Sundials Make Interesting Freshman Design Projects

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Abstract: The design of sundials makes an ideal design project for students enrolled in Intro to Engineering courses for several reasons. First, the task requires some computation, but the level of computation is accessible to any engineering freshman (nothing beyond trigonometry). Second, the project requires the use of simple hand-tools and some simple mechanical design. For example, one may add Vernier scales to the main scales. Third, the project involves some interesting, underlying science that is within the grasp of engineering freshmen. Fourth, there is a wealth of information on the subject on the worldwide web, giving students experience in searching the web for information, and obviating the need for the instructor to provide printed material to the students. There are many sundial types from which to choose (equatorial, horizontal, vertical, and analemmic to name a few) which makes it easier to keep students from "recycling" projects from one semester to the next.

Introduction: We shall discuss four different sundials: equatorial, horizontal, vertical, and annalemic. In addition, we discuss finding true north, finding latitude and longitude, sundial corrections and the equation of time. Finally we include a brief discussion of the results of using this material in a section of Fall 01 Intro to Engineering.

Equatorial sundials: The equatorial sundial is the simplest of the dials we shall discuss since the hour lines are equiangular[1]. The dial consists of a circle (or semi-circle) with the ends of the hour lines spaced equi-angularly around the circumference. (6 AM and 6 PM fall on the diameter of the circle.) The gnomon consists of a thin rod placed perpendicular to the circle and passing through its center. (See Figure 1.) To use the equatorial dial it is necessary to know the direction of true north (The gnomon must point north.) and the location of the ecliptic. (The plane of the dial must lie in the plane of the ecliptic.)

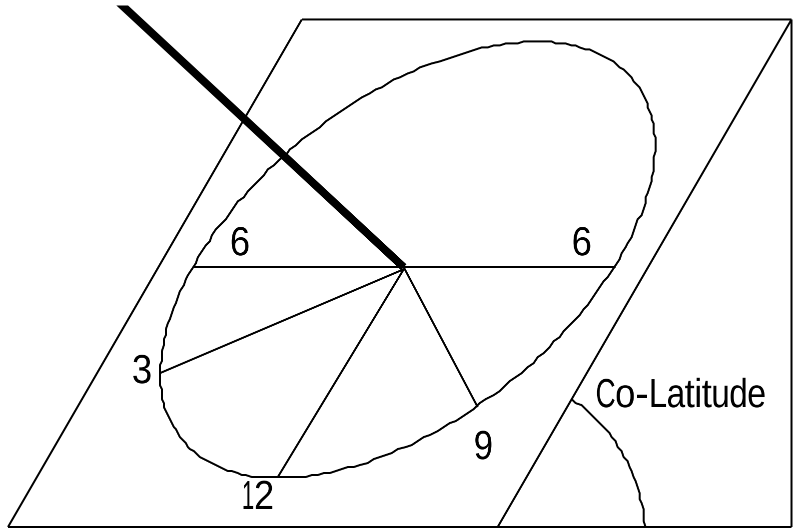

Figure 1 **Equatorial Sundial**

A discussion of the difference between true north and magnetic north[2] is followed by a discussion of magnetic declination[3] for those who wish to use a compass to find true north. (Of course, determination of the magnetic declination requires latitude and longitude information.) A second method for finding true north is to take a nighttime sighting of the star Polaris; which is within 1 degree of true north[1]. A third method[4] is to drive a stake into the ground, and at some time before noon mark the end of the shadow of the stick. Mark a circle in the ground centered at the stick and passing through the mark made at the end of the shadow. Some time after noon, the shadow of the stick once again lies exactly on the circle, at which point another mark is made on the circle. The line lying on the bisection of the angle formed by the center of the circle and the two marks is a true north-south line. The plane of the dial must lie in the plane of the ecliptic, so the gnomon must lie on a line that makes an angle with the horizon that is equal to the latitude. The latitude may be found by taking a nighttime sighting of Polaris: the angle between the horizon and Polaris is the latitude. Alternatively, latitude and longitude information can be found on the web[5]. (Longitude information is necessary for the corrections to the sundial described below in the section on the equation of time.)

