown in Figure 1, the first group may have been asked to solve for the resultant force, while the second group may have been asked to simply provide the i,j,k components of the 33-lb force and then move to another problem solving the next step towards the resultant force. The goal behind the problem segmentation was to aid in the identification of phenomena tied to specific parts of the problem set. The latter group was also asked to write down or verbalize the steps they would use to solve an equilibrium problem before working on any of the segmented problems. Ultimately, the first group of students was asked to solve five Statics problems while the latter group of students was asked to solve ten Statics problems.

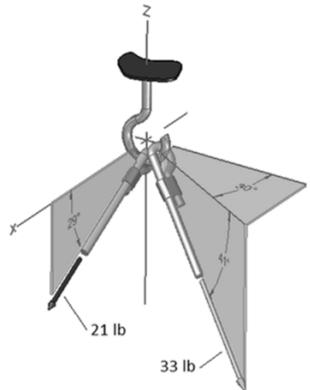


Figure 1. Sample of 3-D Statics Equilibrium Problem

The students were given a practice Statics problem to familiarize them with the types of questions that would be asked as well as the format of question delivery. All students were given sufficient time to attempt the sample problems, and no time limit was enforced on the solved problems. The students were able to end the study at any time, but all of them completed all of their assigned problems. While the two groups solved the same types of problems, revelations into the problems solving process elicited by observation of the first group led to a more refined problem development in the second group. All problems for both groups focused on 2 or 3-D equilibrium. Essentially problems were broken into subsets allowing the researcher to get a more focused data collection process developed. The first group required 35 minutes on average for their whole set of problems, while the second group required 36 minutes on average for their whole set of problems - including the verbal reflection interview following completion of the problems. It is worth noting that the first group had more variation in the amount of time required to solve the problems in terms of both differences at the extremes (41 min vs 23 min) and standard deviation (12 min vs 8.5 min). As expected, the students in the first group required significantly more time per problem than students in the students in the second group, but both groups spent about the same amount of time on the problem set as a whole.

The problems were targeted to a performance type of response and did not use any selection item format. This format was chosen to facilitate problem deconstruction and interpretation specific to the stages required to solve the problem providing better insight into the conceptual knowledge utilized by the students. That said, 2-D and 3-D Statics equilibrium problems are not “open-ended” due to the existence of a correct answer at some level, with a relatively straight-forward approach to the solution having been taught in class before the students participated in the study. Rather they are considered knowledge-rich and well-structured^{8,9}.

Sampling

Participants were recruited from two sophomore Statics classes at a western research university – one in the spring and one in the fall. The students were asked to volunteer for participation after being told that their participation would help a research effort and improve understanding in the field as well as have the possibility of benefitting future Statics students. Eleven students from

the spring class provided data used in this analysis, as did seven from the fall class, which totals to approximately the minimum sample size recommended by Creswell⁶.

Students in the two classes were given a voluntary demographics survey. Of those who responded, the percentage breakdown by major and by gender are shown in Tables 2 and 3, respectively. Students who participated in the study and declined to answer the survey question on major are listed as “Other” in the table. Ethnicity was not requested as part of the survey, but the local undergraduate population in the engineering departments of the targeted university is predominantly Caucasian.

	Aerospace	Biological	Civil	Environmental	Mechanical	Other
Spring Class	4.2%	12.5%	18.8%	4.2%	56.3%	4.2%
Fall Class	0.0%	5.6%	16.7%	2.8%	72.2%	2.8%
Study Participants	0.0%	16.7%	11.1%	5.6%	38.9%	27.8%

Table 2. Composition of Study Group and Source Classes by Reported Major Discipline

	Male	Female
Spring Class	80.9%	19.1%
Fall Class	91.7%	8.3%
Study Participants	77.8%	22.2%

Table 3. Composition of Study Group and Source Classes by Reported Gender

Data Analysis

Transcription and coding were generally done individually. Keyword searches and word frequency were initially used in an attempt to identify codes. It was found that watching the videos and coding during the viewing often provided some of the best insight – at least for those who followed the think-aloud protocol.

