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Abstract

Over the years more and more students are taking calculus in high school. This has caused a problem at colleges and universities because every high school calculus course is different and some students have AP calculus while others don't.

In this paper we will discuss a summer class we have taught since 1982 called Fast Track Calculus. The class was specifically designed for students who have had a year of high school calculus, but have either not had an AP calculus course and thus can not receive college credit; or have taken an AP calculus course, but want to move on more quickly.

The class lasts five weeks and covers all of the material we cover in our calculus sequence. In the paper we will discuss the requirements for admittance into the program, the syllabus, the advantages to the students who take the course, and the outcomes of the students who have taken the course.

Introduction

Rose-Hulman Institute of Technology is a small engineering school. All of our students major in engineering, mathematics, or science. Over half of our incoming freshman class has had a high school calculus course and between eighty-five and ninety percent of our freshman class were in the top twenty percent of their high school graduating class. Like a lot of schools who have students who have taken a calculus course in high school, Rose struggled with the problem of how to best meet the needs of these students.

Fast Track Calculus was started in the summer of 1982, as a way to help students who had taken a year of high school calculus take advantage of their knowledge. The idea was to offer a course during the summer that covered our freshman calculus sequence. Thus, students could begin their freshman year by taking our sophomore level differential equations sequence. By finishing their mathematics requirements a year early, it was our hope to provide these talented students a better opportunity to pursue second majors, or minors, earn technical certificates, or possibly allow them to graduate early.

It was felt that in order to ensure the students had an excellent chance of passing the Fast Track Calculus (FTC) course, only students with an SAT mathematics score of 700 or more would be allowed to enter the program. The students would also need a letter of recommendation from their high school calculus teacher. Students would also have to write a short essay on why they wanted to be admitted to the program.

The most important criteria for selection into the program is the letter of recommendation from the student's calculus teacher. The profile we are looking for is a mathematically talented student who is a hard worker, who is willing to ask questions, and is willing to work with others.

Over the years the standards have remained basically the same. In 1995, the standards for SAT scores were modified. The students must now have either a 700 or more on the mathematics portion of the SAT, (equivalently a 31 on the mathematics portion of the ACT), or a 680 score of the mathematics portion of the SAT and a 700 or better on the verbal part of the SAT. Equivalently, student could have a 30 on the mathematics portion of the ACT with a 31 or better on the verbal part of the ACT. This modification was made because it was felt that students who were strong in mathematics, and who have excellent verbal skills could also be successful in the course.

Two major changes in the course have occurred. One was that in 1989 we began to use a computer algebra system (CAS) in the teaching of our calculus and differential equations. The second occurred in 1995 when we became a co-ed institution and began to require students to have a laptop computer. While the overall content of the course did not change, with either of these changes, the order and way we presented the material did change.

Prior to 1995 the class size was about 20-25 students. From 1995 on we have had two sections of the course with each section having between 20-25 students.

To insure uniformity, both sections are run in parallel, covering the same material and giving the same homework assignment each night. Students who are roommates are placed in different sections of the course so that there is a good cross mix of ideas between the students. This also helps students in that different instructors may present the same material in different ways. Thus, between roommates both ways of presenting the material is covered.

It should be noted that we assign students their roommates for the five weeks they are in the program. We try to pair up students who are from different parts of the country. We also try to put students with different academic backgrounds in the same room. Thus, a student who has taken an AP calculus course usually rooms with a student who has not taken an AP calculus course. A student with an 800 SAT score may room with a student with a 700 SAT score.

Syllabus

Because the students have already had a year of calculus, we believe it is important to begin the course at a rather rapid pace. The idea is to help the students realize that they are no longer in high school and thus, must learn to work more efficiently. We know that the students come from different backgrounds. Some have had an AP calculus course some have not. Thus, our objective is to review all the concepts of calculus that would be taught in our regular calculus sequence, but to change the order to make learning more efficient. This also tends to level the playing field, since early on in the course we talk about some topics from our Calculus III course. Thus, everybody has to learn something new. This means that students from an AP calculus course don't have the luxury of just reviewing material they already know.

The general overview of the course is that we first introduce students to functions in two space. We then introduce the students to functions in three space. Once the students have been introduced to functions in two and three space, we talk about the concept of the derivative, both in two and three dimensions. We then quickly review sequences and series, and then cover the concept of integration in both two and three space.

We spend two and a half weeks on functions and differentiation and two and a half weeks on integration. Thus, there are times during the course when the students are reviewing what they were supposed to have learned in high school, and times when they are learning new material that is related to what they already know.

