Study of Hydraulic Losses in Gravity-Driven Pipe Flow: An Exercise Combining Theory and Experiment for Engineering Technology Students

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Abstract

An exercise is described that combines theoretical predictions and experimental measurements of gravity-driven pipe flow. This exercise is used to reinforce lecture material on viscous incompressible flows in a junior level thermal and fluids engineering course in a baccalaureate engineering technology program. The configuration studied consists of the flow of water through a length of small diameter tubing supplied by an elevated reservoir. The theoretical component of the exercise involves modeling the system using the incompressible energy equation combined with analytical models for viscous losses. The analytical model results in an equation that must be solved iteratively. Computing an iterative solution in a spread sheet gives the students valuable experience with modeling techniques commonly used in solving problems they will encounter in industry. The experimental component provides ample opportunities for students to gain experience with pre-test uncertainty estimation, formulating and executing a test plan, and with post-test statistical analysis of the measured data. The experimental apparatus is made up of inexpensive items found in home improvement stores. The apparatus is small in physical size and requires only a limited amount of water, meaning it can be used in minimally equipped instructional spaces. On account of the simple apparatus, parameters such as tubing length and the elevation of the water reservoir are easily varied. This allows students to carry out such valuable exercises as calibrating their analytical models to experimental results on a baseline configuration, and then investigating how well the calibrated model can predict the flow when the geometry is modified. The paper includes a description that will allow others to easily reproduce the apparatus, and also reflections on the utility of the exercise as an educational tool.

Introduction

Developing an ability to use a combination of analytical and experimental tools to solve technical problems is an important part of the education of engineering technology students. Often in design and analysis work encountered in industry, an approach employing a combination of analytical and experimental techniques is the best approach to achieving a practical solution. In many cases, a strictly theoretical or computational approach is not adequate due to the complexity of the physical phenomena involved (such as complex flow phenomena) or geometric complexity. Likewise, it may not be possible or economical to take a strictly experimental approach to a problem. In such cases, the required approach is to develop an analytical model, calibrate the model to the available experimental data, and then use the calibrated model to predict behavior in response to changes in geometry or operating conditions. In the course of developing this ability, students need to gain experience both in analytical modeling, and in making experimental measurements. In industrial scenarios in particular, it is often important that experimental measurements be planned and executed in manner that causes minimal disruption of operations in addition to being carried out with sufficient accuracy for the results to address the interests of the business.

Development of these skills in the course of baccalaureate engineering technology programs addresses three of the outcomes from the *Criterion 3. Student Outcomes* section of the 2019-20 ABET documentation for accrediting engineering technology programs [1]. These Outcomes are:

(1) an ability to apply knowledge, techniques, skills and modern tools of mathematics, science, engineering, and technology to solve broadly-defined engineering problems appropriate to the discipline;

(4) an ability to conduct standard tests, measurements, and experiments and to analyze and interpret the results to improve processes; and

(5) an ability to function effectively as a member as well as a leader on technical teams.

This paper describes an exercise that combines theoretical calculation and experimental measurements of flow rates in gravity-driven pipe flow. The exercise is currently used in a junior level course in thermal and fluids engineering course in a mechanical engineering technology program. This activity is aligned with the part of the course devoted to viscous flow and the incompressible energy equation. The configuration studied essentially simulates flow from a water tower in miniature. It involves the gravity-driven flow of water through a length of tubing in which a constant hydrostatic head differential is maintained between the entrance and exit of the tubing by using a feed reservoir in which the water is maintained at a constant level. The configuration studied is simple enough to be analytically tractable with material covered in the course. Typically, the flow in the pipe falls in the turbulent regime with hydraulically smooth walls. The theoretical component involves modeling the system using the incompressible flow energy equation along with analytical calculation of the head losses. Incorporating such loss components as Reynolds number dependent skin friction, results in an equation for the flow rate that needs to be solved iteratively. The iterative solution is easily carried out in a spread sheet. Computing the solution in a spread sheet gives students valuable experience in performing the kind of flow modeling that is often useful in industry.

