



## Strength of Materials Through Economical Activities

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## Abstract

The content of a typical strength of materials course offers many challenges to students. Understanding requires retention of core knowledge from multiple previous courses, quick acquisition of new concepts and vocabulary, and synthesis of old and new content to address relatively practical, often realistic problems. Several simple, low-cost activities were developed to promote mechanics concept comprehension, and link these concepts to practical applications and prior knowledge. Through activities incorporating various types of loading such as axial forces, direct shear forces and torque, students are encouraged to think about fundamental stresses, deformations, and their relationship to product design of common items.

The low-cost strength of materials activities described in this paper were implemented in a laboratory setting with mechanical engineering technology students in a sophomore-level course. The activities could be conducted by groups of students in a recitation or studio setting, or adapted to be a combination of demonstration and student group work in a larger lecture setting or flipped classroom. Activity success at improving understanding and application of strength of materials topics, engaging students, and incorporating design thinking is being assessed directly through student surveys and indirectly through regular examinations. Evaluation of assessment data and corresponding improvement efforts will be reported.

## Background

Strength of materials consists of stress, strain, and stability, and how material properties and geometry affect them. This sector of mechanics serves as the foundation for several disciplines. As a result, strength of materials knowledge is required for accreditation in several engineering technology disciplines, and is often included in a variety of affiliated programs, as listed in Table 1.<sup>1</sup> Engineering technology's foundational focus is on practical application of engineering principles and sets it apart from engineering.<sup>2,3</sup> Traditionally, this focus on application has relied on the existence of well-equipped industrial-type laboratories. As equipment costs increase, corporate donations dwindle, and university revenues are directed elsewhere, programs are challenged to have multiples of one piece of any high-cost test equipment. Students subsequently have more demonstration-style laboratory sessions, spend too much of a laboratory session waiting for their turn on the sole piece of necessary test equipment, or otherwise have less opportunity for experiential learning of their discipline.

Table 1: Programs with Strength of Materials content

ETAC Program Accreditation Requirement	Often included but not an ETAC Requirement
Aeronautical	Architectural
Civil	Automotive
Electromechanical	Construction
Marine	Industrial
Mechanical	Manufacturing

In parallel, the drives for online learning for place-bound students and active learning in the lecture setting for in situ students provide motivation for trying other less laboratory-oriented approaches to experiential learning of engineering principles and their application.<sup>4,5</sup> An additional potential benefit to alternative ways of practical learning is the chance to connect these engineering principles to more consumer-oriented products, showing their usefulness to a broad spectrum of society and complementing the choices of women and under-represented engineering (and engineering technology) students.<sup>6</sup> Mott's fifth edition of *Applied Strength of Materials* pioneers this approach through its inclusion of hands-on concept-developing introductory activities that can be done in class or at home for each chapter.<sup>7,8</sup>

The following activities were developed and implemented within a scheduled laboratory setting to ensure students would connect strength of materials concepts and practices to common consumer products.