In reviewing the written solutions, two approaches were used. As mentioned above, a “correct answer” grading was followed to aid in the extreme groups split. For some of the problems, a correct solution was represented by simply having the correct answer written on the page. For other problems (particularly FBDs), particular features were sought after. These “correct answer” scores were positive, and are very similar to standard grading practices in a Statics class. Upon further review of the data, a secondary scoring system was developed and applied to the written solutions for detailed coding of the mistakes and misconceptions. In this latter approach, mistakes were tallied instead of correct answers. Upon observation of a mistake, an entry was made, and that entry was then a reminder to look for that same mistake to cascade through other written solutions. This approach more closely followed the pattern of grounded-theory coding done with the videos and transcripts and continues down the path established by Grigg and Benson’s¹⁰ discussion of solution states beyond “Correct” and “Incorrect”.

Coding was conducted by researchers reviewing recorded data. Prevalent codes were cross-referenced with other evaluators to identify common themes. For a few of the participants’

records, and in an effort to verify inter-rater reliability, the coding was checked with two researchers observing the video records or reviewing the written solutions simultaneously and voicing their observations dynamically - pausing the video when necessary for discussion to refine the insight or express questions about coding. This team approach to coding seemed to be more efficient and was more enjoyable than the “lonely, isolated time of struggling and pondering the data”⁶ so often invoked in thoughts about or descriptions of qualitative data analysis. This team-coding approach reinforced the sense that saturation had been achieved.

Ultimately, the codes of the written solutions aligned very well with a subset of the codes created during reviews of the video-recorded data. Multi-source (two sources for each participant, and multiple participants) analysis thus provided the desired state of saturation. As discussed below, some themes were more predominant between the first and second groups. No significant differences appeared in the codes between the groups, although the stress levels in the second group anecdotally appeared to be lower, which may be the reason for the Spring Study skew toward below-average scores seen in Table 1.

Results

A number of themes were quite plainly found in the participants’ verbal expressions and actions. These are grouped under the heading Expressed themes. Other themes were also found in the participants’ expressions, but were less straightforward. These extracted insights could generally be tied to participants’ strategies, and are thus grouped under the heading Strategy. Finally, certain themes were identified based on the participants’ performance, and were placed under the heading of Task Performance. These were identified through coding of the video-recorded performances as well as the results written on the paper. The primary themes are shown in Table 4 below.

Expressed Themes	Strategy Themes	Task Performance Themes
Environment	Confident & Expert Problem Solving	Mistakes
Remembering	Procedural Regurgitation	Shortcuts
Description	Logical vs Guess-Driven Problem Solving	Misconceptions
Uncertainty/Understanding	Spatial-Kinesthetic Conceptualization	
	Self Evaluation	

Table 4. Primary Themes for 2-D and 3-D Force Equilibrium Problems

Expressions about the *Environment* were common. These included statements about not feeling comfortable with the equipment, to behavior that demonstrated how aware they were of their surroundings. In general, it appeared that the environment did elevate the level of stress to some degree for participants, although many took it in stride. One participant even went so far as holding their calculator up to the camera when they did not write all of their work down on the paper. Another participant explained that they suppressed the desire to use hand motions in solving problems because they felt restricted by the environment. *Remembering*, as an Expressed theme, was invoked when the participant was failing to remember a mathematical formula or procedure. *Description* covers cases whenever a participant was defining the problem out loud –

the mathematical functions, the direction of vectors, or the interaction of forces – or describing the solution. *Uncertainty/Understanding* is generally tied to *Remembering*, because students often expressed doubt when they could not remember a formula or procedure. Additionally, uncertainty was expressed when a solution seemed too easy, if the participant was not sure they understood the problem statement, or if they were unsure the process they had applied was correct. At times the uncertainty was due to the simplicity of the problems – many students added extraneous information in developing the FBD that was not required in a direct analysis – most of the students who expressed uncertainty about the simplicity of the problem were those in the spring group solving the segmented version of the problems. A desire to finish was obvious, as some participants expressed uncertainty as they moved onto the next problem. The Expressed themes were much more apparent in the first group of students, due to the use of the think-aloud protocol. Similar codes were identified during the analysis for both groups, but the post-problem set verbal reflection offered relatively scant validation for the findings from the think-aloud protocol.

