Supporting Student Learning Before, During, and After Lecture in a Probability Course

Dr. Chao Chen, Purdue University Fort Wayne

Dr. Chao Chen is currently an Associate Professor of Computer Engineering in the Department of Electrical and Computer Engineering at Purdue University Fort Wayne, where she has been since 2005. She received her M.S. and Ph.D. degrees from Georgia Institute of Technology in 2003 and 2005 respectively. She also earned B.E. and M.E. degrees from Shanghai Jiao Tong University, China in 1998 and 2001, respectively.

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Abstract

Most students consider probability and statistics a hard subject, partly because it requires a combination of math theory and real world thinking, often not in a very intuitive way. This paper describes the effort of a probability course instructor to actively involve students in their own learning process and enhance the teaching and learning effectiveness. Specific strategies explored include encouraging participation and feedback, adding online quizzes for better preparation and review, and adding instructional resources to support learning. The main goal of these strategies is to actively engage and efficiently support student learning before, during, and after lecture. Assessment data and student feedback show that such approaches are effective and welcomed.

1. Introduction and Literature Review

Probability is a fundamental course for students in both electrical and computer engineering majors at Purdue University Fort Wayne. This course serves as an introduction to probabilities and statistics, as well as their applications to engineering problems. Most students consider probability and statistics a hard subject, partly because it requires a combination of math theory and real world thinking, and the connection between the two is often not very intuitive. It is also challenging for students to apply the theory to problem solving, especially on how to interpret what is given in a scenario, identify the goal, and connect the two using probability tools.

This paper summarizes the effort of a probability course instructor in spring 2022 and spring 2023 semesters to actively involve students in their own learning process and enhance the teaching and learning effectiveness. More than half of the students enrolled in this course are working either full time or part time. Therefore, the goal is to design a pedagogical framework with materials and strategies to efficiently engage them before, during, and after lecture, but not overwhelm them with too much workload. The strategies explored include the following: 1) Encouraging participation and feedback. Participation credits are added encompassing both lecture attendance and online quizzes. Each quiz also provides an opportunity for students to leave feedback. 2) Adding pre-lecture online quizzes for better preparation and review. These online quizzes focus on timely review of past learning and preparation of new learning. 3) Adding instructional resources to support learning. Interactive documents such as MATLAB live scripts are written to allow students to view probability features interactively. In addition, pre-recorded short instructional videos are used as optional review resources. Questions are embedded in some short videos to stimulate learning and self-assessment. We conducted analytic study from student participation data and survey feedback. Feedback from students show that these methods are effective and welcomed. These strategies can be tailored to other engineering courses.

There have been various pedagogical approaches specifically designed for probability and statistics courses for engineering students. For example, the use of technology was discussed in [1], where the author also illustrated the helpfulness of laboratory-like exercises through computer simulations in a probability and statistics course in Texas A&M University. A set of constructivist exercises have been developed in teaching probability and statistics in the University of South

Florida [2] to promote realistic mathematics education and inquiry-oriented teaching and learning. These exercises encourage students to work in teams, create their own knowledge, and develop their own understanding. The author in [3] practiced active learning exercises to boost student accountability in a course on probability and statistics at Valparaiso University. Students spent more than half of lecture time working on in-class projects with warm-up concepts as well as a few questions for routine calculation and more challenging application problems. Moreover, the flipped classroom approach has been used in a probability and statistics courses at University of Pittsburgh, where the lecture was removed from class time and replaced with more active instructional opportunities [4].

The flipped classroom method facilitates active learning by utilizing online materials to supplement face-to-face time lectures, and typically involves students learning content in advance and taking quizzes to assess their understanding before class. This approach enables lectures to be focused on problem solving and interactive activities, thereby maximizing the effectiveness of inperson learning time. Flipped classroom has been used favorably in engineering education (e.g., [5-6]) and shown to enhance in-class interaction and foster a better learning experience. Nevertheless, a key challenge of its overall effectiveness has been the lack of a clear pedagogical understanding of how to effectively translate the flipped classroom into practice. For example, the pre-class activities should be seamlessly incorporated into the in-person sessions. Furthermore, it is essential to establish a conceptual framework that allows for a cohesive approach to activities before, during, and after in-person learning sessions [7].

