Paper ID #32026

Synthesis of a Correcting Equation for 3 Point Bending Test Data

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I am interested in the mechanics side of mechanical engineering. More specifically, I seek to pursue a career that deals with the design, optimization, and analysis of products. My professional development at Kennametal over the past two summers has focused on analyzing both the static and fatigue properties associated with products during operation. Verification of results and generating technical analysis reports followed all completed analyses. Currently, I am working on a senior design project focused on the design and analysis of robotic end of arm tooling to help Kennametal automate a specific pick and place process in a manufacturing facility. I seek to attend graduate school to obtain a Master of Science in Mechanical Engineering part time while working full time as a Mechanical Engineer.

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Synthesis of a Correcting Equation for 3 Point Bending Test Data

Abstract

A frequent requirement of a Mechanics of Deformable Bodies course is for students to complete an experiment using a compression/tension test fixture incorporating a 3-point flexure fixture. The prominent goals of such an assignment are to understand the basic concepts of load/deflection relationships for pure bending situations, to calculate and correlate theoretical analysis with experimental results, and to use computer software to plot and analyze data. Once completed, students should be able to compare theoretical and experimental modulus of elasticity values. The student should then be able to not only determine the type of material used in the 3-point bending test but also discuss any discrepancies between theoretical and experimental values.

During the preparation of lab materials for such an experiment, it was noticed that the data collected from the instrument produced incorrect modulus of elasticity values for all specimens. Extension values suggested that there was more deflection occurring in the test specimen than predicted resulting in modulus of elasticity values lower than expected. In fact, the modulus of elasticity values could be so low that a student could not correctly determine the material type of the specimen being analyzed. In order to correct the extension data, a system deflection analysis was initiated.

First, internal displacements occurring within the 3-point flexure fixture members were determined by calculating the deflections produced during experimental loading. Next, the remaining sources of internal displacements within the system were investigated with the results being fitted to a curve. A correction equation with support separation distance, applied load, and specimen weight as independent variables was generated by summing all the internal displacements found. The original extension data could then be adjusted by subtracting this correction displacement. The resulting correction equation was validated using specimens with a known modulus of elasticity. Successful completion of this project would allow students to appropriately correct the 3-point bending extension data collected during an experiment and more accurately calculate an unknown specimen's modulus of elasticity.

Introduction

Typical curricula for students pursuing a degree in an engineering mechanics field includes the study of load/deflection relationships for certain materials. These relationships include, but are not limited to, axial, torsional, and lateral loading of members. Deflection equations are developed during instruction that allows students to calculate the theoretical deflection of objects so loaded. Regardless of the form these deflection equations take, students can use them to determine the modulus of elasticity for the material comprising the member, an important concept when dealing with unknown materials. While some students can grasp the fundamental concepts of load and deflection just by studying theoretical material, some students learn more easily by completing hands-on experiments. Students who learn more easily using hands-on methods are referred to as kinesthetic learners. Whatever mode of learning works best for these students, all students benefit from performing physical experiments that apply the theoretical material to a physical experiment. The truth in this fact is because "Over 80 percent of college faculty use lecture as their primary instructional method. At its core, kinesthetic learning gives students the opportunity to move out from behind their desks and to interact with their surroundings" [1]. Therefore, even if laboratory experiments are not required by a traditional class curriculum, incorporating them is beneficial for illustrating a concept. An additional benefit is that engineering students may be introduced to the types of load/deflection tests that they may deal with in their professional careers after graduation.

The strength of a material is inherent in the material itself and must be determined by experiment. Hibbeler [2] explains, "One of the most important tests to perform in this regard is the tension or compression test." There are others but utilizing a 3-point bending test provides additional benefits. Three of these benefits are due to test specimen geometry, machining capability, and student instruction. University engineering mechanics laboratories differ concerning the type and capacity of test equipment available. To test material such as steel using a tension test, the diameter of the test sample would be determined by the load capacity of the test equipment available. If the diameter of the test specimen were too large, the test may fail due to no yielding or fracture occurring. So, if a smaller test specimen is required, it would need to be machined to a smaller, non-standard, diameter to conduct a successful experiment. Surface imperfections play a large part in inaccurate results from tension tests due to the stress concentrations they cause which are traditionally reduced by polishing the surface to a smooth mirror finish. This treatment is difficult for specimens small enough to be used in some of the lower capacity test equipment available.

Using a 3-point bending test, the material cross-section can be non-circular and of a larger size in comparison to an equivalent pure tension test. Therefore, less machining capability would be needed to yield a successful test. Good results could be obtained using commercially available stock of various materials, sizes, and geometries. This has the benefit of the 3-point bending test being more usable across a wider array of testing equipment for less setup and material cost. Also, student measurement of the cross-section is made simpler since nonuniform dimensions would be less likely. One of the main instructional results of the 3-point bending test would be to analyze the experimental data to determine the test sample's modulus of elasticity. Determining the modulus of elasticity would require the manipulation of the deflection equation and evaluating it using the specimen's geometric properties along with the experimental loads and resulting deflection values obtained during the test.

There are three main outcomes from incorporating a strength testing lab into an engineering mechanics curriculum. First, the student will gain familiarity with strength testing. Up to this point in the student's study, they would most likely have only been exposed to the pure tension test. Through exposure to the 3-point bending test method, the student would be made aware of alternative strength testing types used in industry. Next, the modulus of elasticity of an unknown material can be determined using experimental data. Typical engineering mechanics problems supply the student the material to be used with its respective modulus of elasticity value so they may determine a deflection for a given load. Requiring the student to determine the modulus of elasticity of an unknown material given the load and deflection turns typical problems upside down. Lastly, this type of experiment necessitates the completion of an in-depth error analysis. This lab requires the student to compare theoretical and experimental modulus of elasticity values. The probability that theoretical and experimental values will match up with no error is very low. Therefore, the reasons behind the discrepancy need to be investigated by the student, encouraging them to perform error analysis. This task challenges the student to evaluate each detail of the experiment, apparatus and test specimen to determine what types of errors may have occurred to cause the difference between experimental and theoretical modulus of elasticity values.

During the preparation of lab materials for such a 3-point bending test, it was discovered that the testing system reported data that produced incorrect modulus of elasticity values. The deflection values reported suggested that all the test samples were deflecting more than they experienced. Therefore, a lower modulus of elasticity values was produced for all test samples. At times, the values were so low that the student was unable to make an accurate determination of the test specimen's material. Initial analysis suggested that there was a proportional correlation in the size of the error to the stiffness of the specimen (defined as the modulus of elasticity multiplied by the specimen's second moment of area). Therefore, an investigation as to the major possible errors existing in the 3-point bending lab setup was undertaken to determine a correction factor/equation for the deflection data.

Background

Materials testing is a fundamental concept that must be understood by those practicing in the field of engineering mechanics. The strength of a material depends on its ability to sustain a load without undue deformation or failure. Although several important mechanical properties of a material can be determined from materials testing, it is used primarily to determine the relationship between the average normal stress and average normal strain in many engineering materials such as metals, ceramics, polymers, and composites.