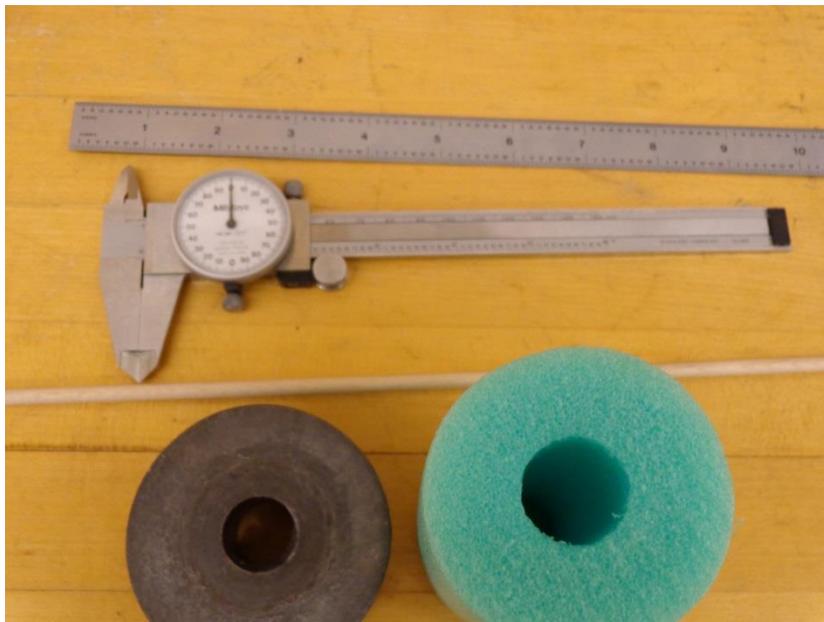


Figure 1: Axial deformation activity items

### Axial deformation activity

This activity incorporates short tubular foam pool noodle sections, known masses, and dial calipers, as shown in Figure 1. The primary objectives are to gain practice with calculating axial stress and with use of the standard equation for axial deformation, especially to recognize that geometry and material stiffness affect how components respond to axial loading. Secondary objectives are to review mass to weight conversions including changing systems of units, to ensure confidence with use of dial calipers for dimensional measurements; to see why significant figures are important; and to recognize manufacturing concerns limit the applicability of theoretical equations. The basic procedure is to measure the length and diameters of the noodle, then add a known compressive mass centered at the axis of the noodle and re-measure the noodle's length. Activity results are calculations of applied force in metric and U.S. customary units, axial stress in the tube, and approximate modulus of elasticity of the foam. The relevant equations for axial stress, axial deformation, and conversion from mass to weight are listed in Table 2.

Table 2: Axial Deformation Activity Equations

Cross-sectional area $A = \frac{\pi}{4} [D^2 - d^2]$	Where: D = outer diameter; d = inner diameter
Axial stress $\sigma = \frac{F}{A_{\perp}}$	Where: F = force (weight); $A_{\perp}$ = cross-sectional area
Axial deformation $\delta = \frac{FL}{AE} = \sigma \frac{L}{E}$	Where: F = force (weight); L = length; A = cross-sectional area, and E = modulus of elasticity
Force (Weight) $F = mg$	Where: m = mass (known); g = gravitational constant (with appropriate conversion factors applied, as needed)

Three-person student teams completed the axial deformation activity during the first laboratory session of the semester. It served as an ice-breaker, providing students a non-threatening time for getting acquainted and returning to the expectations of a rigorous engineering technology course after their summer break. The extent of need for review of very basic engineering practices was greater than anticipated. Multiple teams requested faculty confirmation regarding how to convert mass into weight and how to calculate the cross-sectional area of a hollow circle. (It should be noted that the spring semester students' need for review of basic engineering practices following the winter break was much lower).

Use of pool noodle sections contributed several benefits. Noodles are a common gender-neutral consumer product that nearly every student knows, illustrating the relevance of strength of materials to normal life. Noodles are extruded in a way that makes their central axis an arc rather than a straight line and causes some variation in their diameters, while cutting them into short sections results in cross-sections that are not fully perpendicular to the axis. These manufacturing constraints become obvious when applying an axial compressive load and making the length and diameter measurements, forcing students to think about differences between the assumptions inherent to the axial stress and deformation equations and the real objects to which they are

applied. For mechanical engineering technology students at Purdue University, the foam noodles also link their strength of materials course to a freshmen-level material processing course where foams and extrusion were introduced, contributing to student understanding of the breadth of their discipline. Cost of the noodles is a final perk, with late summer clearance prices of one or two dollars per 5-foot long noodle, a quantity that provides sufficient tubing for about twelve student teams.