The experimental component provides ample opportunities for students to gain experience with pre-test uncertainty estimation, formulating and executing a test plan, and with post-test statistical analysis of experimentally derived flow parameters. As an example, flow rate is determined by measuring the time it takes to collect a measured volume of water discharged from the tubing. Each of the two variables, volume and time, are prone to sources of experimental uncertainty that are easily visualized by the students. Such tangible examples go a long way in helping students to understand techniques like the Kline-McClintock [2] method for uncertainty estimation.

The experimental apparatus is made up of inexpensive items found in home improvement stores. These include such items as small diameter polyethylene tubing used in home ice makers, tube fittings, a plastic bucket for the feed reservoir, measuring vessels, and stopwatches. Typical head differentials involved are about 1 meter, and with the tubing sizes used, the flow rates are usually 1 liter/min or less. The quantities of water used are small, meaning that the testing can be carried out in almost any available instructional space. On account of the simplicity of the apparatus and test procedure, it is practical to vary parameters like reservoir height and tubing length. Measurements can be repeated quickly, facilitating the gathering of enough data to carry out a meaningful post-test statistical analysis. Comparison of the theoretical and experimental results helps students gain insight into the advantages and limitations of both approaches.

Analytical Treatment

The basis for analyzing the flow of water through the system is the one-dimensional incompressible energy equation without addition or extraction of work [3]:

(1)
$$\frac{P_1}{\rho g} + \frac{\mathcal{V}_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{\mathcal{V}_2^2}{2g} + z_2 + h_L$$

Here, P denotes pressure, $\mathcal{V}$ the fluid velocity, and z the height coordinate. The symbol h_L represents the head loss in the system, ρ the fluid density, and g the gravitational constant. The variable subscripts 1 and 2 denote for this problem the surface of the fluid reservoir and the pipe discharge location respectively. See Figure 1.

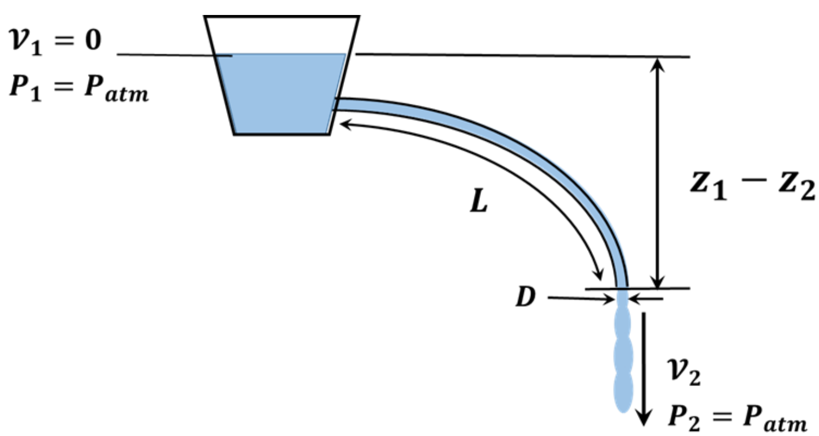

Figure 1 Schematic of water flow setup showing nomenclature used.

For this configuration, the pressure at the top of the reservoir and at the discharge end of the tube is equal to the atmospheric pressure:

(2) $$P_2 = P_1 = P_{atm}$$

The velocity at the surface of the reservoir is negligible on account of the area being much larger than that of the tubing:

(3) $$\mathcal{V}_1 = 0$$

The velocity at the pipe discharge station is equal to the velocity in the pipe, which we can write as simply $\mathcal{V}$:

(4) $$\mathcal{V}_2 = \mathcal{V}$$

With these simplifications, the energy equation reduces to:

(5) $$z_1 - z_2 = \frac{\mathcal{V}^2}{2g} + h_L$$

This form of the equation has the simple interpretation that the change in potential energy of a fluid particle is equal to the sum of the kinetic energy that it acquires plus the energy dissipated by head losses.