The purpose of this study was to determine a correction factor/equation for the 3-point bending test data obtained through experimentation. For these experiments, the deflection data was obtained from an Instron 3345 Single Column Universal Testing System (hereafter referred to as the Instron) with a 5 kN Static 3-Point Flexure Fixture (Figure 1). Both the Instron and the 3-Point Flexure Fixture limited experiments to loads of no more than 5kN. This limitation was a major contributing factor to the necessity of the 3-point bending test experiment (fixture set-up during operation can be seen in Figure 3).

In the 3-point bending test procedure, a specimen of a known cross-section is positioned between two supports, and a load is applied at its center [3]. As mentioned, this test can be used to determine the modulus of elasticity of a material. This value is the constant of proportionality which relates stress and strain within the elastic region of a stress-strain curve (Figure 4). There are several drawbacks to using a 3-point bending

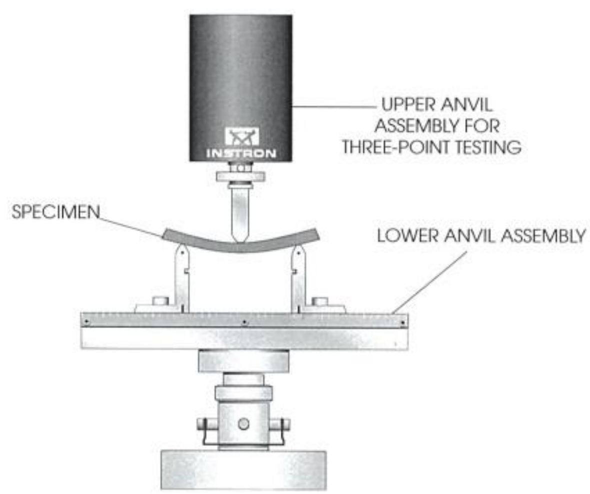


Figure 1 - 5kN Flexure Fixture [6]

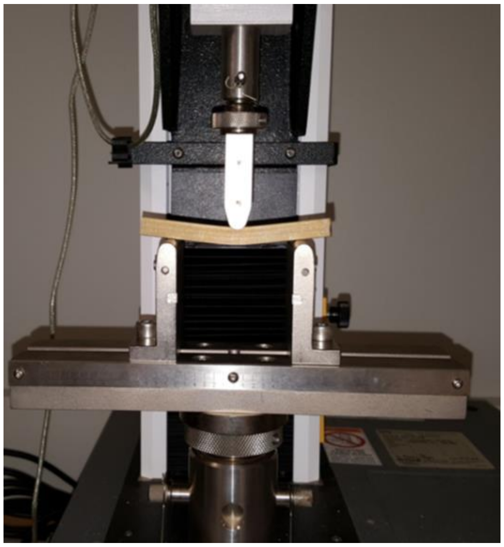


Figure 3 - Fixture during operation.

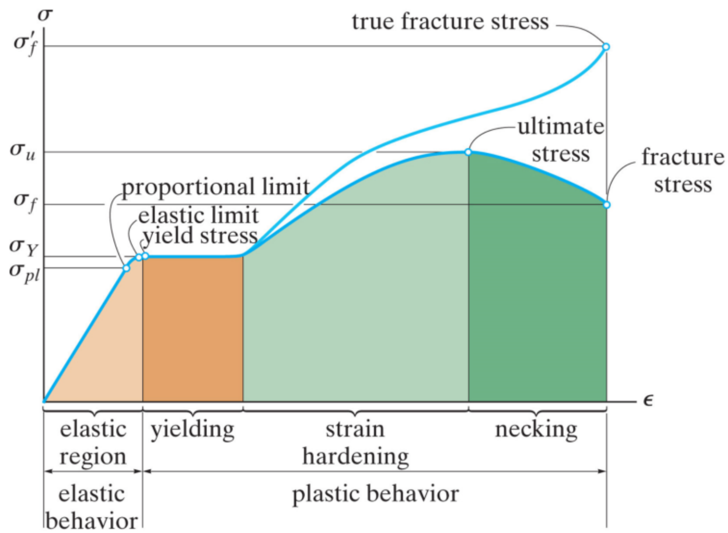


Figure 4 - Generic stress-strain diagram.

test, one of which is that unless a steel test specimen has low stiffness, it does not show the classic yield point phenomenon since the material is not yielding uniformly throughout its cross-section (Figure 2). While the yield point can be identified by the point at which the stress-strain curve becomes nonlinear, its determination isn't as exact as in tension tests. Finally, all deflection equations in common usage assume small deflections, so it is unlikely that a determination can be made for the ultimate strength or fracture strength of the material with any confidence.

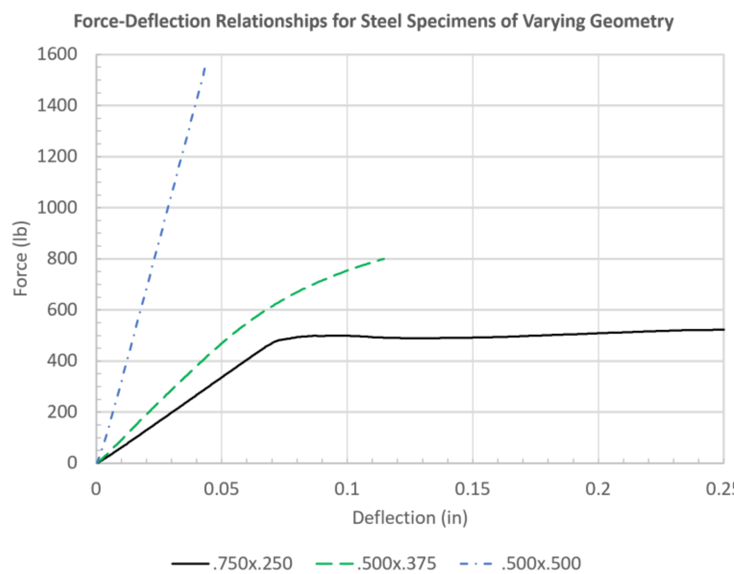


Figure 2 - Types of steel yield point phenomena: .750x.250 (distinct), .500x.357 (indistinct), and .500x.500 (none).

Before going any further, it is important to delineate the types of deflections existing in the test setup since they make up a majority of the predictable error. Axial deflection was determined to be the dominant source of error in the 5kN flexure fixture. Hibbeler [2] explains the axial loading process experienced in most components of the 5kN flexure fixture by saying, "In many cases, the bar will have a constant cross-sectional area A; and the material will be homogeneous, so E [the Modulus of Elasticity] is constant. Furthermore, if a constant external force is applied at each end, Figure 5, then the internal force P throughout the length of the bar is also constant." Equation 1 (models the deflection that occurs in an axially loaded member. It was used to determine many of the individual deflections found in the 5kN flexure fixture under load.