The instructor carefully examined these teaching strategies at other academic institutions and adapted or redesigned them to suit the needs of our student population. Many students enrolled in Purdue University Fort Wayne hold part-time or full-time jobs and not always have ample time to engage in high load of pre-class readings. This makes it challenging to adopt the flipped classroom model. Instead, the instructor chose a more streamlined method of creating course materials that aid students in their learning prior to, during, and after lectures. The goal is to provide efficient support for student learning while minimizing the need for student preparation, fostering effective classroom engagement, and offering supplementary resources for further assistance as needed. As this course is taught specifically towards ECE students by an ECE faculty, the in-class problem solving examples and exercises are designed with engineering application in mind. Interactive laboratory-like demos are also included to facilitate student understanding. In addition, students work on guizzes before coming to the class, but completion of these guizzes does not require them reading new lecture contents beforehand. Supplementary optional material such as web resources, recorded videos, and additional examples is provided, particularly for students that need extra help. The instructor also worked with instructional consultants at the campus learning and teaching center on innovational ways to incorporate these new pedagogical designs as well as assessment tools into the online learning management system.

The remainder of this paper is structured as follows. We first introduce the context of our study, including student background and challenges. We then explain the detailed design and implementation of the above-mentioned pedagogical strategies. The effectiveness of these strategies has been assessed through student participation and performance data over both semesters, as well as from student feedback throughout the semester. Finally, we summarize the effort and discuss further improvement.

2. Context of Study

The probability course (ECE 30200) in the ECE department at Purdue University Fort Wayne is a required course for both electrical engineering and computer engineering majors and has been taught annually for many years. It is a 3-credit course taught in a 75-minute lecture setting twice a week. This course aims to serve as an introduction to the concept of probabilities and statistics and their applications to engineering problems. The course learning outcomes are listed below:

On successful completion of this course, students should be able to:

- 1. Model uncertainties with probability theory and solve basic probability problems.
- 2. Describe different types of random variables and solve problems with important distribution functions.
- 3. Solve problems with joint distributions of two random variables.
- 4. Derive the distributions of functions of random variables.
- 5. Solve problems with conditional probability models.
- 6. Compute point estimates and confidence intervals for parameters of interest.
- 7. Perform simple statistical inference such as hypothesis testing in the presence of uncertainty.
- 8. Understand the statistical properties, such as mean, autocorrelation, and autocovariance, of random processes.

ECE 30200 is scheduled in a face-to-face setting except in the spring 2020 semester when campus was closed for COVID-19. The conceptually difficult material in this course requires that students fully comprehend the theory before applying it to solve real-world problems. Besides the mathematically heavy content, there are additional challenges for students enrolled in this course:

- *Lack of motivation*: There has been a noticeable decrease in student motivation to attend lectures, interact with the instructor and fellow classmates, and seek academic assistance. This decline has been particularly ovservable in the last two years partly due to the impact of COVID-19, which has disrupted everyone's lives. As a result, the quality of student learning is negatively impacted, especially for those at risk.
- *Limited class time*: Didactic lecture is still the most commonly used method in teaching probability courses. During lectures, to illustrate the application of probability in real-life and engineering scenarios, the instructor would spend a moderate amount of time demonstrating the problem solving process systematically. Due to the lecture time limit, students often do not get sufficient time themselves to inspect the problem setting, interpret the problem through the lens of probability theory, and take an active role in their learning.
- *Busy life*: The average numbers of total credit hours taken by the students are 14.4 and 15.8 in spring 2022 and spring 2023, respectively. In addition to taking classes, more than half of the students enrolled in this course are working either full time or part time. Figure 1 shows the student working hour survey data for spring 2022 and spring 2023 semesters. Considering the busy working and class schedules for most students, it is not realistic to flip the classroom or overwhelm them with much workload outside of classroom. The course lecture time is set to 9-10:15am in the morning, which also makes it hard for students to spend time right before class for preparation.



Figure 1. Student working hour survey result.

3. Methods

The main pedagogical strategies that we added to the instruction of ECE 30200 are summarized as "Be there - Get ready - Resources are plenty." More specifically, the instructor used lecture attendance and online quizzes to encourage participation. The online quizzes are also designed to help getting students prepared for the lecture. In addition, supplementary instructional resources were added to support learning.

3.1 Be there

In Purdue University Fort Wayne, most classes were transitioned back to in-person in fall 2020, following a period of online instruction for half a semester in the spring of 2020. Nevertheless, the mandate of physical distance, assigned seats, and facemasks discouraged student attendance to some extent, which greatly impacted the classroom atmosphere and interaction. With COVID-19 slowly settled down, the first thing that was put in place in ECE 30200 starting spring 2022 was the lecture attendance, which accounted for 5% of the overall grade. Of course, attendance of some lectures for individual student was excused for illness or other legitimate reasons. The main purpose is to have students physically be there and actively participating in the classroom. The instructor can also directly observe students' understanding and struggles during class. Student class attendance rates over a total of 28 lecture sessions (exam sessions are excluded) in spring 2022 and spring 2023 semesters are included in Figure 2. Although attendance data from previous years are not available for comparison, an overall average attendance rate close or above 85% (for example, 84.9% in spring 2022 and 85.4% in spring 2023) is satisfactory.