<u>Horizontal and vertical sundials[1,4]:</u> The horizontal and vertical sundials each consist of a rectangular plane with the hour lines inscribed on it with a triangular gnomon set perpendicular to it. (See Figure 2.) The central issues in the construction of such dials are the positioning of the hour lines and the shape of the gnomon. The gnomon for these dials is essentially a right triangle placed perpendicular to the plane of the hour lines with the angle between the hour-line plane and the hypotenuse equal to the latitude for a horizontal dial and equal to the co-latitude for the vertical dial. The hour lines for the horizontal dial are computed from $\tan(D) = \tan(t) \cdot \sin(f)$, where D is the angle that the desired hour line makes with the 12:00 noon line, t is the time of the desired hour line in degrees, and f is the latitude. To convert time in hours to time in degrees we use $1 Hour = \frac{360}{24} = 15^\circ$.

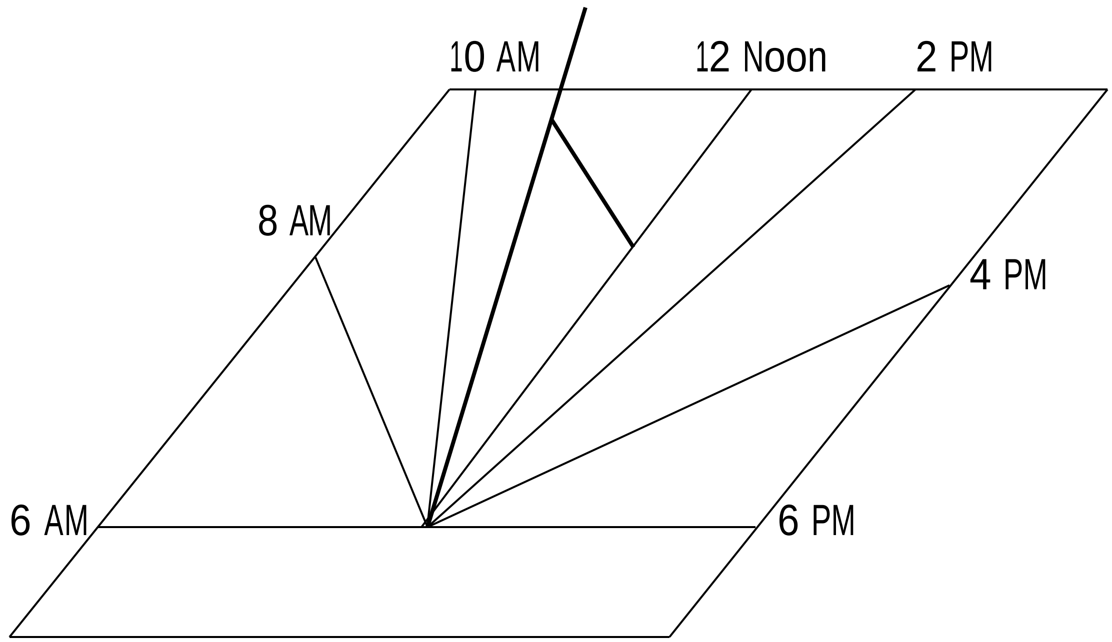

Figure 2 Horizontal Dial

<u>Corrections to the hour lines:</u> The hour lines of any sundial must be corrected for daylight saving time and for difference between the longitude of the place where the dial is used and the meridian of the time zone in which the dial is used. The correction for daylight saving time is a simple renumbering of the hour lines. The correction for longitude is made by simply subtracting the meridian longitude from the location longitude, preserving the sign, and directly applying the correction to the hour angle D. The longitude correction may be permanently built in to the dial as described above, or the dial hour lines may be left uncorrected and the time corrected by $1 Hour = \frac{360}{24} = 15^\circ$ each time the dial is used. In addition all the dials we describe, except for the analemmic dial, must be corrected by the equation of time as described below.