The deflection of one member of the 5kN flexure fixture, as well as the test sample, are not represented by Equation 1. These components can be modeled as a simply supported beam

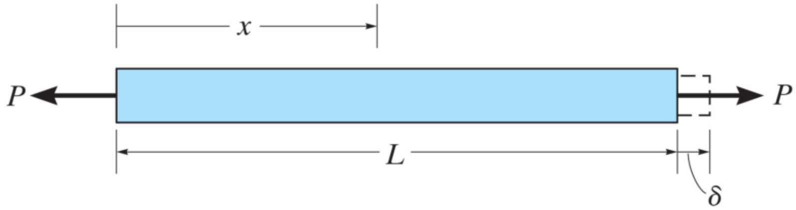


Figure 5 - Elastic deformation of an axially loaded member [2].

$$\delta = \frac{PL}{AE} \tag{1}$$

Where:

δ	=	displacement of one point of the bar relative to another point
L	=	original length of the bar
P	=	internal axial force at the section
A	=	cross-sectional area of the bar
E	=	modulus of elasticity for the material

with an idealized concentrated center load. Hibbeler [2] explains loaded beam analysis as, "Using tabulated results for various beam loadings, it is therefore possible to find the slope and displacement at a point on a beam subjected to several different loadings by algebraically adding the effects of its various parts." If there is only one load on a beam, there is no need to use a superposition method. And, if that load is located centrally on the beam, a simpler form of the deflection equation results. The deflection equation (Equation 2) for this condition can be found in tables (Figure 6) or derived. The maximum deflection could then be determined by substituting the necessary variable values.

$$\delta = \frac{PL^3}{48EI} \tag{2}$$

Where:

δ	=	maximum displacement of the beam compared to its unloaded state
L	=	original length of the bar
P	=	external load acting at the center of the beam
I	=	moment of inertia of the beam
E	=	modulus of elasticity for the material

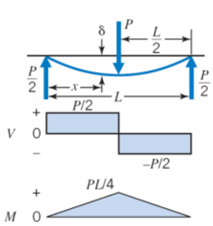
	Slope at Ends, θ	**Maximum Deflection, δ_{max}**	**Deflection δ at Any Point x**
1. Concentrated center load 	$\frac{PL^2}{16EI}$	At center: $\frac{PL^3}{48EI}$	For $0 \le x \le L/2$: $\frac{Px}{12EI}\left(\frac{3L^2}{4} - x^2\right)$

Figure 6 - Tabulated equations for a concentrated center loaded simply supported beam [4].

The maximum deflection equation found in most tables is only part of the actual maximum deflection equation since there exists a second term (Equation 3) found through derivation using Castigliano's method (or others).

$$\delta = \frac{PL^3}{48EI} + \frac{3PL}{10GA} \tag{3}$$

Where:

A = cross-sectional area of the beam

G = modulus of rigidity for the material

It is not that there is an error in the textbook, but it was determined by the authors that the second term wouldn't have much of an effect on the final answer. In their textbook, Juvinall and Marshek [4] demonstrate that this is the case and further bolster their opinion by stating "For rectangular-section beams of length at least eight times depth, transverse shear deflection is less than 5 percent of bending deflection." Still, by removing that second term, the model is no longer an idealization model, it becomes an approximation model. Since the deflections in this lab are on such a small order of magnitude, considering the more exact maximum deflection was appropriate.

Having defined how the deflection of the test specimen could be theoretically obtained (Equation 2), the method used to determine the specimen's modulus of elasticity could be presented. The student must follow a prescribed process (Table 1) to complete the lab. Using this method, the student could complete one of the main outcomes of the experiment by calculating the measured modulus of elasticity value from experimental 3-point bending data.

Table 1 - Lab procedure to determine modulus of elasticity.

1. Determine the necessary properties:
   a. Width of beam: b
   b. Height of beam: h
   c. Theoretical modulus of elasticity for the beam material: E
2. Complete beam characteristic calculations:
   a. Calculate moment of inertia: $I=b\cdot h^3/12$
3. Determine the measured spring constant of the material
   a. Determine Δy from the linear portion of the given Force vs Deflection graph
   b. Determine Δx from the linear portion of the given Force vs Deflection graph
   c. Determine k_{meas} by calculating the linear portion's slope $\Delta y/\Delta x$
4. Calculate the modulus of elasticity by rearranging Equation 2 and substituting k_{meas} for P/δ
   a. $E_{calc} = (L^3\cdot k_{meas})/(48\cdot I)$

Method

The methodology to determine an error correction equation consisted of two parts. The first dealt with determining the predictable internal displacements occurring within the 3-point flexure fixture members by calculating the deflections produced during experimental loading. The second was to obtain the repeatable errors that existed in the remaining experimental setup through experimentation. Combining the findings of the predictable and repeatable displacements would form the error correction equation to be applied to data resulting from 3-point bending tests.

5kN Flexure Fixture Displacements

The reasoning behind investigating the 5kN Flexure Fixture displacements is explained by Hibbeler [2]. "Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes are referred to as deformation, and they may be either highly visible or practically unnoticeable." So, even though deflections in the 5kN Flexure Fixture would be unnoticeable to the human eye, they would likely affect the Instron's extension data. Determining the internal displacements of the fixture required that the dimensions of the flexure test components be known. These dimensions would both aid in hand calculations and the three-dimensional modeling of the system. Using a digital dial caliper and a standard ruler, the necessary measurements of the 5kN Flexure Fixture components (shown in Figure 1 and Figure 3) were recorded in SI units (Figure 7).

Hand calculations to determine the internal deflections within the 5kN Flexure Fixture (Figure 9) for every component between the load cell and the Instron base plate was performed. All components of the flexure fixture were made of steel, therefore a modulus of elasticity of $2x10^{11}$ Pascals was used for these calculations. The axially loaded member's deflections (circled in red) were calculated using Equation 1 while the deflection for the lower anvil assembly experiencing bending (circled in yellow) was calculated using Equation 3. The eight individual deflections (Figure 8) were then summed to create the deflection equation (Equation 4).