Below is the day by day syllabus for the course:

Day One:	Functions, implicit functions, and parametric equations:
Day Two:	Inverse functions including inverse trig functions, logarithms and exponentials, introduction to vectors.
Day Three:	Dot products and cross products of vectors, and planes in space.
Day Four:	Lines in space, quadric surfaces, average rate of change, and the derivative.
Day Five:	Chain rule, differentiation of parametric equations and implicit functions; motion, and partial derivatives.
Day Six:	Chain Rule for multivariable functions, formal max/min theory, curvature, and curves in space.
Day Seven:	Unit tangent and unit normal vectors.
Day Eight:	Normal and tangential components of acceleration.
Day Nine:	Gradient, directional derivative, Taylor polynomials, Newton's Method, L' Hopital's Rule.
Day Ten:	Taylor's Series in three space, max/min in three space, and Lagrange multipliers.

Day Eleven:	Continue Lagrange multipliers, least squares.
Day Twelve:	Sequences and Series. Anti-differentiation, area, and Fundamental Theorem of Calculus.
Day Thirteen:	Techniques of Integration , numerical integration.
Day Fourteen:	Volumes of revolution, surface area of revolution, arc length.
Day Fifteen:	Exponential growth and decay, the logistics equation.
Day Sixteen:	Separable differential equations and modeling.
Day Seventeen:	Projectile motion, linear differential equations.
Day Eighteen:	Line integrals, work, and flux.
Day Nineteen:	Double integrals.
Day Twenty:	Centers of mass, moments of inertia.
Day Twenty-one:	Triple integrals.
Day Twenty-two:	Moments in space and change of coordinate systems.
Day Twenty-three:	Area of polar curves cylindrical coordinates.
Day Twenty-four:	Spherical coordinates.

Day Twenty-five: Final exam.

Weekly Schedule

In order to cover all the material each day is tightly organized. The students are in class from 8:05 AM until 12:00 noon and from 1:30 PM until 5:00 PM each day. The schedule for Monday through Thursday is:

8:05AM -9:15AM:	Lecture and discussion of new material.
9:15 AM-10:00AM:	Students work on the daily homework assignment. Instructors are in the classroom to answer any questions the students may have.
10:00 AM-11:00AM:	Lecture and discussion of new material.
11:00AM-12:00 PM:	Students work on the daily homework assignment. Instructors are in the classroom to answer any questions the students may have.
12:00 PM-1:30 PM:	Lunch and study time.
1:30 PM – 3:00 PM:	Problem session. A student led discussion of the previous day's homework assignment.
3:00 PM-4:00 PM:	Lecture and discussion of new material.
4:00 PM -5:00 PM:	Students work on the daily homework assignment. Instructors are in the classroom to answer any questions the students may have.
5:00 PM -	Dinner and students study and finish their daily homework assignment.

On Friday, after the problem session, the students take a weekly test.

To help the students with their homework during the evening hours we hire about seven student counselors. The counselors are all students who have taken the Fast Track Calculus course and did very well in the course. Thus, they know how to do the problems and can show the students where they are going wrong, or at least point them in the correct direction. Thus, the students can get help twenty-four hours a day, seven days a week.

We have found that at the beginning of the course, the above schedule is followed. However, as the course progresses and the students get used to the pace, we usually are

able to cover all the new material for the day during the morning. This leaves the entire afternoon for the students to work on their homework.

Homework and Problem Session

The students are given a rigorous assignment everyday consisting of between 20 and 40 problems. Because of the nature and speed of the course, and problems, students are encouraged to work together on the homework assignments, both during class time and in the evening.

Each day the previous day's assignment is turned in by 8:05 AM, the start of class. The counselors grade the assignment during the morning, and return the assignments to the students at 1:30 PM each afternoon, at the problem session. The assignments are graded as E for excellent above 90% correct; S for satisfactory, above 80% correct; and U for unsatisfactory for below 80% correct. Any student who receives a U on an assignment must redo all the problems the student missed and resubmit the assignment. The assignment is not considered passed until all missed problems are done correctly. All U's must be made up, prior to the final exam. Thus, a student may have several outstanding U's on any given day. To try to insure that students do pile up an insurmountable number of U's, each weekend there are mandatory sessions for students who have "too many" outstanding U's. During the first weekend this may include students who have three or four U's, while on the last weekend it may be any student who has one or more outstanding U's

During the problem session, students may ask to see any problem on the previous homework assignment worked out. Students who have the problem correctly done volunteer to put the requested problems on the board. This gives every student the opportunity to speak before a group and defend their solutions. It is also intended to help students who received a "U" on the homework assignment to see the problems they missed done correctly.

