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# System Analysis Methodology for Teaching Algebra: A Foundation in Engineering Education

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# System Analysis Methodology for Teaching K-12 Algebra: An Instrument for Introducing the Engineering Design Approach in K-12 Education (<u>Fundamental</u>)

***Abstract*****—**The public school system in Virginia is implementing more challenging standards and assessments to ensure that all of their graduates possess the knowledge and skills needed for success in college and the workplace. One of the standards calls for increased rigor through an emphasis of solving multistep problems and applications. The implementation of this approach will require a change of thinking in the way the teachers approach subject matter for preparing students to study engineering and other related fields. We proposed a 'system analysis' approach as a model solution to address developing and teaching word problems in algebra courses beginning at the middle school level. The initial implementation of the approach was tested during teachers' circle workshops and 2-week summer institutes for mathematics, technology, and art middle school teachers from three school divisions in the southeastern region of Virginia. In addition, mathematics and engineering/technology faculty members from community colleges were also part of the institute. This paper focuses on the development of the system analysis approach for teaching algebra using bridge design and shipping terminal design problems as applications. Our approach illustrates to teachers how to solve word problems algorithmically using equations and variables, and how to teach this method to their students. It demonstrates how an engineering methodology can be applied effectively in K-12 education. As a result of the program, the teachers could teach their students the difference between true problem solving and the trial-and-error approach.

***Index Terms*****—Algebra, problem solving, system analysis.**

## Introduction

Strategies to boost mathematics understanding to help students prepare for careers in STEM fields are a necessity due to a deepening problem over the years, culminating in many college freshmen matriculating without sufficient tools to succeed in college[1]. Recently, the Commonwealth of Virginia started testing students on more rigorous content standards to meet national and international benchmarks for college-and-career readiness in mathematics among other subject matters as part of preparing students to compete in today's global economy[2]. The commonwealth's standards such as solving real-world problems involving equations and systems of equations in algebra are comparable to common core standards such as "creating equations that describe numbers or relationships and understanding solving equations as a process of reasoning and explain the reasoning"[3].The envisioned level of knowledge for solving real-world problems requires an integrative engineering approach that applies the fundamentals of mathematics.

The approach discussed in this paper focuses on applying a basic engineering method, namely, 'system analysis' to avoid a trial-and-error approach for obtaining a solution to a technical design of a real-world problem. The trial-and error method does not allow for the analysis of the system and to reach conclusions on how independent variables affect the performance of the system, resulting in an inefficient use of time. But the approach being presented takes the design and/or problem-solving process from a blind-folded endeavor to a systematic, efficient process. It focuses on teachers gaining deeper understanding in representing a problem and its constraints by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context; rearranging formulas to highlight a quantity of interest, using the same reasoning as in solving equations; and understanding solving equations as a process of reasoning and explaining the reasoning as required by the algebra standards of learning.

This approach also supports the 'Next Generation Science Standards' (NGSS)[4] which include a commitment to integrate engineering design into K-12 science education. The purpose of this inclusion has been clearly described in 'A Framework for K-12 Science Education'[5] where it states that students are expected to be able to define problems - situations that people wish to change by specifying criteria and constraints for acceptable solutions; generating and evaluating multiple solutions; building and testing prototypes; and optimizing a solution[4]. These goals are exactly the same as goals that are addressed in this work.