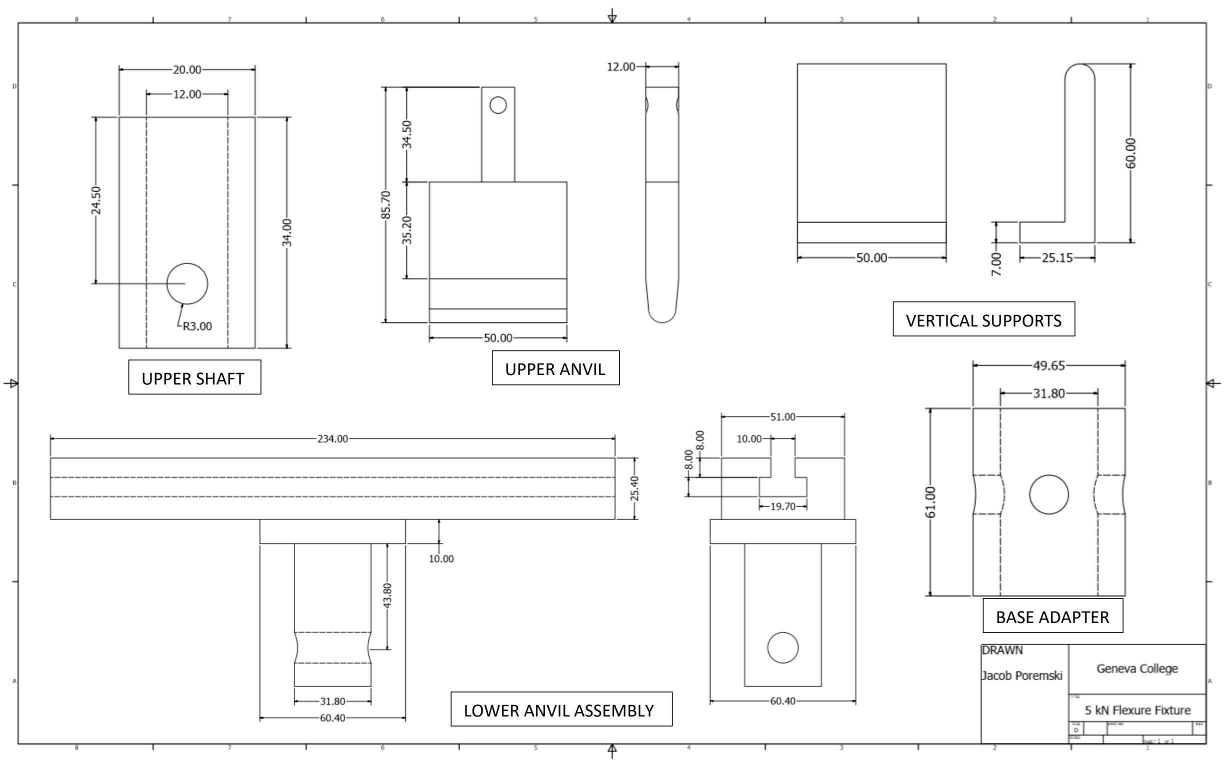


Figure 7 - Dimensions of 5kN Flexure Fixture components in millimeters.

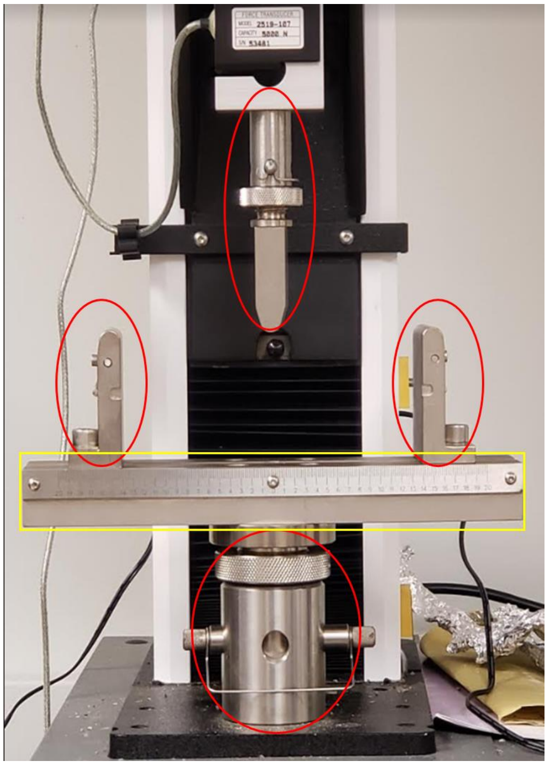


Figure 9 - 5kN Flexure Fixture.

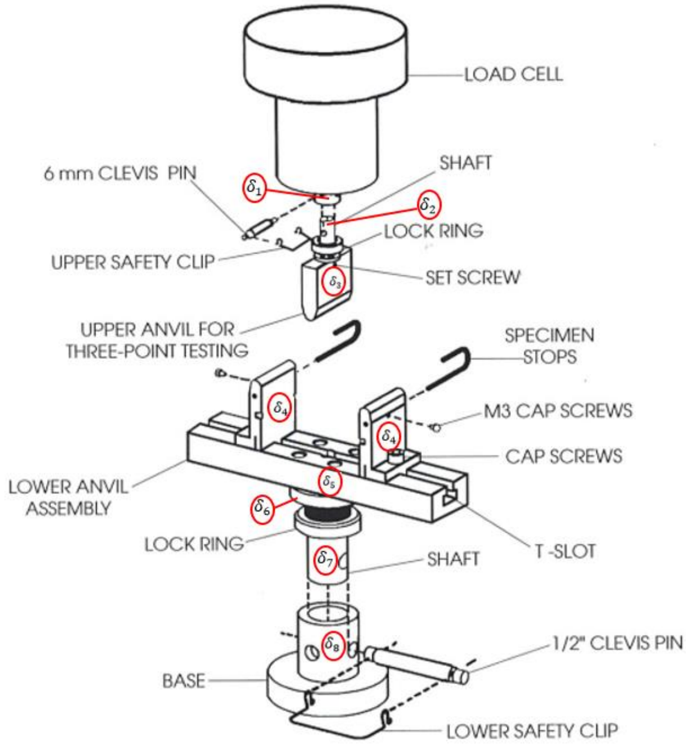


Figure 8 - Individual deflections considered [6].

All deflections except δ_5 were due to axial loading, therefore the individual deflections could be determined using the total load being applied as well as the cross-sectional area, length of the component, and its modulus of elasticity. Deflection δ_5 was due to the lower anvil assembly being loaded in bending as a center loaded simply supported beam, so the calculation additionally required the moment of inertia and the modulus of rigidity.

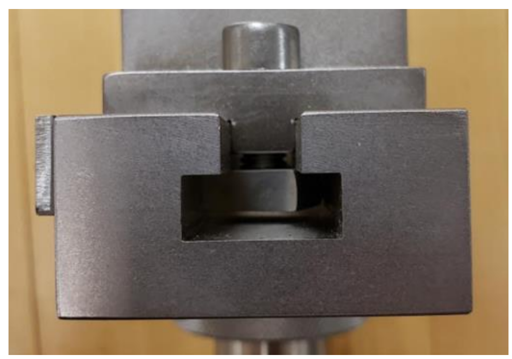


Figure 10 - Lower Anvil cross-section.

$$\delta_{total} = \sum_{n=1}^{8} \delta_n \tag{4}$$

The total load for each component was adjusted to include the weight of the components above it to more accurately calculate the predictable error. The moment of inertia for the lower anvil cross-section (Figure 10) was calculated using a composite body method. The modulus of rigidity for steel is generally known, but it can also be calculated (Equation 5) from the modulus of elasticity if Poisson's ratio is known (generally accepted to be 0.3 for most steels).