<u>The equation of time:</u> Most students are surprised to learn that the length of the day, as measured from noon to noon, differs significantly depending on the date. If one notes the location of the sun in the sky at noon, as measured by a very accurate clock, throughout an entire year, and joins the points together one would get an elongated figure eight in the sky, the so-called analemma. The analemma can also be drawn by noting the position on the ground of the shadow of a rod driven into the ground at noon throughout a year. What this means is that noon standard time and noon sun time (local apparent time) may differ by as much as 14 minutes throughout the year. Thus, local apparent noon (the sun is at its highest point in the sky) may occur before or after noon standard time (as corrected for longitude). This difference between standard time and sun time is caused by the fact that the earth's orbit is elliptical rather than circular, and by the fact that the earth's axis of rotation is tilted from the ecliptic by 26 degrees. The equation of time correction that must be added to, or subtracted from the sundial time can be found by simply plotting average data[6] as shown in dashed blue in Fig. 3, or from first principles[7] (calculated from an idealized model) as shown in solid red in Fig.3.

Tilt of the earth: $tlt = 23.433^{\circ}$ Eccentricity of earth orbit $ecc = 0.016713$ $e = \frac{360}{366}$

$$eqn_{Tilt} = \frac{p}{4} - \tan^{-1}\left(\cos(tlt)\cdot\tan\left(\frac{p}{4}\right)\right)\cdot\frac{24\cdot 60}{2\cdot p}\cdot\left(\sin\left(2\cdot\frac{2\cdot p\cdot(n-81)}{366}\right)\right)$$

$$eqn_{Ecc} = \frac{24\cdot 60}{361}\cdot\left(\frac{-360}{p}\cdot ecc\cdot\sin\left(e\cdot(n-2)\cdot\deg\right)\right)$$

$$eqt(n) = eqn_{Tilt}(n) + eqn_{Ecc}(n)$$

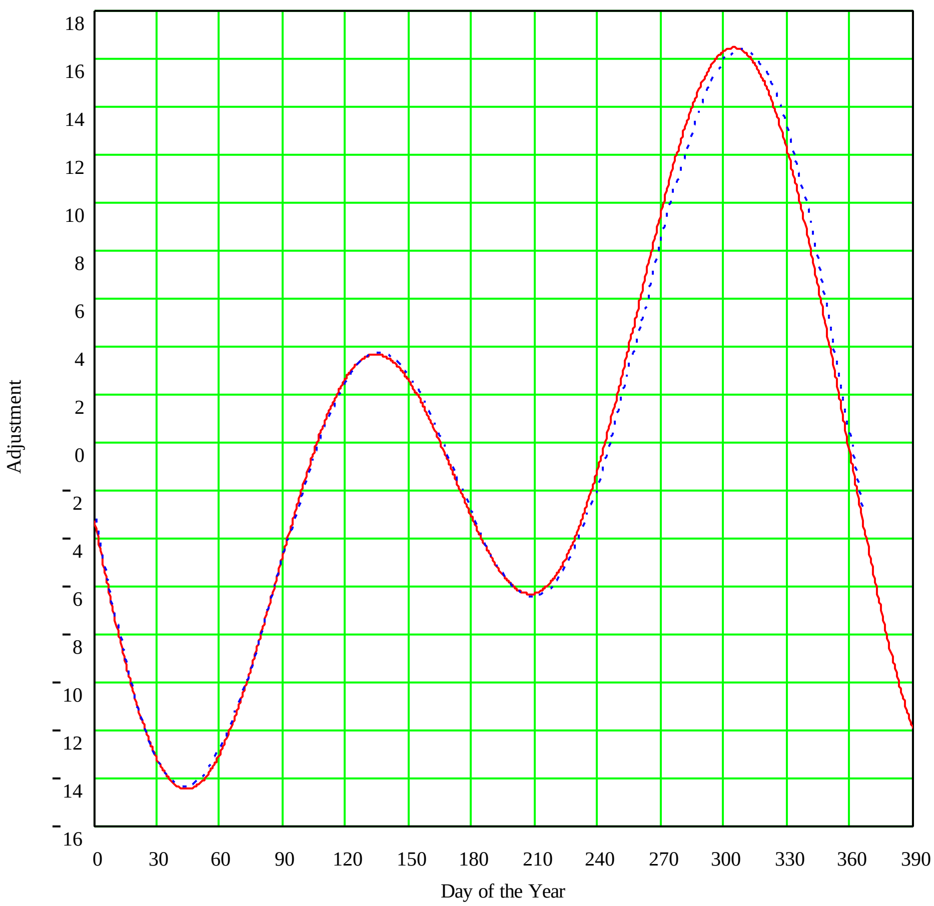

Figure 3 Plot of Equation of Time from First Principles[7] in Solid Red, from Averaged[6] Data in Dashed Blue(Positive value means Dial Fast Negative Value Means Dial Slow)