Implementing the axial deformation activity in other settings would require some adaptation. For active learning in a large-lecture setting, the measurements portion may have to be done as a demonstration (preferably with student volunteers to provide some of the insights that come during the measuring experience). No modifications should be needed for the related calculations. For distance students, developing a short video of the measurements portion would suffice. Calculations could be done individually, but working in small teams via the Internet or otherwise is suggested to facilitate student interactions. In all settings, one implementation challenge comes from the slight arc of the noodle's longitudinal axis. When adding the compressive mass, a mechanism for ensuring the mass stays on the noodle may be required. When hollow-disk masses are used, a short section of dowel rod, long pencil, or similar item will suffice. When the masses are solid, enclosing the whole unit within clear tubing may be necessary (making deformed length measurement more difficult) or simply having a student prepared to steady the unit may resolve the issue.

#### Simple or direct shear stress activity

Determining the relevant shear area is often a point of confusion for strength of materials students when considering direct shear loading. This activity was developed to clearly show the difference between single and double shear areas and how the loading and shear area interact, with secondary objectives of reviewing analog micrometer use and considering ultimate shear strength. Small craft sticks were joined by applying a dot of standard white school glue to one overlapping end of the sticks to create either single or double shear specimens. The adhesive was assumed to form a circular area equal in diameter to the width of a craft stick. Enough sets of sticks were bonded to have at least one single and one double shear specimen per three-student team. Representative components are shown in figure 2. Teams used micrometers to measure the width of the sticks at multiple locations, averaging their width values to establish a shear (adhesive) area diameter and calculate their single and double shear areas. Student pairs then pulled on the craft sticks until shear failures occurred. Forces were not measured. Instead, based on given forces of  $F$  for single shear and approximately  $2F$  for double shear, simple shear stresses were calculated using the teams' shear areas. Related equations are given in Table 3. Linking the resulting stress to material properties was encouraged. Manufacturing concerns were again noted to connect application of theoretical equations to practice. In this case, the nature of

the craft stick wood was sometimes a big factor, with bent and twisted sticks making the application of forces parallel to the adhesive area challenging. The gluing process resulted in additional variations that were occasionally far from ideal, with angles in both the longitudinal and lateral planes of the specimens. Reading analog micrometers nearly always requires some student review. This can, of course, be omitted if the instructor chooses to provide dimensions or has digital micrometers. Having the students obtain the width measurements helps them recognize that material rigidity affects the dimensional reading shown, so including students when making these measurements is recommended whenever practical.



Figure 2: Simple (direct) shear stress items

Table 3: Direct Shear Stress Equations

Direct shear stress $\tau = \frac{F}{A_s}$	Where: F = given shear force; $A_s$ = shear area (area that resists sliding and is parallel to the force)
Single shear area $A_s = \frac{\pi}{4}(D^2)$	Where: D = diameter (and stick width)
Double shear area $A_s = \frac{\pi}{4}(D^2)(2)$	

Costs involved in the simple shear activity are fairly low, but involve much more repeated preparation time than the axial loading activity. A gross of craft sticks can be purchased for about \$10, and school glue bottles are generally under \$3 apiece. Depending on how many sets of specimens are needed, gluing may take an hour split over two sessions (one bond at a time for double-shear specimens). If average widths are pre-determined, no other adaptation is required to

implement this activity in a lecture setting. Distance learning students may have to prepare their own craft stick specimens and find a friend to help pull specimens to shear failure. A short movie of the activity could be posted as a backup method of incorporating this into a distance learning mode for those students who prefer to skip preparation of specimens.