The summation of all eight deflections (Equation 4) produced an equation with three independent variables: the applied load P, the weight of the test sample W_{Bar}, and the span of the vertical supports L. All non-variable factors in the summation, consisting of geometric and material properties, became coefficients for the variables of the simplified 5kN Flexure Fixture correcting equation (Equation 6).

$$G = \frac{E}{2(1+v)} \tag{5}$$

Where:

G = the modulus of rigidity

E = the modulus of elasticity

v = Poisson’s ratio

$$\begin{aligned} 5kN\ Flexure\ Fixture\ Correction \\ &= 1.67 * 10^{-6} PL^3 + 1.67 * 10^{-6} W_{bar} L^3 + 1.57 * 10^{-5} L^3 \\ &+ 3.69 * 10^{-9} PL + 3.69 * 10^{-9} W_{bar} L + 3.46 * 10^{-8} L + 2.98 \\ &* 10^{-9} P + 7.47 * 10^{-10} W_{bar} + 8.87 * 10^{-9} \end{aligned} \tag{6}$$

To verify this correction equation, a bar that was unlikely to bend due to a very high moment of inertia (weighing 75.4978 N) was chosen to be the test specimen (Figure 11) for both the hand calculation and an ANSYS Workbench analysis (Figure 12).

The force placed on the top face of the 5kN Flexure Fixture was 4000 N. Standard Earth gravity was applied so that the weight of the bar and the individual components figured into the final deflection solution since the correction equation considered them as well. ANSYS Mechanical was set up to solve for two directional deformations: the y-axis for the entire body and just the top face. The top face was where the deflection values would be comparable to the correcting equation results. When the solution was complete, contour plots showing the directional deformation of the body (Figure 13) and the top face (Figure 14) were produced. The result of the comparison of the top face directional deformation with the error correction calculation was then performed (Table 2). The maximum percent error between ANSYS and the hand calculation values was 1.598% with the average percent error being 0.457% validating this correction equation component.

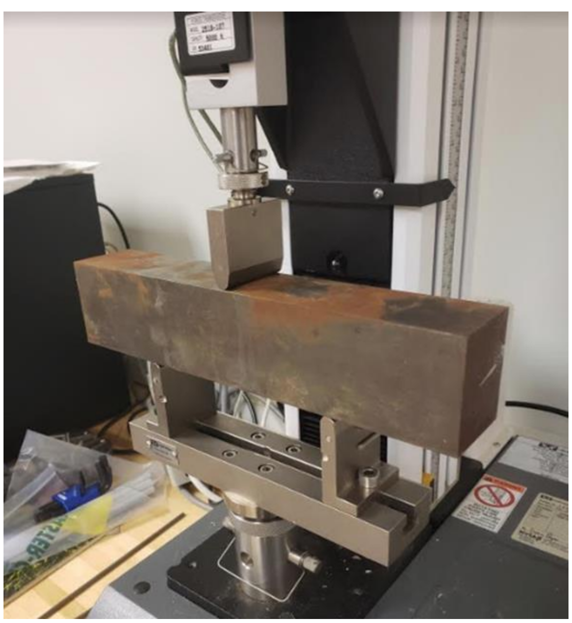


Figure 11 - Verification setup.

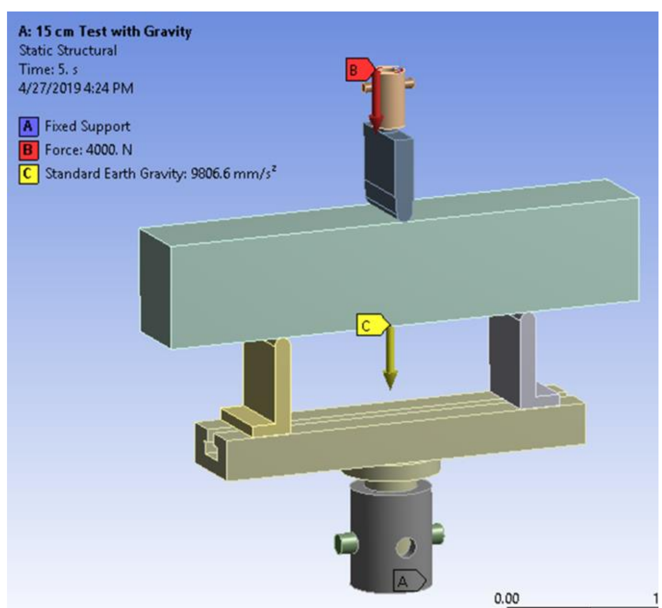


Figure 12 - ANSYS model boundary conditions.

Table 2 - ANSYS and Hand Calculation Comparison.

Hand Calculation Verification of Results									
Hand Calculation (mm)	0.039306	0.039306	0.039306	0.039306	0.039306	0.039306	0.039306	0.039306	0.039306
ANSYS Result (mm)	0.039934	0.039438	0.039278	0.039372	0.039754	0.039415	0.039125	0.039282	0.039772
Percent Error	**1.598**	**0.336**	**0.071**	**0.168**	**1.140**	**0.277**	**0.460**	**0.061**	**1.186**

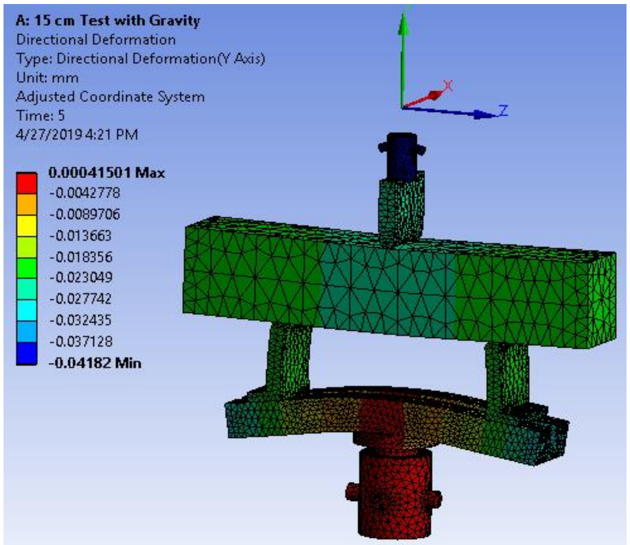


Figure 13 - Total system directional deformation.

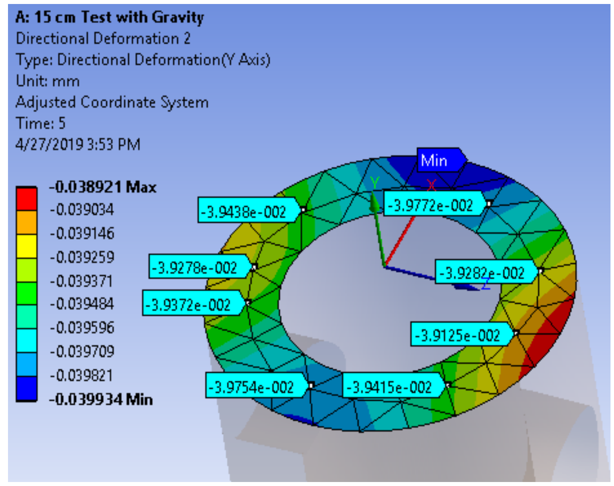


Figure 14 - Top face directional deformation.