The head loss has two components, skin friction on the pipe walls and a so-called minor loss associated with the entry to the pipe. The loss associated with the flow entering the pipe can be expressed as the product of the dynamic pressure of the flow in the pipe and an empirical loss coefficient, K_{ent}:

(6) $$h_{L,ent} = \frac{1}{2g}\mathcal{V}_{pipe}^2 K_{ent}$$

The skin friction head loss is calculated using the Darcy-Weisbach equation:

(7) $$h_{L,fric} = \frac{1}{2g}\mathcal{V}_{pipe}^2 \left(\frac{L}{D}\right) f$$

If flow conditions are such that the flow is expected to be laminar, the Darcy friction factor can be calculated using:

(8) $$f = \frac{64}{Re}$$

For turbulent flow, in a pipe with a hydraulically smooth wall, the skin friction factor can be calculated using the Petukhov formula [3]:

(9) $$f = (0.79 \ln Re - 1.64)^{-2}$$

Alternatively, skin friction formulas such as the Prandtl power law formula [3] can be used. For walls that aren't hydraulically smooth, the Colebrook-White formula [3] or the Haaland formula [4] can be used to calculate f.

Calculations are easily carried out in a spread sheet, or with a simple computer program. One way to proceed is to set the calculation up so that it iterates on the velocity $\mathcal{V}$ required to give values on the right side of the equation that are equal to the height difference, $z_1 - z_2$.

Experimental Apparatus

The apparatus and measuring equipment required for this lab experiment is both simple and inexpensive. The apparatus consists of a length of small diameter smooth-walled tubing connected to an elevated water reservoir as depicted in Figure 2. In the test setup shown in the Figure, the tubing is ¼" diameter stiff polyethylene plastic of the type used for supplying water to ice making machines. The tubing selected has an inside diameter 0.170 inches (4.32 mm). For the configuration shown, a head difference between the tubing discharge and the surface of the reservoir of about 0.8 to 0.9 meters can be achieved. Provided the tubing lengths are not excessive, this head difference is sufficient to produce turbulent flow in the tubing. The stiffness of the tubing used ensures large radius curves, meaning that there are no significant losses associated with bends in the flow path.

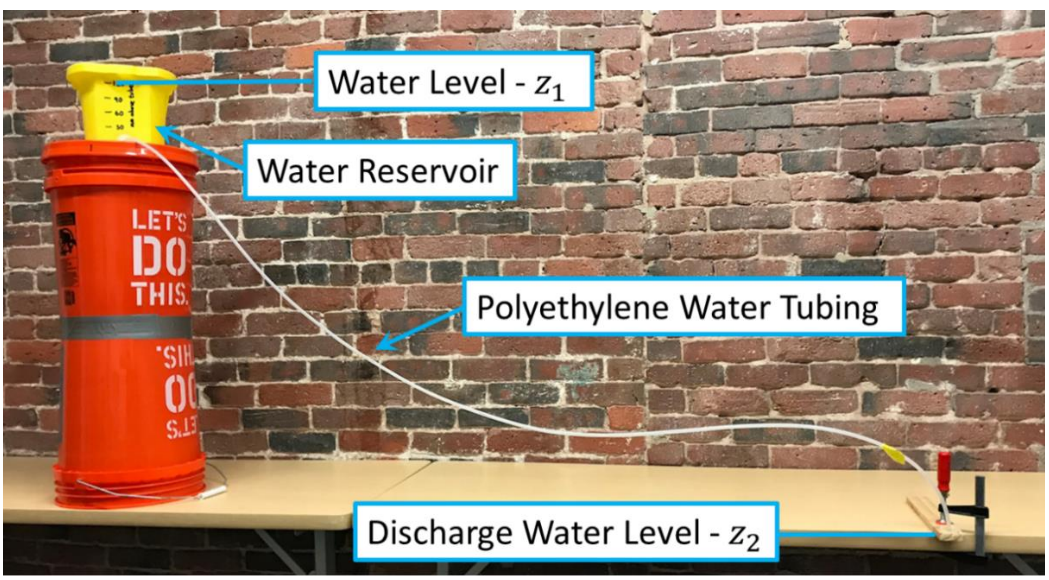

Figure 2. Experimental apparatus.