### Torsional shear stress and design activity

Small plastic water bottles are used to consider torsional loading and the contrasting design needs of cap connections intended for quick failure and water bottles which require unscathed survival upon opening. Representative components are shown in figure 3. Activity objectives are to gain experience with calculations related to torsion and to improve understanding of design constraints for common consumer items. Three-person teams work together to measure geometry of bottle cap connections and water bottle, then calculate polar area moments of inertia, the cap connector's breaking torque, the maximum torsional shear stress at a given location on the bottle, and the actual design factor for the bottle based on a known material strength.<sup>9</sup>



Figure 3: Items for the torsional shear stress and design activity (plastic strip omitted)

Working from the highly accessible action of opening a water bottle, students measure the cap diameter and dimensions of the approximately rectangular connections. They also measure the bottle's outer diameter slightly below the neck (where it is purely circular in cross-section, if possible) and thickness from a pre-cut sample of another bottle. Calculations of polar area moment of inertia for the cap connections and the bottle are made. The given breaking strength for the cap connections are used with polar area moment of inertia in the standard torsional shear

stress equation to determine the approximate torque to open the bottle cap. (The thread angle is assumed to be small enough to neglect the axial loading present in the connections. This could be revisited when considering principal stresses, maximum shear stress, and related theories of failure). The approximate torque is subsequently incorporated into the calculation of torsional shear stress in the bottle. Finally, the bottle's design factor is found from the bottle material's tensile strength and its torsional shear stress. The equations used for the bottle activity are provided in Table 4.

Table 4: Torsion Equations

Polar area moment of inertia for connections (for the cap), $J = \sum_i nR_c^2 A_c$	Where: n = number of connections; $R_c$ = radius of the connections; $A_c$ = shear area of the connections
Polar area moment of inertia for hollow circles (for bottle cross-section), $J = \frac{\pi}{32} [D^4 - d^4]$	Where: D = outer diameter, d = inner diameter
Torque from torsional shear stress in the cap, $T = \frac{\tau J}{R_c}$	Where: $\tau$ = ultimate shear strength of the plastic = $S_{us}$ ; J is for connections, and $R_c$ is the connection radius
Torsional shear stress in the bottle, $\tau = \frac{Tc}{J}$	Where: T = torque from the cap; c = 0.5D; J is for hollow circles
Design factor, $N = \frac{S_{us}}{\tau}$	Where: $S_{us}$ is ultimate shear strength and $\tau$ is the torsional shear stress in the bottle

A number of benefits occurred through completion of the torsional stress activity. At the most basic level, students were reminded that inner and outer diameters differ by two wall thicknesses rather than one. The polar area moment of inertia calculations reinforced the link between this quantity and its calculus base through the cap portion. The design factor for the bottle turns out to be much larger than the typical recommended published values, leading students to consider what other constraints might be important to the bottle's design.<sup>7</sup>

Several implementation surprises arose with this activity. First, even though the water bottles appeared to be the same, there were variations in the size and number of cap connections. Second, students tended to clamp down when measuring the cap connections, permanently deforming them when the least expensive water bottles are used. For the initial implementation, it was not possible to reuse the caps, an unfortunate fact to discover when multiple consecutive laboratory sessions were underway. With more sturdy (and expensive) water bottles, this problem was alleviated. Implementation cost is primarily the water bottles: this could be negligible if the instructor plans ahead and saves bottles from a single manufacturer. Otherwise, cost should be on the order of five dollars, depending on class size. If this activity were done in a large lecture setting, having students bring in their own water bottles and simply pre-measuring

and checking material properties for several brands and types might be best. Otherwise, a student-assisted demonstration of the measuring and counting of connections, followed by small group calculations should work well. Similarly, for distance students, preparing a video of the demonstration as backup for those students who lack appropriate measuring instruments would be helpful. Most distance students should be able to do the complete activity at home alone or in a small team if appropriate instructions are provided, e.g., “use caution when using box cutter to cut water bottle”, and so on. An online chat to delve into design considerations could enhance the activity, as well.