Repeatable Errors

With the 5kN Flexure Fixture deflections accounted for, the remaining repeatable deflections in the system needed to be determined. An experimental setup (Figure 15) was generated to determine the remaining system deflection. The bar of very high moment of inertia was once again utilized since its internal deflections due to axial loading could be considered negligible due to its large cross-sectional area. Due to the geometric trait that the compression test plate's diameter was large compared to its

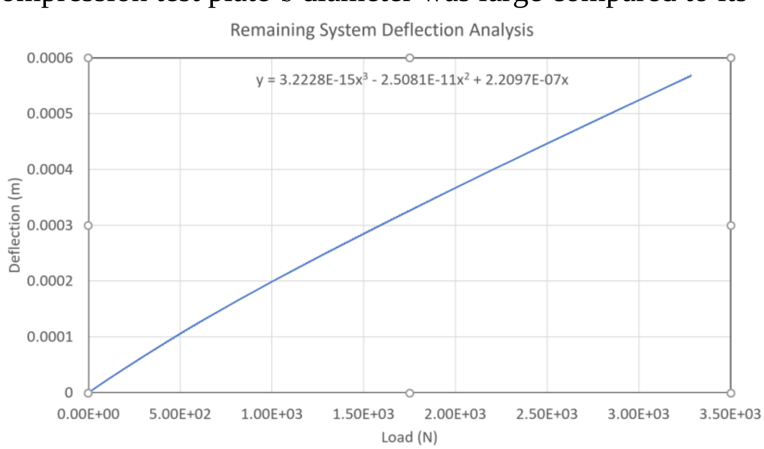


Figure 16 - Remaining system deflection analysis results.

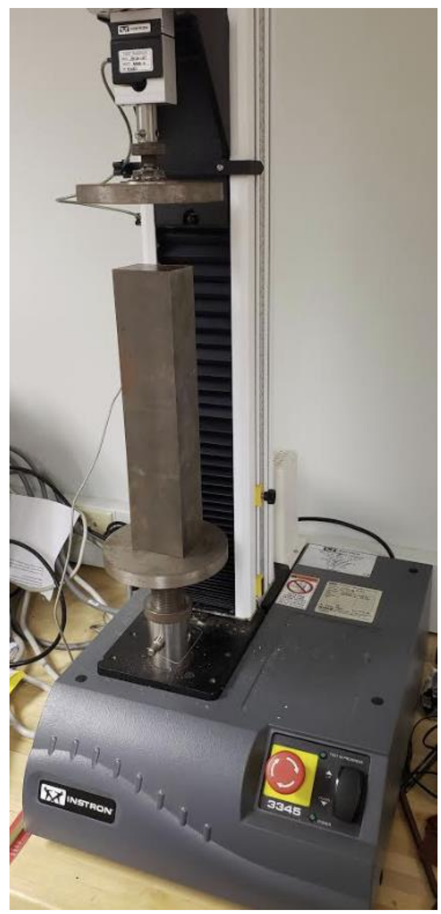


Figure 15 - Experimental setup for remaining system deflection.

height, its internal deflections could also be deemed negligible. Therefore, most of the reported deflection should be from the remainder of the system. It was expected that the remaining repeatable system deflection would be comprised of the deflection in the load cell and the Instron's column. Due to the nature of the suspected remaining deflections, it was suspected that the results would be nonlinear, and a cubic polynomial approximation appeared to fit best. Also, given that there should be no deflection when there was no load, a zero intercept was specified. The resulting deflection data was processed, and a zero-intercept linear regression analysis was performed (Figure 16) to generate the remaining system correction equation (Equation 7).

$$\begin{aligned} &\mathit{Remaining\ System\ Correction} \\ &\qquad = 3.2228 * 10^{-15}P^3 - 2.5081 * 10^{-11}P^2 + 2.2097 * 10^{-7}P \end{aligned} \tag{7}$$

Results

With the total system deflection analysis completed, the total system correction equation (Equation 8) was created by summing Equations 6 and 7. The impact of the test specimen's weight was investigated by letting $W_{bar} = 0$. Since the test specimen would not typically weigh more than 2 N, and since doing so resulted in a difference approximately equal to 0, a simplified total system corrections equation resulted (Equation 9).

$$\begin{aligned} &\mathit{Total\ System\ Correction} \\ &\qquad = 1.67 * 10^{-6}PL^3 + 3.22 * 10^{-6}P^3 + 1.67 * 10^{-6}W_{bar}L^3 \\ &\qquad + 1.57 * 10^{-5}L^3 - 2.51 * 10^{-5}11P^2 + 3.69 * 10^{-9}PL + 3.69 \\ &\qquad * 10^{-9}W_{bar}L + 3.46 * 10^{-8}L + 2.24 * 10^{-7}P + 7.47 \\ &\qquad * 10^{-10}W_{bar} + 8.87 * 10^{-9} \end{aligned} \tag{8}$$

$$\begin{aligned} &\mathit{Total\ System\ Correction} \\ &\qquad = 1.67 * 10^{-6}PL^3 + 3.22 * 10^{-6}P^3 + 1.57 * 10^{-5}L^3 - 2.51 \\ &\qquad * 10^{-11}P^2 + 3.69 * 10^{-9}PL + 3.46 * 10^{-8}L + 2.24 * 10^{-7}P \\ &\qquad + 8.87 * 10^{-9} \end{aligned} \tag{9}$$

A graphical representation of the total system corrections equation was generated (Figure 17) using MATLAB to investigate the sensitivity of the correction equation to the different independent variables. As expected, the correction equation is more sensitive to load than the span of the vertical supports.

With the final total system correction equation developed, verification was sought through experimental testing. Several materials were selected for testing

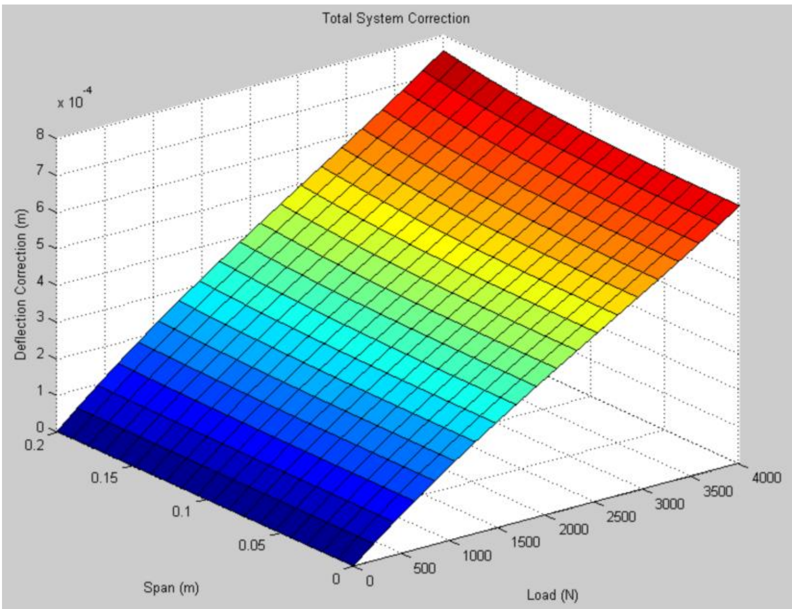


Figure 17 - 3D Plot of Total System Correction Equation.

Table 3 - Test Sample Geometric and Mechanical Properties.