Analemmic sundials: The analemmic sundial, which must be corrected for daylight saving time and longitude, need not be corrected for the equation of time. The tradeoff is, of course, a more complicated construction. (The analemmic sundial is placed horizontally, with the minor axis on a North – South line.) The analemmic sundial is constructed as shown in Fig.4, 5, and 6 below[4,8].

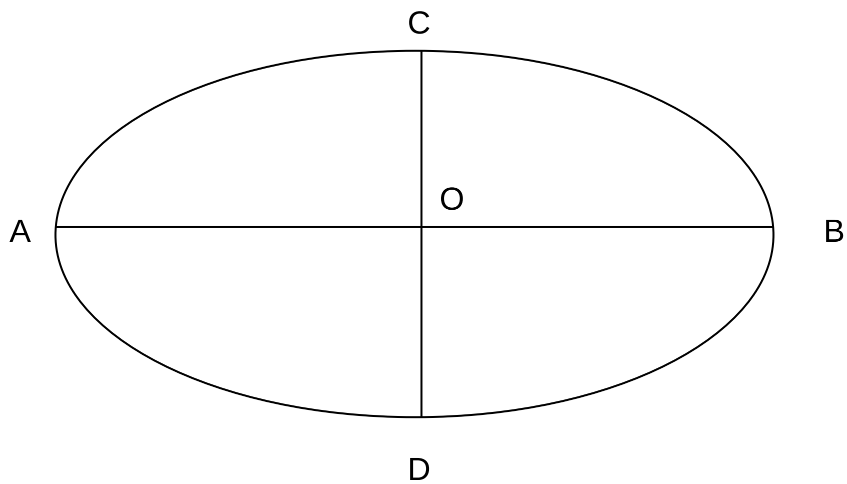

Figure 4 Major and Minor Axes of Annalemic Sundial

$CD = 2(\sin(f)(AB))$ where f is the latitude.

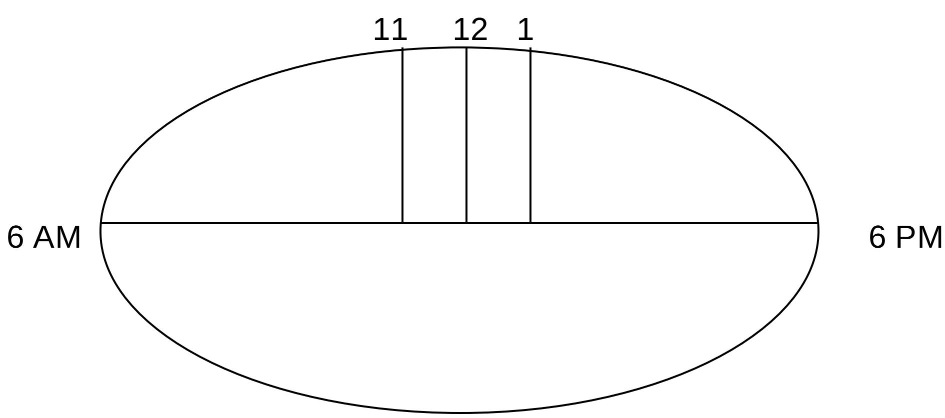

Figure 5 Hour Lines of Annalemic Sundial

The horizontal distance H from the 12:00 noon line to an hour line is
$H = \sin(t)(AB)$, where t is the time in degrees of the hour line desired.

The gnomon of an analemmic dial is a thin vertical rod, which must be placed in the appropriate hole in the date dial corresponding to the calendar date.