Strategy themes were assigned to different behaviors largely based on the confidence a participant demonstrated. A lack of hesitation and a lack of expressing uncertainty were deemed to indicate confidence. Some participants expressed confidence on nearly every problem. If they achieved the correct answer (as did the participant mentioned above who avoided showing their work on the paper by flashing their calculator screen at the camera), then a level of expertise is assumed. Thus one theme was *Confident & Expert Problem-Solving*. Granted, some participants did not express uncertainty, nor did they hesitate, and they still calculated an incorrect solution. These were labeled as having incorrectly applied a process – *Procedural Regurgitation*. It was noted that those who expressed frustration in *Remembering* seemed to want to simply regurgitate a process without understanding how it worked or how to apply it (i.e. a lack of conceptual understanding), and often set themselves up to achieve an incorrect answer in so-doing. It is assumed that those who showed confidence and achieved the correct answer had the correct conceptual understanding – some of them did indicate correct conceptual understanding in the out-loud thoughts and work indicated solid conceptualization. The *Logical vs Guess-Driven Problem Solving* strategy contains the experiences of others who were slower at working through the problem. The slower problems solvers either seemed to be reasoning out a conceptual understanding of the problem, or they seemed to be trying to pick an appropriate-sounding procedure from their memories – more cases of *Procedural Regurgitation*. A special Strategy theme was created for those who used their hands in solving the problem (herein assigned the theme *Spatial-Kinesthetic Conceptualization*). The practice did not appear to be correlated with success at achieving the correct answer, but clearly represents a physical manifestation of an otherwise unobservable – presumably spatial – thought process. These hand motions could precede any of the Strategy themes listed above. Finally, if the participant was describing the solution verbally (*Description*, above), they were generally performing *Self Evaluation*. *Self-Evaluation*-practitioners generally demonstrated conceptual knowledge of the problem, but occasionally were misguided. Self-evaluation seemed to be more dependent on the participant than on how well the participant did on a problem (i.e. some participants just made it a matter of practice to check their solutions while others did not). Interestingly, nobody included “gut check” (or the like) in their list of steps, for those who were asked to list the steps required to

solve equilibrium problems. As was the case for Expressed themes, the Strategy themes were much more obvious when participants were using the think-aloud protocol than the post-problem set verbal reflection.

Task Performance themes are based on specific errors that the participants made in providing their solution. Note that these perceived errors are more errors of the process than of the result. These errors were re-cast in the broader categories *Mistakes*, *Shortcuts* and *Misconceptions*. It is noted that participants may have been fully aware of their choice when they did something that was later coded as a *Mistake*, or they may not have agreed with labeling a particular practice as an error. However, if it would have resulted in a negative mark from a strict grader (or one who was looking for conceptual flaws), then it was included. Generally, all errors are thought of as *Mistakes*, although some may also be indicative of *Shortcuts*. Root-cause *Misconceptions* were identified to help gather together some of the *Mistakes*. The subdivisions of these themes are discussed below and then displayed in Table 5. In coding the problems, it was observed that the second problem set simply contained fewer Mistakes per problem, due the reduced scope per problem. This improved differentiability within the Task Performance themes, but no new themes were revealed.

Some of the *Mistakes* were associated with FBDs: “Inconsistent Labeling of Axes” (either they do not match other steps within the same problem, or the labels are left off), “Incorrect Drawing of Angles” (between the proper coordinate-system axis and the proper vector) “Incorrect Drawing of Vectors” (including improper placement of the vector, failure to place an arrowhead on a vector, and drawing the vectors such that they are not proportional based on magnitude), and “Failure to Label the Force” (with its magnitude or variable name). These were placed in the *Misconception* “Limited Spatial Conceptualization”, along with the “Incorrect Sign Convention” *Mistake*. These errors often resulted in an incorrect answer. One other FBD error – the “Absence of an FBD” – does not necessarily indicate a lack of spatial understanding because several participants were observed who left off the FBD and still arrived at the correct answer.

Mistakes in balancing forces were costly, in terms of arriving at the correct answer. Some participants “Did Not Include the Reaction Force in the FBD”, and some “Failed to Balance Forces” when solving the equations. The latter problem nearly always resulted in an incorrect answer, and the former sometimes did. These were cast as “Incorrect Physical Conceptualization” *Misconceptions*.

Three other *Mistakes* fall in the *Misconceptions* category of “Mathematical Errors”. Those three are “Incorrect Vector Math” (including summation of vectors and finding the magnitude of a vector), “2-D Trigonometry Problem” (i.e. involving a single trigonometric function), and “3-D Trigonometry Problem” (i.e. involving multiple trigonometric functions). The “3-D Trigonometry Problem” is also considered to be a “Limited Spatial Conceptualization” *Misconception*.