Material	Width (in)	Thickness (in)	Modulus of Elasticity (psi)
4140 Steel	0.25	0.25	2.97×10^7 [5]
	0.25	0.5	
	0.5	0.25	
6061 T6 Aluminum	0.25	0.25	1.00×10^7 [5]
	0.25	0.5	
	0.5	0.25	
360 HO2 Brass	0.25	0.25	1.41×10^7 [5]
	0.25	0.5	
	0.5	0.25	

with different geometric and modulus of elasticity properties (Table 3) to be evaluated at different vertical spans to determine how close the correction equation modified data predicted the test specimen's known moduli of elasticity.

The testing sequence desired was as follows:

1. Set vertical support span to 10 cm
2. Bend 9 test samples (three geometries of three materials) until yielding or 4000 N
3. Export data to correcting excel spreadsheet
4. Repeat steps 1-3 for vertical spans of 12 cm and 14 cm

Table 4 - Modulus of elasticity verification test results.

Material Type	Span (m)	Geometry (in x in)	Theoretical E (psi)	Original E (psi)	Corrected E (psi)	% Error Original	% Error Corrected	% Difference Original	% Difference Corrected
4140 Steel	0.1	0.25 x 0.25	2.97E+07	2.52E+07	3.24E+07	15.15	9.09	16.39	8.70
		0.5 x 0.25		2.12E+07	3.24E+07	28.62	9.09	33.40	8.70
		0.25 x 0.5		NA	NA	NA	NA	NA	NA
	0.12	0.25 x 0.25		2.71E+07	3.14E+07	8.75	5.72	9.15	5.56
		0.5 x 0.25		2.44E+07	3.25E+07	17.85	9.43	19.59	9.00
		0.25 x 0.5		NA	NA	NA	NA	NA	NA
	0.14	0.25 x 0.25		2.78E+07	3.10E+07	6.40	4.38	6.61	4.28
		0.5 x 0.25		2.62E+07	3.14E+07	11.78	5.72	12.52	5.56
		0.25 x 0.5		NA	NA	NA	NA	NA	NA
6061 T6 Aluminum	0.1	0.25 x 0.25	1.00E+07	9.17E+06	1.00E+07	8.30	0.00	8.66	0.00
		0.5 x 0.25		8.20E+06	9.66E+06	18.00	3.40	19.78	3.46
		0.25 x 0.5		5.98E+06	1.00E+07	40.20	0.00	50.31	0.00
	0.12	0.25 x 0.25		9.40E+06	9.92E+06	6.00	0.80	6.19	0.80
		0.5 x 0.25		8.60E+06	9.43E+06	14.00	5.70	15.05	5.87
		0.25 x 0.5		6.94E+06	9.65E+06	30.60	3.50	36.13	3.56
	0.14	0.25 x 0.25		9.54E+06	9.92E+06	4.60	0.80	4.71	0.80
		0.5 x 0.25		8.78E+06	9.27E+06	12.20	7.30	12.99	7.58
		0.25 x 0.5		7.64E+06	9.56E+06	23.60	4.40	26.76	4.50
360 H02 Brass	0.1	0.25 x 0.25	1.41E+07	NA	NA	NA	NA	NA	NA
		0.5 x 0.25		8.92E+06	1.06E+07	36.74	24.82	45.00	28.34
		0.25 x 0.5		6.66E+06	1.19E+07	52.77	15.60	71.68	16.92
	0.12	0.25 x 0.25		NA	NA	NA	NA	NA	NA
		0.5 x 0.25		9.33E+06	1.04E+07	33.83	26.24	40.72	30.20
		0.25 x 0.5		7.68E+06	1.09E+07	45.53	22.70	58.95	25.60
	0.14	0.25 x 0.25		NA	NA	NA	NA	NA	NA
		0.5 x 0.25		9.56E+06	1.06E+07	32.20	24.82	38.38	28.34
		0.25 x 0.5		8.41E+06	1.06E+07	40.35	24.82	50.56	28.34

Two of the material/geometry combinations did not yield useable results. The 0.25 x 0.5 4140 steel specimens were too stiff to give useable results. In retrospect, milder steel should have been chosen so that useable results could have been obtained from this geometry. The 0.25 x 0.25 360 H02 brass specimens did not appear to be the same material as the other two brass geometries since the modulus of elasticity was half that of the others and none were close to what was expected from the specific material ordered. This was possibly due to a quality control issue. These values were all reported as N/A in the verification of test results (Table 4).

The percent error improved by 13.98% (on average from an original percent error of 23.21% to a corrected percent error of 9.92%). The percent difference improved by 17.02% (on average from an original percent difference of 27.79% to a corrected percent difference of 10.77%). Overall, a reasonable improvement. If one were to discount the brass tests (due to suspected quality control issues regarding the material received), the percent error improvement would be 11.8% (on average from an original percent error of 16.40% to a corrected percent error of 4.62%) and the percent difference improved by 13.99% (on average from an original percent difference of 18.55% to a corrected percent difference of 4.56%). Much better results, as the remaining errors were under 5%.

Several other observations were obtained from the results of this verification testing. The first being that the Load–Deflection curves (Figure 18) showed linear segments whose slopes were the spring constant (k) for that material and geometry combination. Each of the three test specimens for this material and span showed different k values because they had different stiffnesses. Second, the stiffer the specimen, the more error there was to be corrected.

Next, the corrected curves show a higher modulus of elasticity (slope) and lay close to each other (Figure 19). The uncorrected curves show that the stiffer the specimen, the lower the slope value. Only the linear portion of the curves from the load-deflection results was used for this graph. Note: the horizontal axis represents the deflection modified by a constant determined by Equation 10 and has the units of in^2, which when combined with the vertical axis units of lbf, results in the slope of the curves having the units of psi, for modulus of elasticity.

Lastly, with the material and specimen geometry held constant, the effect of the span could be observed (Figure 20). As with the previous results, the corrected curves were very close to each other. The remaining

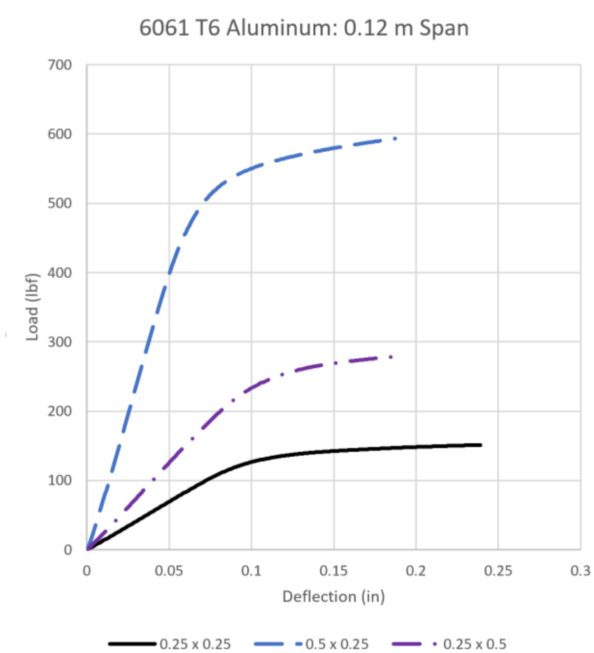


Figure 18 – Load-deflection curves.