In addition to lecture attendance, 5% of overall grade is on participation of online quizzes. After each lecture, an online quiz will be made available and due before the start of the next lecture. These quizzes normally contain 2-5 simple questions (e.g., multiple choice, multiple answers, true/false, matching, fill-in-blanks), mainly to preview some problems in the coming lecture, and review the concepts on what was covered in the previous lecture. Grading is on participation only, not on correctness. Figure 3 depicts the student participation rates on the pre-survey, a total of 26 online quizzes, and the post-survey in spring 2022 and spring 2023. An overall participation average of above 90% (for example, 93.3% in spring 2022 and 92.0% in spring 2023) shows that most students did finish the quizzes before each lecture.



Figure 2. Student class attendance for each lecture session.



Figure 3. Student participation rates on online quizzes and surveys.

3.2 Get ready

In addition to being used as a measure of participation, the online quizzes also help students to get ready for each lecture. Two types of questions are included: preview questions and review questions. The preview questions allow students to think through some in-class illustrative problem before lecture. Students attempt to interpret the probabilities and identify the goal. This way they will come to the class more prepared and better understand why the instructor chooses a certain approach in solving the problems. The review questions are also included to help students assess their previous learning in a timely manner. In addition, the instructor is able to review the question statistics before the lecture, hence to understand the common mistakes and misunderstandings, and adjust the lecture coverage and deliverance accordingly.

The preview-deliver-review cycle behind the quiz design is illustrated in Figure 4(a) with an example given in Figures 4 (b-d). The example in Figure 4(b) is a typical detection problem to illustrate the use of Bayes' theorem in a binary communication system to calculate posterior probability of P[A|B] based on the prior probability P[A] and the conditional probability P[B|A]. The instructor plans to cover the Bayes' theorem and walk through this example problem in class. In the pre-class quiz problem shown in Figure 4(c), students are presented with the setting of this example. However, they do not need to do any actual calculations, but to focusing on interpreting some of the key probability notations that will be necessary for solving the in-class problem. This practice of translating the word descriptions of probability-related information into mathematical expressions is crucial for the practical application of probability theory in real-world scenarios. Moreover, this gives them a chance to think through the problem setting by themselves before the instructor "discloses" the answer and guides them through the problem-solving process. Since this

is a simplified version of the in-class example presented in the form of a matching problem (or multiple-choice, multiple-answer, true/false, fill-in-blanks quiz problems in other cases), it does not demand a significant time commitment from students. After the lecture, a review question like the one in Figure 4(d) is often included in the next quiz to give students another opportunity to reflect what they have learnt during class. This type of just-in-time learning periodically refreshes the knowledge and provides practice opportunities when information is readily available.







Figure 5. Student accuracy rates on online quizzes.

The student accuracy rates on the online quizzes in spring 2022 and spring 2023 are presented in Figure 5. The average accuracy percentage rates for the quiz questions are 51.4% and 62.3% for spring 2022 and spring 2023, respectively. This is understandable since most quiz questions were related to harder in-class examples. The inclusion of these questions was not to evaluate students' comprehension, but rather to offer them an opportunity to grasp and interpret the setup of the examples.

3.3 Resources are plenty

In addition to providing regular course content through PowerPoint slides, practice problems and solutions, etc., the instructor also included extra interactive resources to enhance learning.

An important concept in probability theory is the distribution of random variables, especially how different parameter values would change the distribution, and how additional knowledge about a random variable would affect its probability metrics. All these can be depicted through customizable figures or animations. For this purpose, interactive and reconfigurable documents such as MATLAB live scripts were written to allow students vary certain parameters and observe their effects on probability features instantly. Figure 6(a) shows one section of a MATLAB live script on the distribution function of the Gaussian random variable. It shows an example code to call the specific MATLAB function to generate and plot the distribution function. The important parameters (e.g., mean and standard deviation) of this function can be adjusted and the effect on the distribution will show immediately on the figure.