The final two *Mistakes* are “Incomplete Solution” and “Failure to Follow Instructions”. These may be related. However, the “Incomplete Solution” may also be indicative of a *Misconception* (unspecified) or is simply a *Shortcut* gone awry. Other perceived *Shortcuts* include “Absence of

an FBD”, “Failure to Label the Force”, “No Written Calculations” (i.e. not showing their work), and “Inconsistent Labeling of Axes” (in the case where the labels are left off). Note that “No Written Calculations” is not coded as a *Mistake*, but would result in a reduced assessment score if partial credit were provided.

Task Performance Themes		
Misconceptions	Mistakes	Shortcut?
Limited Spatial Conceptualization	Inconsistent Labeling of Axes	X
	Incorrect Drawing of Angles	
	Incorrect Drawing of Vectors	
	Failure to Label the Force	X
	Incorrect Sign Convention	
	3-D Trigonometry Problem	
Incorrect Physical Conceptualization	Did Not Include the Reaction Force in the FBD	
	Failed to Balance Forces	
Mathematical Errors	Incorrect Vector Math	
	2-D Trigonometry Problem	
	3-D Trigonometry Problem	
unspecified	Incomplete Solution	X
	Failure to Follow Instructions	X
		Absence of an FBD
		No Written Calculations

Table 5. Subdivision of Task Performance Themes into Misconceptions, Mistakes, and Shortcuts

Discussion

It is interesting to view the results with regard to the literature on Statics education, as there is much in common. Steif¹¹, at least in general terms, identified the *Mistakes* indicated above, as did Litzinger et al⁷. The narrow focus of this paper to 2 and 3-D equilibrium problems promotes the identification of problem-specific errors that often fit under cited themes, but with subtly different nuances. It is in the revelations garnered from data with subtle differences or nuances where new findings are most insightful. Case & Marshall¹², Litzinger et al⁷, and Grigg & Benson¹⁰ provide other useful presentations of student learning and problem-solving, which will be included in the comparison below.

Steif’s¹¹ *Concepts of Statics and Common Errors* most closely align with the Task Performance themes described above. Primarily, the conceptual foundation of the targeted portion of Statics included in this study belong to Steif’s fourth concept: “Equilibrium conditions are imposed on a body.” The Expressed theme of *Misconceptions* from this study do not match his concepts, but may be said to lie at a more focused level. His third and tenth errors cover multiple errors identified in this study. Error 3, "Leaving a force off the free body diagram (FBD) which should

be acting," aligns with the FBD *Mistakes* listed above - particularly the "Incorrect Drawing of Vectors" (in the case that a vector is left off the FBD). Error 10, "Failure to impose balance of forces in all directions and moments about all axes," aligns with the *Mistakes* regarding the balancing of forces. It is noted that Steif's Error 3 may act as a precursor to both of this study's errors in that category ("Did not Include the Reaction Force in the FBD" and "Failed to Balance Forces"). Two of this study's Misconceptions – "Limited Spatial Conceptualization" and "Incorrect Physical Conceptualization" – share elements with both of Steif's *Common Errors* listed above. Steif also indicates that resolving and combining forces are critical mathematical errors, but does not identify the specific errors as done in this study. Most of the errors identified in this study and by Steif would be coded as "Conceptual errors" by Grigg and Benson¹⁰. It is noted that some of the errors coded in this study may also result from their "Management Errors" – namely "Inconsistent Labeling of Axes", "Incorrect Drawing of Angles", "Incorrect Drawing of Vectors", "Failure to Label the Force", and "Incorrect Sign Convention". Some errors identified as "Mechanical Errors" by Grigg and Benson may have also been identified – "Incorrect Sign Convention" and the mathematically-founded errors listed above. We feel that a trigonometry problem is better classified as a "Conceptual Error" than a "Mechanical Error" (it is unclear how a trigonometry problem would have been coded by Grigg and Benson as their records only indicate algebra problems). This is based on the assumption that through visualization (a conceptual process), students can identify proper application of trigonometric functions, and that improper application of a trigonometric function is not due to a mechanical mistake such as typing it into a calculator incorrectly, or plugging an angle into the function incorrectly.

We found that the think-aloud protocol provided a lot of insight into the cognitive and meta-cognitive strategies used by the participants. In fact, without the think-aloud protocol it was difficult to identify meta-cognitive processes. Only a few of the post-assessment reflection interviews provided metacognitive insight. Litzinger et al⁷ provide good insight on cognition and meta-cognition in solving Statics problems. They apply inferential statistics to the use of a think-aloud protocol and the resulting coded indicators of meta-cognitive control, and self-explanation. Litzinger's findings align well with the findings in this paper. Case and Marshall¹² identified approaches to learning also align well with their findings and this paper's findings. The commonality is discussed below.