The feed reservoir was made from a plastic bucket having a flat side as shown in Figure 3. Having a flat surface facilitated the connection of the tubing to the reservoir. The connection was made by modifying a push-to-connect fitting made for use with polyethylene water pipe. A fitting with one end with a male pipe thread was chosen to for the connection. The threaded end passes through a hole drilled in the reservoir. A female threaded end from a similar tube fitting was cut off and made to serve as a nut. As shown in the second picture in Figure 3, a rubber washer was used to help seal the connection. A plastic washer is placed between the nut and the rubber washer to provide an even distribution of the compression force. Application of some silicone seal in this area can also be helpful. In use this connection, although somewhat makeshift, has been found to provide a good seal.

Figure 3. Water reservoir showing push-to-attach water tubing fitting. Picture at right shows interior of reservoir with washers used for sealing.

The discharge end of the tubing needs to be fixed at a constant height. Figure 4 shows a simple fixture made from a block of wood clamped to a table. The tubing end passes through a hole in the block that has been drilled to a size to provide a friction fit.

Figure 4 Clamp and block arrangement for clamping discharge end of tube.

The flow parameter to be measured in the experiment is the volume flow rate of the water. This is obtained by measuring the time it takes to capture a volume of water. The equipment required is a timing device such as a stop watch or clock with sweep second hand, and a means of measuring the amount of water collected such as a graduated cylinder.

Test Procedure

In carrying out the testing, it is important to maintain a constant rate of flow of water. A constant flow rate is established by maintaining a constant height difference, $z_1 - z_2$, between the water level in the reservoir and the discharge end of the tubing. With discharge end of the tubing clamped at a fixed height, constant head difference can be maintained by having one member of the experimental team replenish the water in the feed reservoir to keep the water level constant. At the start of a measurement, the reservoir can be filled while one person caps the discharge end of the tubing with a finger. Once the reservoir is at the desired level, the water can be allowed to flow. After a short time has been allowed to for the flow to establish itself, a vessel can be moved into place and the time required to collect a sample measured. The amount of water to be collected to obtain a target level of uncertainty should be established beforehand using a pre-test uncertainty analysis as discussed below.

Measuring flow rate by means of timed fill of a vessel is an example of a positive displacement technique [5]. When performed properly this technique has high accuracy. The technique is also considered a standard means to calibrate other types of industrial flow meters that provide continuous measurements of fluid flow rates [5]. The accuracy of measurements obtained by the timed fill method can be increased by taking measurements over longer time intervals (i.e. by taking larger volume samples) or by using more precise devices to measure volume and time. The factors that affect the uncertainty in the measurement are easily visualized by the students.

Running the experimental tests requires a team of least two people, but can easily accommodate a team of three or four. One person can have the primary task of maintaining the water level of the reservoir, and two more can be usefully occupied collecting the water discharged from the tubing and measuring the collection time. If there are additional team members they can perform such tasks as recording data, coordinating the starting and stopping of the timed volume collection, and watching for any anomalies. A sample lab procedure sheet is included as an Appendix.

In the author's classes, it is usual to perform tests for configurations that produce at least two different flow rates. Changing the length of the tubing or the height difference between the water surface in the reservoir and discharge end of the tubing can be used to vary the flow rate. The students are instructed to perform multiple flow rate measurements at each flow rate in order to be able to perform post-test statistical analysis.

Experimental Uncertainty Analysis

The flow rate measurements required for this exercise can be the basis for both pre-test and post-test analysis of measurement uncertainties using well-known statistical techniques [2], [5], [6]. Application of these methods to timed-fill flow rate measurements, using a level of mathematical preparation typical of that possessed by engineering technology students, has been described in a previous paper by this author [7], and is discussed only briefly here.