Special Projects

In addition to their regular homework students are given four group projects during the course. These projects are to be done by groups of from three to five students without any aid from the counselors, the faculty, or members outside their group. The group projects are graded on the same scale as the homework problems. If a group gets a U on the project, they must fix their mistakes and resubmit the project.

Testing

During the course students are given a test each Friday, after the problem session. One of the faculty members grades all of the exams. The graded exams are returned to the

students during Monday's problem session. A student passes the exam if the student receives a score of 75% or better and has at least one problem done completely correctly. If a student does not pass the exam, the student must take a make-up exam over the same mathematical concepts. The criteria for passing a make-up exam are exactly the same as those for passing the original exam. Make-up exams are usually given during Tuesday evening. Thus, if a student needs to take a make-up exam, the student is unable to use that time to do their homework problems. If a student does not pass the make-up exam, the student must take a second make-up exam. This process continues, until the student passes an exam on the required mathematical content.

There is also a final exam which the students must pass. The final mainly covers the material in our Calculus III course as this is the really new content for students. For the final exam a passing grade is seventy percent, as the students don't have the opportunity to take a make-up exam.

Grading

The course is graded on a "credit", "no credit" basis. To receive credit for the course the student must successfully complete all homework assignments (all U's must be completed successfully); the student must pass all exams; successfully complete all special projects, pass two proficiency exams (one on differentiation, and one on integration); and the student must present at least three problems at the board during the twenty-four problem sessions.

A student who successfully completes the program receives 15 hours of credit (for Calculus I, II, and III) and is able to take our sophomore level differential equations course during their first term.

Success Rate

Since 1982, when the program began, only two students have "completed" the course and not received their 15 hours of credit. Neither of those students completed their freshman year and left school.

There have been several students (fewer than 10 since 1982) who have decided that the program was too difficult and have left the program. This usually happens during the first week. Many of these students returned to school in the fall, were successful, and have graduated.

Reasons for taking the course

We advertise the course for highly motivated students who want to get ahead in their studies. We stress that taking the course will allow the students greater options during their stay in school. Some students get double majors or multiple degrees, while others go on internships and still graduate in four years. Some students study abroad for a term or year, or get technical certificates, or area minors. In addition since the students are

ahead mathematically, they are more apt to be accepted in the research experiences for undergraduates (REU) programs after their sophomore year. Some students decide to use their credit to graduate early.

Assessment of the program

This paper was originally undertaken to see if our claims were actually true. Thus, we looked at the students who started college in 2000, 2001, and 2002. All of these students have had four years in which to graduate. In this group of students, 137 students took Fast Track Calculus and there were 1156 students who did not take Fast Track Calculus. We will be looking at several different areas and will use the z-statistic to compare proportions. In order to be more statistically correct, we will use the Bonferrni method with alpha equal 0.05 in a two-tailed test. This means that for each area we are testing, six in all, we will be using an alpha value of .05/7 = .0071 in our two tailed test. This translates into a critical z-value of 2.45. Thus, if our calculated z-value is greater than 2.45, or less than -2.45, the difference between the proportions is significant at the Bonferrni family level of 0.05.

What follows are the preliminary results of our assessment.

SAT Scores: The student who took Fast Track Calculus had an average SAT math score of 723. The non Fast Track students had an average SAT math score of 673. The Average verbal SAT score for the Fast Track students was 650. The average verbal SAT score for non Fast Track students was 614.

As the reader can see, the Fast Track students are selected to be our better students. Thus, in a sense the comparisons that follow, while true, are what one might expect when comparing some of your best students to the general population.

It should be noted that well over 50% of our entering students have had a calculus course in high school. Also our beginning mathematics course for freshman students is calculus. Also recall that between 85% and 90% of all our freshman students are in the top 20% of their high school graduating classes. Thus, we expect that a high percentage of all our students will graduate.

Graduation Rate: The number of Fast Track students who graduated was 121 or 88.3%, while 879 or 76% of the non Fast Track students graduated. This gave us a z-value of 3.247. Thus, the proportion of Fast Track students who graduate is significantly higher than for our general population.

Some parents, teachers, and some of our colleagues are concerned that because of the pace of the program, students won't actually retain the material they have been taught. We believe this is a valid concern. While we don't have the data at the present time, we do know that all of our students graduate as engineering, mathematics, or science majors. Since Fast Track students graduate at a higher rate than our other students, this leads us to

believe that the Fast Track students retain enough of the mathematics they learn to graduate.