As students were observed who used logic to tease out the solution, we identified them as using the logical side of the *Logical vs Guess-Based Problem Solving Strategy* listed in this paper, and we found overlap discussed by Litzinger et al⁷ as Principle-Based explanations with problem-representation explanations. This also aligns with Case and Marshall's¹² deep approaches to learning. Meanwhile, Grigg and Benson¹⁰ utilize the Problem Decomposition code (and possibly the Forward Chaining code) at the logical end of the spectrum and Means-End Analysis code at the more guess-based end of the spectrum. Evidence of Clustering, as used by Grigg and Benson was not considered, and thus was not identified. The presence of *Spatial-Kinesthetic Conceptualization* is a potential aid to achieve the deeper level of comprehension identified by Litzinger.

The *Confident & Expert Problem-Solving* Strategy observed in this study align with anticipative explanations⁷ as well as the deep approaches to learning¹². Based on the coding done in this study, it is difficult to differentiate, but future work may target this aspect more precisely.

As students verbally described their efforts to confidently remember the proper course of action, the *Remembering* and *Uncertainty/Understanding* themes were applied in this study. Those combined themes are closely related to the label Monitoring used by Litzinger et al⁷. And their juxtaposed code of Evaluation aligns with the *Self Evaluation* Strategy and overlaps with the *Description* expressions from this study. Depending on the intention expressed by the students, these could align with surface (e.g. if the self-evaluation was simply a statement of what had been done or included incorrect information that was rattled off without hesitation) or deep (e.g. if the self-evaluation involved expressing understanding of the problem or reasoning about how to know if the solution was correct) approaches¹². The *Self Evaluation* Strategy would also be identified by Grigg and Benson's¹⁰ "Guess-and-check" code.

As a lack of conceptual knowledge was observed, *Remembering* became more important with the apparent goal of *Procedural Regurgitation*. This aligns well with Grigg & Benson's¹⁰ Plug-and-chug code, and has strong similarities with Case and Marshall's¹² surface approach and procedural surface approach. As they mention, the only difference between the two approaches is the strategy used. To make the transition from the procedural surface to the procedural deep approach required a change in intention. Specifically, a participant would seek to understand the use of the procedures rather than simply recall them from memory. This aligns with the perceived difference between those who logically applied the problem-solving process from those who were simply trying to regurgitate the procedures needed to finish the problem-solving process. It is beneficial to view progression through these strategies or approaches to learning in the context of the progression from novice to expert in Litzinger et al⁴.

Spatial approaches to problem-solving (particularly represented by *Spatial-Kinesthetic* Conceptualization) did not generally seem to be tied to a specific strategy, they were indicative of a deeper approach. Based on Case and Marshall's¹² descriptions, it would seem that spatial problem-solving would be more aligned with the conceptual deep approach, but observations in this study indicate that procedural approaches were still common following behaviors indicating spatial thought processes. The use of spatial ability in Statics problem solving was not discussed in the publications reviewed.

Practical Implications

The practical implications of this research will be characteristic to certain types of recipients. Those recipients may be grouped into those interested in research, and those participating in similar studies. Those who may be interested in the research include other researchers and instructors. These findings may serve as a baseline against which other findings may be differentiated – for content specific to Statics coursework (such as that recorded by Steif and Dantzer¹³ as well as physical and mathematical concepts that are critical to Statics coursework. Additionally, the specificity of the listed *Mistakes*, *Shortcuts*, and *Misconceptions* that lead to errors is intended to provide reference material for new instructors who may not anticipate all of

the mistakes listed, and can thus plan interventions before the behavior become apparent in class. It is noted that most experienced Statics instructors would likely anticipate the errors identified in this study. The presentation of student errors and behavior in the context of *Misconceptions* and *Strategy* may also aid in the eventual improvement of instruction which benefits the students – an instructor can diagnostically anticipate typical mistakes and even interpret behaviors to understand where and when there are difficulties. For further insight on improvement of instruction, other publications are available¹³.

The *Strategy* themes may be used to characterize the types of learning approaches utilized in a given class. As Case and Marshall¹² mention, it is likely the course context that is driving them to a specific learning approach. They recommend that for foundational courses, the context of the course be reviewed to encourage conceptual deep approaches. Statics is such a foundational course. Student scores and behaviors may be applied in formative or action research manner by an instructor to evolve their course's content to encourage their desired learning approach.