During the pre-test phase of the lab, the students make uncertainty estimates for the two measured variables that are used to determine the flow rate. These are the sample volume collected, and the time it takes to collect a sample. Students can estimate how large a sample needs to be collected in order to obtain the flow rate with a desired level of uncertainty. They can be asked to reflect on performing similar tests in industry situations where operational constraints may motivate performing the measurements as quickly as feasible, while keeping measurement uncertainties within desired limits.

Since the measurements needed can be made quickly, students can repeat the flow measurements several times. This allows a large enough data set to be obtained so that meaningful averages and standard deviations can be calculated. Chauvenet's Criterion [5] can be used to identify and reject outliers. The post-test calculations of scatter in the flow measurement can be compared with the pre-test uncertainty estimates to see whether the pre-test estimates were reasonable, or whether the estimates of how well time and volume can be measured need to be revisited. Longer or shorter test times can be tried and the effect on the post-test statistical quantities observed.

Sample Results: Baseline Configuration

Figure 5 shows results of a typical analytical calculation of the variation of the flow rate with length of water tubing. Turbulent flow was assumed. In the example shown, the height difference between the top of the reservoir and discharge is, $z_1 - z_2 = 0.89\ m$. Also shown is an experimental measurement made for the same height difference and a tubing length of 2.0 meters. Comparison with the calculation shows good agreement, with the measured flow rate about 6% less than predicted. This level of agreement would not be untypical when the experiment is carried out with reasonable care. At this operating point, the Reynolds number is found to be consistent with an assumption of turbulent flow.

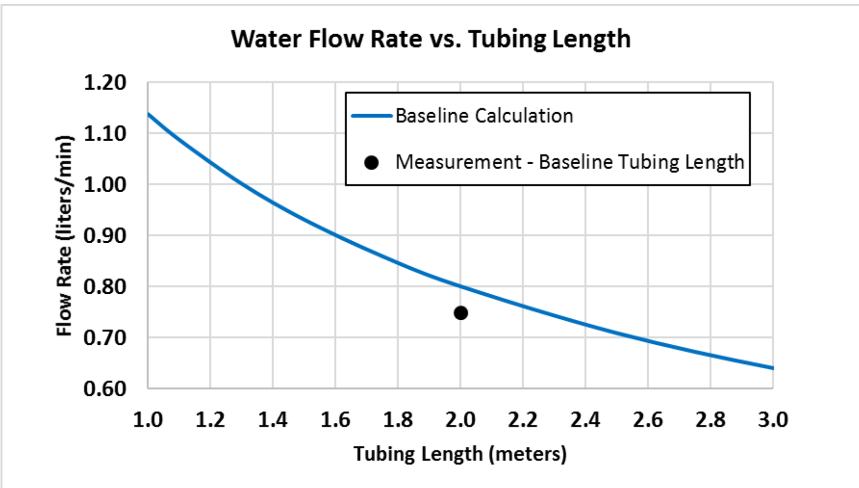

Figure 5 Calculated variation of flow rates with tubing length and comparison with experimental measurement for 2 meter long tubing.

It should be noted that trials with much longer lengths of tubing (about 7.5 meters) produced flow rates much lower than predicted, even when a laminar friction factor (consistent with low Reynolds) numbers was used. It was observed in these instances that air bubbles tended to become trapped in the line. The discrepancy in flow rate is attributed to the trapped air bubbles creating constrictions in the flow path.

Sample Results: Predictions with a Model Calibrated to Experiment

A second phase of this exercise involves modifying the analytical model to match the measured flow rate at the first operating condition, and using it to predict the flow rate that will occur at a new operating condition. The new condition can be obtained either by a change in height difference, $z_1 - z_2$, or by altering the length of the tubing. Once the new configuration has been chosen, the students are asked to predict the flow rate that will be achieved using the information available from the analytical model and experiment.

They are first asked to "calibrate" their analytical models to the first data point and use it as a baseline. The students can be given the freedom to use the method of their choice. One strategy might be to adjust the prediction by the percent discrepancy in flow rate seen in the baseline case. Another would be to multiply the predicted friction factor by a "fudge factor" until the baseline experiment is matched, and use this multiplier for subsequent calculations.