Grade Point Average: The average grade point average for a Fast Track student was 3.35. The average grade point of a non Fast Track student was 3.02. This gives us a z-value of 6.43 which is statistically significant. Thus, Fast Track students do significantly better in their courses than non Fast Track students. This again leads us to believe that Fast Track student retain the mathematics they learned. We are now beginning the process of testing for differences in grades in specific courses.

Two or more Degrees: The number of Fast Track students who graduated with two degrees was 5 or 3.6%, while 6 or 0.52% of the non Fast Track students graduated with two degrees. This gave us a z-value of 3.77. Thus, the proportion of Fast Track students who graduate with two or more degrees is significantly higher that the proportion for non Fast Track students.

Double Majors: The number of Fast Track students who had double majors was 24 or 16.8%, while 53 or 4.6% of the non Fast Track students had double majors. This gave us a z-value of 6.048. Thus, the proportion of Fast Track students who got double majors is significantly higher than the proportion for non Fast Track student.

Minors: The number of Fast Track students who graduated with at least one minor was 67 or 48.9%, while 334 or 28.9% of the non Fast Track students had at least one minor. This gave us a z-value of 4.788. Again the proportion for Fast Track students who graduate with at least one minor is significantly higher than for our general population.

Technical Certificates: The number of Fast Track student who graduated with technical certificates was12 or 8.8%, while 58 or 5.0% of the non Fast Track students graduated with technical certificates. This gave us a z-value of 1.83 which meant that there was no statistical difference between the proportions for Fast Track students and the non Fast Track students who graduate with certificates.

Left due to Poor Grades: Of the 16 Fast Track students who did not graduate, 4 are still in school and 12 have left school. Of the 12 students who have left school, 5 were dismissed for poor academic performance. Thus, the 41.2% of the Fast Track Students who left school were dismissed for poor academic performance. For the non Fast Track students 233 have left school. Of these 233 students 16, or 26.2%, were dismissed because of poor academic performance. This gives a z-value of 2.43. This is not statistically significant.

One should note that because the numbers are so small, these statistics may be a bit dubious. However, it shows that just because a student is successful in Fast Track Calculus, it does not guarantee that the student will be successful in school. In fact, of the 12 Fast Track students who left school, only 4 were not on academic probation. Thus, we conclude that the major reason Fast Track students leave school is because of poor academic performance.

Each year our school administers a Freshman Poll to assess the freshman courses. In this poll the freshman students are asked to rate each of their courses on a four point scale. The ratings used are: 0 meaning the course was a "Big Waste of Time"; 1 meaning the course was "Not Worthwhile"; 2 meaning the course was "Not Good, Not Bad"; 3 meaning the course was "Worthwhile"; and 4 meaning the course was "Very Worthwhile". Fast Track Calculus is consistently the highest rated freshman program or course offered. The ratings for Fast Track Calculus for the past seven years are listed in the table below.

Year	2000	2001	2002	2003	2004	2005	2006
Rating	3.61	3.65	3.90	3.76	3.78	3.80	3.85

It should be noted that all freshman are allowed to take the poll. The question about the Fast Track program does ask students not to answer the question if they did not participate in the program. Thus, students who left the Fast Track program may also answer the questions about the Fast Track program. We have no way of knowing if these students do in fact answer the question.

Another aspect we believe speaks well for the program is that at least half a dozen families have sent two of their children through the Fast Track Calculus program. In addition, two families have had three of their children participate in the program.

Conclusion and aspects for further study

Our Fast Track Calculus program is designed to help students with high mathematics ability and high motivation more easily attain their academic goals.

The early data shows that students who take Fast Track Calculus are more likely to graduate, obtain multiple degrees or multiple majors, and have minors than are the students who do not take Fast Track Calculus. Also if a Fast Track student leaves school it is more likely that it is because of poor grades. These results are what we expected to find.

We are now in the process of further analyzing the data to see if there is a difference between students who took Fast Track Calculus and the students who did not take Fast Track but have equivalent SAT scores. At the present time we believe that Fast Track Calculus does no harm to the students' academic progress. However, there is need for further study.

Fast Track Calculus is definitely not a program for your average student. However for students with a strong academic background and strong mathematics skills, we believe that the program allows these students more options in their academic career.

Because of the success we have had with the Fast Track Calculus program, we believe that it may be a program that other engineering schools may wish to try. Thus, the

engineering schools could help their entering students, who have taken a high school calculus course, more easily obtain their academic goals.