$$Adjusted\ Deflection = \delta * \frac{48 * I}{L^3} \tag{10}$$

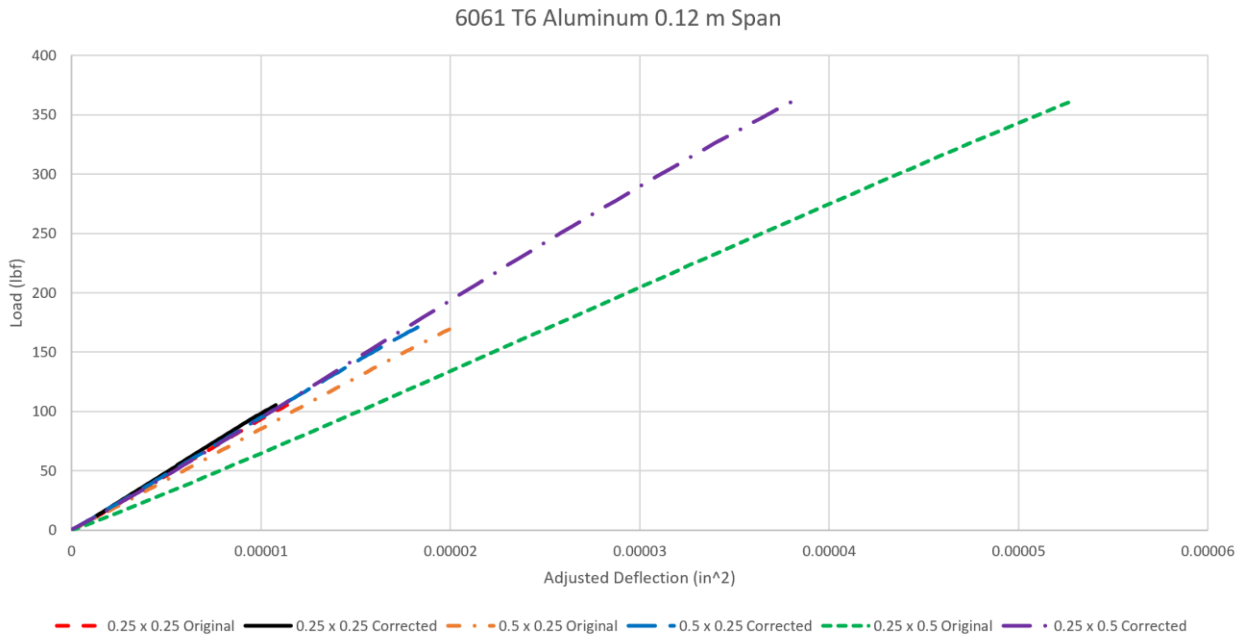


Figure 19 - Modulus of elasticity slopes (original and corrected).

curves had a slope whose values became lower as their span decreased. This was to be expected since the specimen becomes harder to bend and thus behaved as if it was stiffer.

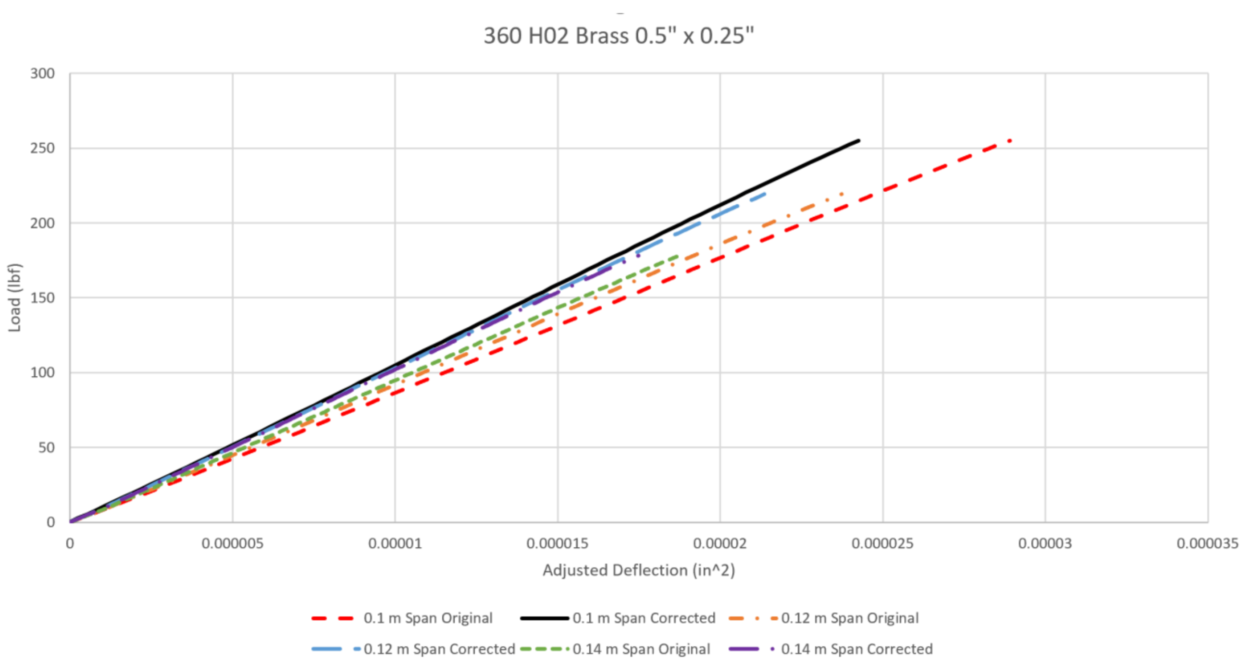


Figure 20 - Modulus of elasticity slopes (original and corrected) for various spans.

Discussion

This verification process showed that an error still existed between theoretical and experimental modulus of elasticity values. This trait of the data was noted but no additional correction was applied to further address the remaining discrepancies. The reasoning behind this decision was that if all the error was accounted for, the students would have no error analysis section in their lab reports. The originally identified purpose of the developed correction equation was to correct most of the errors in the testing system (the predictable and repeatable errors, specifically). Therefore, the ability of the student to correctly determine the unknown material used during testing would be substantially improved using this correcting equation. The minor error remaining could be due to errors in the load cell, the weight of the specimen, variation in the specimen's length during deflection, and others. It remains for the student to acknowledge the existence of these minor errors and report them in their findings along with their reasoning behind the differences between theoretical and measured modulus of elasticity values.

Conclusions

A correction equation was found that reduced errors existing in testing data from an Instron 3345 Single Column Universal Testing System with a 5 kN Static 3 Point Flexure Fixture to under 5%. This allows the student to more accurately determine the modulus of elasticity and, thus, determine the material (or its alloy) being tested. Introducing this type of experiment to the engineering mechanics curriculum would enhance a student's ability to obtain the course objectives by allowing them to demonstrate their level of knowledge of the strengths of materials.

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