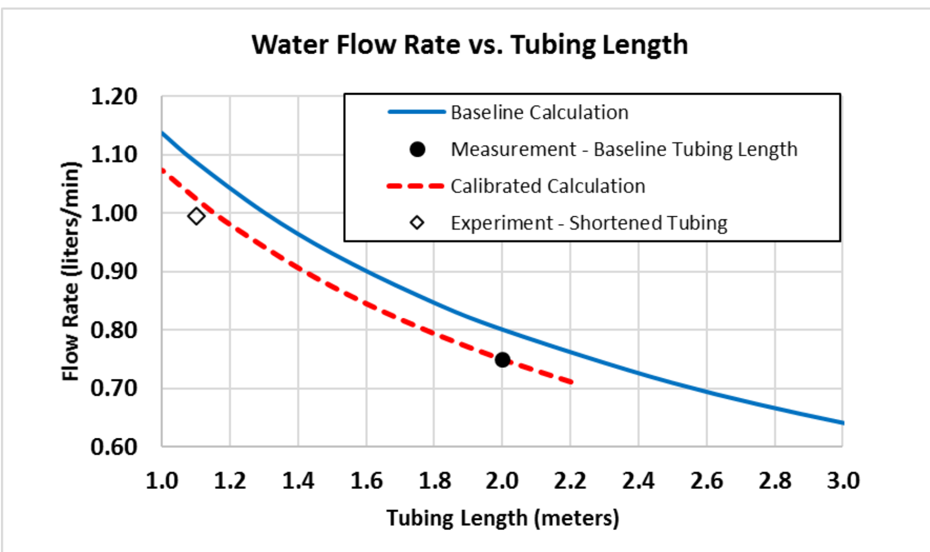

Figure 6 Flow rate prediction after calibration to baseline experiment, and comparison with measured flow rate for 1.1 meter long tubing.

An example using the second method is shown in Figure 6. The modification of the configuration consists of shortening the tubing to 1.1 meters while maintaining the same height difference. For this example multiplying the friction factor obtained from the Petukhov formula by a factor of 1.13 matched the baseline flow rate. The calibrated model was then used to predict the flow rates for various lengths of tubing. The modified analytical model is shown as the dotted line in the figure. The diamond symbol on the plot shows the flow rate obtained in the subsequent measurement with the shortened tubing. The comparison shows that the calibrated model does an excellent job of predicting the flow rate.

When carrying out this part of the exercise with a class, it has been found useful to stage it as a sort of competition among the students to see who can make the best prediction of the new flow rate. The students turn in the results of their predictions and the measurement is then taken. The competitive aspect, sometimes enhanced by token prizes, seems to be helpful in engaging the students.

Reflections and Lessons Learned

This lab exercise has now been in use for more than three years in the engineering technology curriculum. It is now used regularly as part of the junior level fluids and thermal engineering course, but early on had been used in a senior level follow-on course. These experiences have provided much useful feedback which is being used to make adjustments to improve the effectiveness as a learning tool. In the near future it is planned to carry out a formal student survey for this and similar exercises, such as the one involving pump performance [7]. The reflections presented here are based on instructor observations. These observations have provided useful guidance regarding how the exercise should be structured, and also places in the classroom component of the course where more thorough instruction is needed in areas like problem solving techniques. A few key observations are as follows:

1. The students appear to become well engaged in the tasks related to taking measurements in the laboratory. They tend to show enthusiasm for distributing tasks among the team members and in coming up with plans for how they will execute the measurements. They appear to enjoy the data gathering, and also the aspect of a friendly competition during the second phase of the project where they predict and then measure the flow for a modified geometry.
2. In the initial deployments of the exercise, the students were asked to calculate skin friction factor values from the experimental results, and then compare them with values used in the corresponding analytical calculations. This step often seemed to lead to some confusion about which friction factor value should be used for what purpose. As a result, the volume flow rate itself is now used as the basis of comparison between the theoretical calculation and the experiment. The students still need to call upon their knowledge of viscous flow analysis in the calculation. Later, they may also be given the opportunity to make empirical modifications to the loss models to calibrate their calculations to the measurements. The modified calculation formulas can then be used to predict the flow rate of an altered configuration.
3. In reviewing students' work on the theoretical part of the exercise, it has been observed that a significant number of them have difficulty performing the calculation. This appears to stem from having insufficient preparation in two areas; 1) setting up and solving problems that require iteration in order to obtain a numerical answer, and 2) working with software tools such as spreadsheets for making multi-step calculations.