A couple additional possible activities:

Two more strength of materials activities were considered that were determined to too closely duplicate existing traditional experiments in the course at Purdue University that may be appropriate elsewhere. The first of these is related to beam deflection and flexural stress, and would probably work best as a demonstration in a lecture setting (with corresponding video posted for distance students). One or more small plastic bars (or other material of known modulus of elasticity) are mounted as simply-supported and cantilever beams. Fixturing can be done in an incidental fashion (textbooks hold the beam in place at the edge of a table for the cantilever; narrow chair backs or other items provide support at two “points”) or via instructor construction. The length, width, and thickness of the beam or beams are measured, as well as the distance from the beam center or end to the surface below it. Known load can be added at the center of simple supports or the free end of a cantilever beam, as pictured in figure 4. The



Figure 4: Sample beam deflection activity



Figure 5: column buckling activity components

distance to the surface is measured again to determine the deflection of the beam. The beam's rectangular area moment of inertia and beam tables are then used to estimate the beam's modulus of elasticity, which can be compared to published values.<sup>7,8</sup> For a beam of rectangular cross-section, the corresponding equations are shown in Table 5. The activity could be repeated after rotating the beam 90° about its longitudinal axis to observe the effect of changing its geometric stiffness. Flexural normal stress calculations and linking to design constraints for stress and deflection are other ways to increase the substance of this activity. To implement, the most expensive element is the beam or beams, which can be constructed from the plastic sheeting available from many hardware stores at a cost of \$15-\$25 for a sheet with a large dimension of 24 inches, since necessary measuring devices are assumed to be accessible for engineering technology instructors. It is worthwhile to note that a simple beam will not provide the desired link to real consumer products at the level of previously discussed activities, so more instructor effort to focus on meaningful applications will make the beam deflection activity more accessible.

Column buckling is another strength of materials concept that is often much more understandable with simple visual aids. In lecture, a steel rule works well for demonstrating a column with pinned ends that has a weak direction and can be used to show the effects of adding lateral supports (with a little assistance from student volunteers). If the cross-sectional dimensions are measured in class and steel's modulus of elasticity is determined in advance, calculations of slenderness ratio and critical buckling load can be done by small student teams.

Table 5: Bending Activity Equations

Rectangular area moment of inertia (for a rectangular cross-section), $I = \frac{bh^3}{12}$	Where: b = the dimension parallel to the bending axis; h = the dimension perpendicular to the bending axis
Modulus of elasticity from the cantilever beam deflection formula for force at the free end, $E = \frac{48yI}{FL^3}$	Where: y = maximum beam deflection; I = inertia; F = force at the free end; L = length of the beam from support to force
Modulus of elasticity from the simply supported beam deflection formula for a single force at its center, $E = \frac{3yI}{FL^3}$	Where: y = maximum beam deflection; I = inertia; F = force at the center of the supports; L = length of the beam from support to support

Other end conditions require a bit more preparation, but should clarify the need for end-fixity factors. To produce a fixed base end under vertical loading (for pinned-fixed ends), the instructor can either purchase modeling clay for a lump on the base surface (at a cost of about \$4) or cut a slit of appropriate width into a wood block (e.g., piece of 2x4, hopefully from scrap so cost is negligible). Two such blocks could be used to show buckling with two fixed ends. Application of the buckling load can be controlled if clear, relatively large tubing is first placed around the column (cost of about \$10). For a small group activity, if the engineering technology laboratory does not have a sufficient number of steel rules, equivalent activities can be built around drinking straws and/or chenille strips, both of which can be purchased for a class of 30+ students at a cost of two or three dollars. Mass of the items added to apply a compressive buckling load can be predetermined or measured during the activity. While engineering technology students are certainly aware of steel rules, straws and chenille strips offer better linkage to consumer products for those who have not been involved in constructive hobbies, and are shown in figure 5. The lengths of straws, chenille strips, or other columns should be selected to ensure there is a good match between column formula and slenderness ratio. The corresponding equations for long columns are shown in Table 6.