The instructor has created multiple MATLAB live script files on the following topics:

- Pseudo-random number generator: introducing various random number generator functions in MATLAB and the histogram function to plot the distribution.
- Discrete random variables: introducing built-in probability mass functions and cumulative distribution functions in MATLAB for various discrete random variables.
- Generate discrete random samples: introducing methods of generating random samples for discrete random variables with a given probability mass function. In addition, showing built-in sample generating functions in MATLAB for various discrete random variables.
- Continuous random variables: introducing built-in probability density functions and cumulative distribution functions in MATLAB for various discrete random variables. The Gaussian pdf example in Figure 6(a) is part of this script. The inverse transform method to generate random samples based on the distribution function is also included.

These scripts are provided to students under the learning management system. Students have access to MATLAB through university computer lab, their own computer, and MATLAB online via university license. There are also a few MATLAB coding problems in homework assignments. These experiences not only enhance students' understanding of probability concepts, but also let them practice the application skills of generating random samples and analyzing data.

The instructional videos produced during the COVID-induced transition to virtual teaching in the spring of 2020 serve as another valuable resource for students. These videos include concept delivery as well as problem solving demonstrations. Since courses were moved to online after spring break of spring 2020, videos were recorded only on the second half of the semester.

Nevertheless, this content (i.e., multiple random variables, statistics, and introduction on random processes) are in general more challenging for students. Providing extra resources on these materials is more beneficial. There are a total of 45 short videos made, each about 2-10 minutes long. In the semesters after 2020, the instructor used these recorded videos as optional review resources and released them after corresponding lectures. Furthermore, for a few long videos (usually solving more complicated problems), questions are embedded in these short videos to enhance self-learning. For example, Figure 6(b) shows a screenshot of a recorded video of the instructor solving an example problem on statistics. There are two questions embedded in this video, as indicated by the two marks on the progress bar. The screen capture of the first question is included on the bottom of Figure 6(b), which asks the student to calculate an important parameter (α) before using the formula provided in the textbook to find the $(1 - \alpha)$ confidence interval. The video will pause when a question is encountered, the correct answer of this question will be revealed before the video resumes. The purpose of incorporating these basic interactive questions is to stimulate and evaluate learning while a student is watching the video.

The instructor includes the following question "Have you watched any of the recorded videos for in-class examples? Do you think they are helpful?" in the participation quiz twice in the semester, for example, once in week 11 and the other in week 14 during spring 2022. In week 11, only 18.2% of students answered that they have watched the video, whereas in week 14, this ratio increased to 50%.



Figure 6. Examples of additional resources



User #	Plays	Average completion rate	Total completion rate	course grade
1	3	32.3%	93%	A-
2	2	100%	100%	D+
3	2	100%	100%	С
4	1	94%	94%	D
5	1	30%	30%	C-
6	1	66%	67%	A-
7	1	3%	3%	С
8	1	7%	7%	A

(a) Video views

(b) User engagement

Figure 7. Viewer analytics of a problem-solving video.

Figure 7 illustrates the viewing analysis record of one video clip with the instructor solving two problems using the central limit theorem. This record is over spring 2022 and shows twelve plays from eight unique viewers over the duration of one video clip of around 11 minutes long. It can be seen in Figure 7(a) that there are a few sections of the video that are more frequently viewed. These sections correspond to the parts that involve interpretation of the problems as well as carrying out more intricate mathematical derivations. The eight unique viewers accumulated a total of 92.4 minutes viewing time, with an average completion rate of 72.1%. The viewing statistics of the eight students as well as their course grades are included in Figure 7(b). It appears that students with good grades (three out of eight with a grade of A- or above) or those who are struggling (five out of eight with a grade of C or below) tend to spend time reviewing the video. Also, among the eight students, three viewed the video multiple times, whereas some others did not view the entire video, but instead focus on specific sections that they need to review.

4. Student Feedback

Getting student feedback is important for the instructor to assess the effectiveness of the methods and adjust teaching accordingly. The instructor has mainly used two online channels to get student feedback: feedback question within each quiz and the end-of-semester survey.

4.1. Timely feedback

In addition to preview and review questions, each quiz also provides an opportunity for students to leave feedback. This is set up as a simple optional short answer question. In some weeks, the instructor seeks feedback in terms of suggestions and comments on the course overall, other weeks it may be more specific regarding the lecture structure, use of software and class resources, etc. As most students are reluctant in communicating with instructors officially in person or through emails. This way of soliciting feedback as piggybacked in quizzes makes it convenient for students to write down comments without the need of creating extra links or forms. It also gives the instructor quick feedback and a chance to adjust teaching in a timely manner.