While these issues have also been observed in homework problems, it has been easier to see them clearly in a project like this lab exercise. The lesson learned is that more classroom instruction and practice opportunities in this area are needed earlier in the curriculum.

Conclusions

An exercise has been described in which students use a combination of theoretical analysis and experiment to study gravity-driven water flow. It combines material taught in the fluid dynamics portion of a junior level engineering technology class with the application of experimental methods. Synthesis of theory and experiment is utilized to improve the predictive capability of the analytical model. The areas of learning that can be addressed with this exercise include:

1. Application of the incompressible energy equation and techniques for calculating viscous losses in internal flows.

2. Planning and execution of experimental tests combined with coordination of teams to execute measurements.

3. Application of pre-test and post-test statistical analysis. The pre-test uncertainty analysis is used to estimate how long measurements should be taken in order to achieve a desired accuracy level. Post-test analysis applied to multiple measurements can be used to confirm whether scatter in the data falls within predicted levels.

4. Using experimental data to tune analytical models to provide better predictions of behavior for altered configurations and test conditions.

This combination of learning areas can be emphasized as needed to achieve the learning outcomes for the course and for the program at large. Extensions being explored include the addition of other loss-producing components such as tight radius bends, flow constrictions, and valves. These make for a more complex analytical model, and more parameters that might need to be tuned in order to calibrate an analytical model of the system. At the same time these more complex systems are more representative of actual internal flow systems found in industry.

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Appendix: Sample Lab Procedure Sheet

ET 635: Fluid Technology and Heat Transfer
LAB 3A: Hydraulic Loss in Tubing with a Circular Cross Section

Objective:

The objective of this lab is twofold:

1. To experimentally determine the friction factor in a length of tubing.
2. To compare the experimental results with predictions using accepted values of pipe friction factors from formulas and/or graphs (Moody diagram).

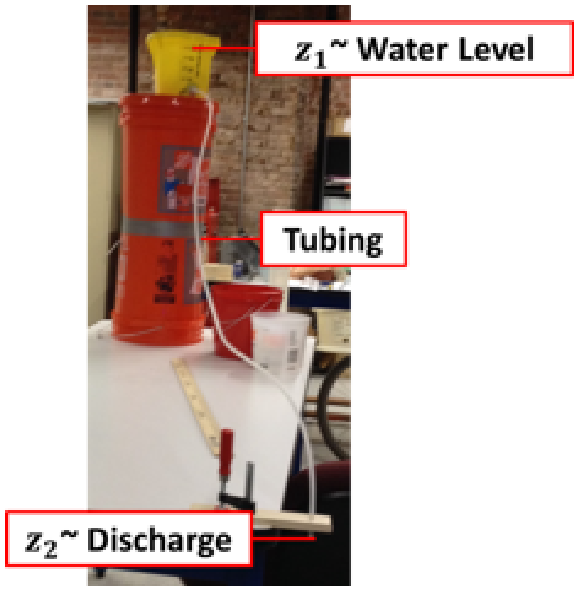

Figure 1. Experimental Setup

Background

The setup shown in Figure 1 can be used to determine the friction loss in a length of tubing through measurement of the flow rate which is driven by the force of gravity. The working fluid for this setup is water, and the tubing has a circular cross section of 0.170" (4.318 mm) inside diameter. The water supply enters the tubing at Location 1 from the reservoir (yellow bucket). The water flows under the force of gravity through the length of tubing and discharges at a lower point at the termination of the tubing, Location 2.