Direct benefit to the participants in this study was limited to the satisfaction of having been involved and to the benefits that come with increased practice¹². Upon reflection, it has been noted that more could have been done to provide benefit to the participants. In Litzinger et al⁷, they indicate that when students are at an impasse, that is when they have an opportunity to really learn. An interaction could have been planned for those participants who reached an impasse during the Statics problems, to increase the likelihood of real learning occurring. Such an intervention could have been part of the reflection interview. It is also noted that conducting such an interview immediately following a problem could provide more benefit to the participants than reflecting on the problems after all of them have been completed. It is hoped that those students who reached an impasse did obtain some insight and deeper learning as they moved through it during the study.

Study Limitations

This study focuses on a targeted portion of the Statics problem set – 2-D and 3-D equilibrium problems. Other works¹³ are recommended to the reader for insight on different Statics problems.

The misconceptions discovered are meant to add to and develop the research currently in place. However, the data analyzed is specific to the types of problems seen here and further research is needed to discover commonality to other problem types within Statics. In addition, the participants represent a small segment of the total possible populations that may be engaged in Statics on a national and international level. Further work would be required to discover if other populations reveal similar misconceptions. The processes defined in a grounded theory study provide a framework upon which quantitative analysis may be built. Such analysis within other populations could improve generalizability and transferability. Finally, the interaction of the researchers with the participants in this study may play a part in the mistakes exhibited, as treatment was similar to that in a classroom during a quiz or test (i.e. the participant was left to solve the problem on their own with minimal dialog and without intervention by the researcher). This maintained some authenticity for a study of learning in the typical academic environment,

but a more involved dialog may improve insight into the cognitive and meta-cognitive processes of students.

Conclusion and Recommendations

Specific findings have been provided to support and expand upon the publications on Statics coursework^{7,13} and student learning strategies and approaches^{4,10,11}, as well as engineering education research methods^{3,5}. The strategies employed, behaviors manifested, and errors made in Statics courses will sound familiar to those with experience in the field. Recommendations exist that encourage the improvement of student performance and comprehension through a change of context¹² – the need for which is recognized in review of student performance as done in this study. It is interesting to note that demonstration of confidence in their implemented procedure seemed to be the goal of many of the students, although it did not guarantee a correct solution – thus a false confidence was observed. An investigation into stress (as opposed to that which is observable by a video camera) may shed more light onto the issue of confidence. Axial coding based on confidence was not performed in this study, and thus the granularity of the confidence coding remains coarse, but it did lead us to separate some strategies from others. Focused research on and coding of confidence and its interplay with stress may yield fruitful results.

Ideas for improvement include having students involved in the grading process and reflecting upon seeing how their performance compared to the standard. These practices could, respectively, help participants receive personal instruction, formative feedback, and correction by the research assistant and enable the expression of participants' impressions on the value of the research more fully. Additional insight would also be gained by the researcher during transcription, analysis, and reflection activities that include the participants.

For future research, investigation into the relationship between spatial ability and competency in Statics is recommended. Relationships across themes may be researched with much benefit, such as identifying any correlations between *Expressed* themes (e.g. *Uncertainty/Understanding*) and *Strategy* themes (e.g. *Procedural Regurgitation*) with *Task Performance* themes (e.g. *Incorrect Physical Conceptualization Misconception*). The expert-novice split also deserves more study, and should be directed along pathways exposed by this research as well as by Litzinger et al⁴. Targeting the amount of time required to solve each problem may shed interesting light on the expert-novice split, and is given some attention in some of Litzinger's other work⁷. Additionally, further investigation into the team-coding approach as a developing methodology for verifying inter-rater reliability and saturation is recommended due to its perceived increase in efficiency compared to parallel individual coding efforts.

Acknowledgements

The authors would like to acknowledge Dr. Idalis Villanueva, Nicholas J. A. Wan, Brad Robinson, Maria Manuela Valladares, and Dr. Kerry Jordan for helping with data collection and providing a quiet lab in which to conduct the study. Garin Savage helped with transcription. Funding for the study came from Utah State University's Office of Graduate Studies via the Presidential Doctoral Research Fellowship for Benjamin Call, the Utah State University

Research Catalyst SEED Grant for Maria Manuela Valladares, attained from Dr. Idalis Villanueva, and the College of Engineering for Christopher Green as the lead Statics teacher's assistant and undergraduate researcher.

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