Student feedback data as submitted with every quiz in spring 2022 and spring 2023 are included in Figure 8. It shows that every time over 50% of students included something in the feedback input box. The overall rates of feedback is 72.6% in spring 2022 and 63.1% in spring 2023, respectively. Most of the feedback are short and just informative, such as "no", "not yet", "so far so good" to the question of "Do you have any comments or suggestions regarding this course so far?" Nonetheless, this type of positive feedback is still an indication that the class is progressing well and the student has not encountered big hurdles so far. In addition, some students did provide valuable suggestions and specific comments in the feedback. Figure 8 includes the percentage of such feedbacks among all students. Over the semester in average 16.8% of all students in spring 2022 and 11.4% of all students in spring 2023 have left specific comments in each quiz. Examples of such comments include "Going over more examples of independent events would be helpful", "More examples on the different types of random variables", "questions 2 and 3 seem unclear", etc.

Please note that the higher rates of specific comments in the some weeks (e.g., 17th and 23rd weeks in spring 2022) are for a different question asking whether students have watched the online instructional videos, as described in Section 3.3.









4.2. End-of-semester survey

At the end of the semester, there is a post-survey for the students to reflect on the class and provide feedback. It is used as a participation activity. This survey starts as follows:

"Your insights into your learning in this course can help me see our course from your side of the desk. Please respond to any three of the statements below (more if you'd like). I will use them as I plan for my courses next semester."

Then students are given the option to choose to answer three out of the six listed questions. Given this option, most students tend to answer in more detail when elaborating their answers. A summary of student feedback in the end of spring 2022 on these questions are given below:

- *Question 1: "This course most helped my learning of the content when ... because ..."* 77.3% of all students answered this question. Many students mentioned going over examples in class was helpful. They also mentioned weekly assignments (homework and quizzes), lecture notes, and videos.
- *Question 2: "It would have helped my learning of the content if ... because ..."* 81.8% of all students answered this question. Most recommended more real-world examples. Some mentioned going over certain commonly missed questions on homework or exams.
- *Question 3: "The approach I took to my own learning that contributed the most for me was ... because ..."* 77.3% of all students answered this question. Most mentioned actually working on examples and homework problems helped.
- *Question 4: "The biggest obstacle for me in my learning the material was ... because ..."* 86.4% of all students answered this question. The response for this question varies. Some mentioned that the concept is hard and confusing at times, others consider time management is challenging, a couple complained about the early class time (9am).
- *Question 5: "A resource I know about that you might consider using is … because …"* 59.1% of all students answered this question but only a few did recommend some resources other than what is already included. For example, one student recommended more MATLAB integration with the course, a few recommended some web links.
- *Question 6: "What I think I will remember five years from now is ... because ..."* 54.5% of all students answered this question. The topics mentioned include probability basics, various distribution functions, conditional probability, as well as using Gaussian distribution for statistics applications.

5. Student performance

The instructor compared the student performance in terms of class overall grade in spring 2022 with that from two previous semesters, i.e., spring 2019 and spring 2021. Some of the statistics are included in Table 1. Please note that due to COVID-19, the instruction over half of the semester in spring 2020 was purely online and much of the assignments was adjusted. Therefore, student performance in spring 2020 was excluded from comparison.

	spring 2019	spring 2021	spring 2022
average	74.50	75.10	75.28
standard deviation	13.34	11.65	9.45
10 percentile	60.14	61.81	64.14
25 percentile	66.99	68.73	66.12
75 percentile	83.70	82.67	84.44
90 percentile	91.86	91.65	89.24

Table 1. Student performance comparison with data from previous years

As shown in Table 1, student performance data in spring 2022 follow a similar distribution as previous years, but with some improvement in the average, standard deviation, and 10 percentile values. This provides some support that the new holistic approach enhances student performance. The instructor plans to collect additional data in the upcoming semesters in order to obtain more compelling evidence of the effectiveness of the pedagogical model.

6. Summary

This paper summarizes the effort of a probability course instructor to design materials and strategies to efficiently engage students before, during, and after lecture. Feedback from students show that the methods are effective and welcomed. These strategies can be tailored to other engineering courses.

The instructor plans to continuously improve this probability course in the future semesters. For example, through reviewing each quiz question, adding feedback to individual quiz questions, and revising the questions if needed. Instructional videos of the content and examples in the first half of the semester will be added, starting from more difficult ones. Scaffolding pedagogical teaching support can be added to help students acquire the knowledge as needed. For example, additional resources can be included in the learning management system and released on a conditional basis if a student answered some quiz questions incorrectly or if a student's performance on some assignments falls below a threshold.

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