Procedure

Lab measurements, calculations, and reporting will be carried out in groups. Each group should make measurements using both of the two test setups marked "A" and "B". Each setup will use a different length of tubing. When making the experimental measurements, one team member should replenish the water supply in the yellow feed bucket so that the hydraulic head remains constant. The length of the tube will be measured as well as the relevant height form the water surface in the feed bucket to the discharge end of the tubing. Repeat the flow rate measurement at least 5 times for each test setup.

Theoretical calculations: Before performing the experimental procedure determine the expected flow velocity and volume flow rates. Take appropriate measurements of the apparatus. Use the energy conservation equation and the head loss equation given below. Combine the two equations and solve for the expected velocity at the discharge point. You will have to make an assumption in regard to the flow regime (laminar of turbulent) to calculate the friction coefficient.

Lab Report:
Each group should submit one report which includes:

1. Brief relevant comments on the experimental procedure used.

2. Summary data obtained (volumes, times, height in fluid reservoir, etc.).

3. Brief relevant comments on the appropriate assumptions regarding the water pressure at the top of the reservoir and at the discharge location.

4. Experimentally derived values of the following test parameters for both setups:
   a. Volume flow rate, $\dot{V}$.
   b. Mean flow velocity in tubing, $\mathcal{V}$.
   c. Flow Reynolds number, Re.
   d. Friction coefficient, f.
   e. Total head loss in tubing, h_L.

5. Comparison of the experimental and theoretical values:
   a. Compare the friction factor (f) values derived from the experiments against calculated values at the same Reynolds number. Use formulas for laminar and/or turbulent flow as appropriate.
   b. Comment on correspondence between theoretical and experimental results, including potential sources of error and uncertainty analysis of the volume flow rate.

Use the reporting format of your choice, but make sure to include all the above in an easily recognizable and readable form.

Useful Equations and Assumptions
Useful governing equations are as follows: These equations will be covered more in class and can be found in Chapter 9 of the text book. They include, but are not necessarily limited to the following:

- Energy Conservation Equation: $\frac{P_1}{\rho} + \frac{\mathcal{V}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\mathcal{V}_2^2}{2} + gz_2 + gh_L$
- Head Loss due to Friction: $h_L = f\frac{V^2}{2g}\left(\frac{L}{D}\right)$
- Friction factor for laminar flow in smooth pipe: $f = \frac{64}{Re_D}$
- Friction factor for turbulent flow in smooth pipe: $f = (0.79 \ln Re_D - 1.64)^{-2}$

- Flow Reynolds number: $Re = \frac{\rho V D}{\mu}$

Note: Reynolds number (internal or pipe flow) to determine flow regime:

$Re < 2100$	Laminar
$2100 < Re < 4000$	Transitional
$4000 < Re$	Turbulent

Velocity will be found using conservation of mass. The following assumptions are useful:

- The surrounding atmospheric pressure is uniform.
- Water in the reservoir is replenished so that the upper surface is at a constant level.
- The reservoir is large enough so that the pressure at each level can be determined from a hydrostatic calculation.

Points to consider:

- What assumption is appropriate regarding the water pressure at the top of the reservoir bucket at Location 1?
- What assumption is appropriate regarding the water pressure at Location 2 where it discharges into the atmosphere?
- It may be most convenient to make the theoretical calculations using a spreadsheet (EXCEL) or other software.

Suggestions for Experimental Procedure

In terms of experimental procedures, the following may be helpful:

- Fill the tubing and reservoir with water. Set the water level in the feed bucket at a height where it can easily be monitored and maintained by pouring in water from a container.
- Allow flow rate to develop and perform flow rate readings based on timed readings of the water level in a graduated cylinder. It is recommended that you do the following:
    - Take at least 5 readings, determine the flow rates for each and then average them. Use good experimental practice regarding rejection of readings that may be "outliers".
    - Use timing periods of at least 20 seconds. You need not stop the flow between readings. One team member can call out when the water level in the cylinder reaches predetermined levels and another records the time value.
    - Measure and record relevant heights, tubing lengths, and reservoir surface levels.