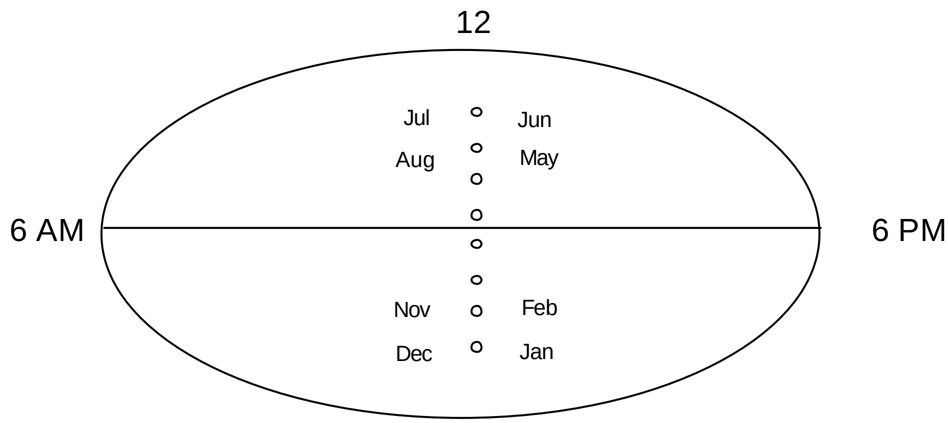

Figure 6 Date Dial of Annalemic Sundial

The distance Z from the center line to the appropriate date hole is

$$Z = \tan(dec)\cos(f\ )(AB)$$

where dec is the declination[6] of the sun on the desired date and f is the latitude.

Conclusions: The construction details of several sundials have been discussed in sufficient detail to permit their fabrication. The sundial project can be assigned as a one week cardboard and tape project, or a longer time can be allotted for a realization of the dial in wood and/or metal.

The initial class of twenty-eight freshmen engineering students at LTU in Fall 2001 enthusiastically completed a sundial project. They especially enjoyed our unusual warm Michigan fall weather. However, the students made several suggestions to facilitate the sundial project:

1. The end of September is more overcast in Michigan than the end of August, so the sundial project should be the first project of the semester (the Introduction to Engineering class meets in the fall, and the semester starts at the end of August), so we will implement this suggestion.

2. The equatorial sundial was by far the easiest to understand, and most successful groups chose this design. In fact, students suggested in-depth discussion of this type, while comparing it to the others [which might be built by the more ambitious students].

3. Students liked this application of mathematics, and were encouraged by the interesting application of trigonometry.

Our suggestions for the class project:

1. Have the student groups of two give a progress report to the class for approximately two minutes on the second day of the project. (This technique is used with several other freshmen projects.) The students typically present their designs in a powerpoint format, which forces them to finalize their ideas and understanding, requires them to draw the figures to scale, and enhances their communication skills. This exercise takes an hour, so students can quietly work on their projects while others are presenting, if necessary. Encourage them to use a 3D modeling program - many will and most should.

2. Have the instructor choose the groups, because if students are not well matched, several students will end up completing it alone.

3. Make small compromises on accuracy - especially if the project's time frame is limited due to other circumstances.

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Dr. Johnston spent 3 yrs in the U.S. Navy as a RADAR Tech before finishing the BSEE from WSU in 1978. He spent 2 years at Motorola as an Electronic Engineer and two years at the Gulbransen Organ Co. as Mgr. of Adv. Ckt. Dsgn. before finishing the MSEE in 84. He spent 6 yrs on the faculty of WSU, finished the Ph.D. in 93 also from WSU, and is currently Assoc. Prof. of ECE at LTU. His research interests include Pwr. Electronics, Var. Speed Drives, and Computer Analysis.

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Dr. Anneberg received a B.S. in I.E. in 79 from U of M. She worked 5 years as a Reliability Engr. for GM. She received an M.S. in Comp. Engr. from WSU in 83, and the Ph.D. from WSU in 91. She has consulted for Daimler Chrysler, St. Clair Intermediate School District, and others. She has been on the faculty at LTU since 1991, and is presently an Assoc. Prof. of ECE. Her research interests include distributed computing, error detection and correction, and software engineering.