Table 6: Column Buckling Activity Equations (for long columns)

Radius of gyration, $R_{sm} = \sqrt{\frac{I_{sm}}{A}}$	Where: $I_{sm}$ = smaller rectangular area moment of inertia; A = area
Slenderness ratio, $SR = \frac{kL}{R_{sm}}$	Where: k = end fixity factor; L = column length; $R_{sm}$ = smaller radius of gyration
Critical buckling load, $P_{cr} = \frac{\pi^2 EA}{(SR)^2}$	Where: E = modulus of elasticity; A = cross-sectional area; SR = slenderness ratio

### Assessment of implemented activities

Assessment of the first offering of the axial, simple shear, and torsional shear activities was done at a very basic level. Using a 5-point Likert scale, students were surveyed to obtain their perception of the activities in terms of being very helpful, helpful, neutral, not helpful, or harmful for specific purposes. Table 7 shows the relevant section of the survey instrument. None of the activities were identified as harmful, and most students indicated they found the activities either helpful or very helpful, as illustrated in Figure 6. Based on student responses and instructor observations, the activities will continue. Assessment of related course learning outcomes is ongoing and appears to support incorporation of the activities. Due to a number of additional course modifications to improve student success, however, learning improvements cannot be tied directly to these laboratory activities. Examination results indicate that student understanding of direct shear, especially distinguishing single and double shear, improved approximately 15% over two previous course offerings. Student comfort level when using micrometers, awareness of the effects of manufacturing variations, and understanding of design concerns appeared to benefit from implementing these activities, as did the group interactions and classroom climate.

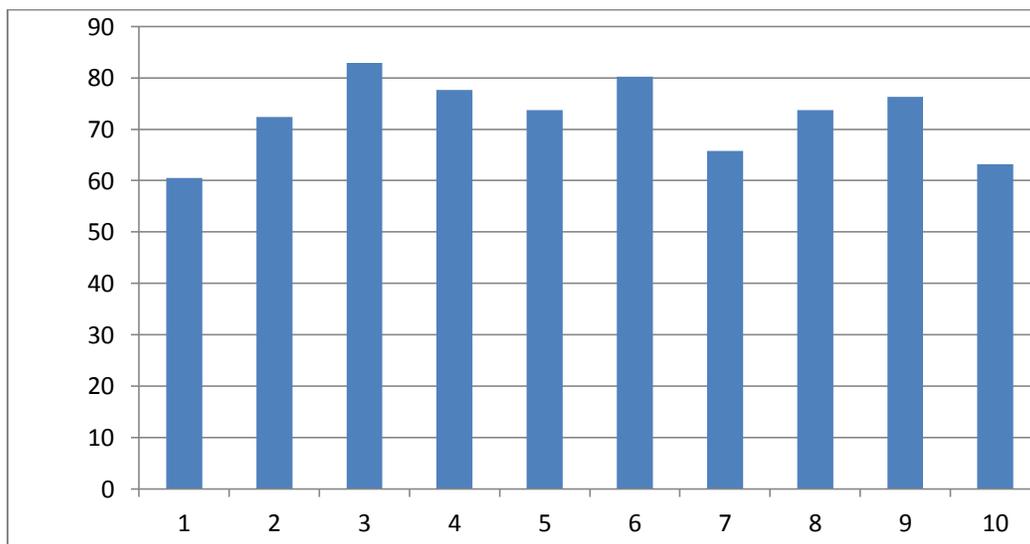


Figure 6: Percentage of students who found the activities to be helpful or very helpful in response to survey items 1 through 10.

Table 7: Listing of survey items 1 through 10 for student survey assessing perception of aspects of low-cost, hands-on laboratory activities

The hands-on lab activity was (student response blank) for (numbered item)	
1	reviewing mass to weight calculations
2	reviewing micrometer and dial caliper use
3	calculating axial (direct normal) stress
4	relating geometry and axial deformation to modulus of elasticity
5	recognizing single shear and double shear
6	calculating shear area for direct shear stress
7	calculating polar area moment of inertia
8	using ultimate shear strength to find loading for failure
9	calculating torsional shear stress
10	determining the actual design factor

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