## Problem-Solving Approaches

In 2013, the Governor of the Commonwealth of Virginia proposed an algebra readiness initiative algebra intervention service for students in grades 6,7,8 and 9 who are at risk of failing the Algebra I end-of-course test because it is generally understood that algebra is a key indicator for success in STEM disciplines[6,7]. Engineering departments are struggling with students matriculating in college without the fundamental math skills expected from students who passed Algebra in high school. Some colleges have ventured to teach special engineering math courses for college freshmen; others still resort to summer refresher and remedial courses, whilst various entities including pre-college institutions are working on integrating technology as a way of interactively addressing the fundamental gap[1, 8]. In most of the activities, the general consensus in the approach to teaching algebra and other subject matter is to add more rigor in preparing students for college and career as highlighted in new standards of learning where the emphasis is on the substance of content. Additional teacher training or professional development is required to adapt teachers to the new rigor in standards and deepening of content knowledge[9,10,11,12]. The most important skills that a student is expected to acquire in the algebra courses is problem-solving and algorithmic thinking. Klein[13]mentions that few states in the US offer clear guides in the development of problem-solving and mathematical reasoning skills, also, stressing that there is a lack of attention to derivations and proofs in mathematics courses. In recent years, the introduction of the Common Core attempts to address these shortcomings. In the Commonwealth of Virginia, elementary school students are required to memorize basic number facts, but the elementary standards fall short in word problems and algorithms, and there is too much emphasis on calculators[13]. Although two sixth-grade standards require the students to solve multi-step problems, there is scant reference to word problems in the standards[13]. Additionally, the emphasis is on numerical calculations, while omitting the requirement for coverage of the standard algorithms, which are essential in understanding mathematical concepts[13]. This practice prevents the progression to mathematical reasoning and algorithmic thinking. The commonwealth has not accepted the Common Core, but it made extensive changes to the learning standards in 2009; these new standards also brought new evaluation measures for the teachers. The problem-solving skill helps develop and strengthen the reasoning skill. An efficient method such as the ‘system analysis’ approach is needed to cultivate effective problem solving skills. This approach emphasizes a path to algorithmic reasoning and integration of mathematics with other subjects to infuse real-world problems in teaching algebra. In algebra classes, students experience difficulty in recognizing how to initiate the problem-solving process because a systematic approach is not introduced as a tool; methods like brain-storming, trial-and-error, etc., are employed[14]. However, ‘system analysis’ approach provides a roadmap to the problem-solving process, by which the reasoning activity is streamlined and generalized. This method may also be employed to demonstrate to the students how to formulate either word problems or a design basis. It also helps the teachers to show to their students how to switch from rote memorization to critical thinking.

**‘System Analysis’ Approach**

The system analysis approach considers the definition of a system as composed of components which interact with each other to produce output(s) based on inputs. The best way to define the behavior of a system is to define the variables which govern the behavior of the system. Some of these variables are independent (or input) variables while others are dependent (or output) variables, whose values depend on the values of the independent variables. To comprehend how a system behaves is to express mathematically how the values of the independent variables affect the values of the dependent variables. The resulting mathematical expressions are referred to as model equations for the system. The analysis of engineering, technology, and physical systems is possible by using those model equations. For example, Newton’s second law of motion is a model equation: $F = m*a$, which explains that mass and acceleration are the variables (independent) which determine the force (dependent) acting on an accelerating body (system). The general system analysis steps to problem solving are outlined in Figure 1.

The initial step is producing a diagram illustrating the system and clearly delineating all the input/output variables (V) that affect the behavior of the system and listing all the pertinent independent equations (E) between the variables. The difference in the number of variables and independent equations establishes the number of degrees of freedom (DoF) in the system. The DoF value can be used as a check point on the mathematical reasoning or formulation of the problem. If the DoF is zero, the solution of the problem is determinate with only one solution; but if $V > E$, there may be several alternative solutions, which defines the problem as a design case. However, if $V < E$, it implies an inconsistency in mathematical formulation. The proposed approach is used as a model to address developing and teaching word problems in algebra beginning at the middle school level. The implementation involves learning to formulate a system of equations and then finding the solutions algebraically. The initial implementation of the approach was tested during a 2-week summer institute for mathematics, technology, and art teachers from three school divisions in the Hampton Roads region of Virginia. The approach was demonstrated to the teachers by using two engineering design problems, namely, a bridge design and a shipping terminal design, which were solved algorithmically using ’system analysis’, and demonstrating clearly the underlying principles for the application of mathematical methods in the development of design projects. These were named as ‘Algebra-in-Action’ projects, which may also be used in the context of project-based learning. Thus, they will provide a deep satisfaction of solving a complex problem resulting in building confidence in tackling word

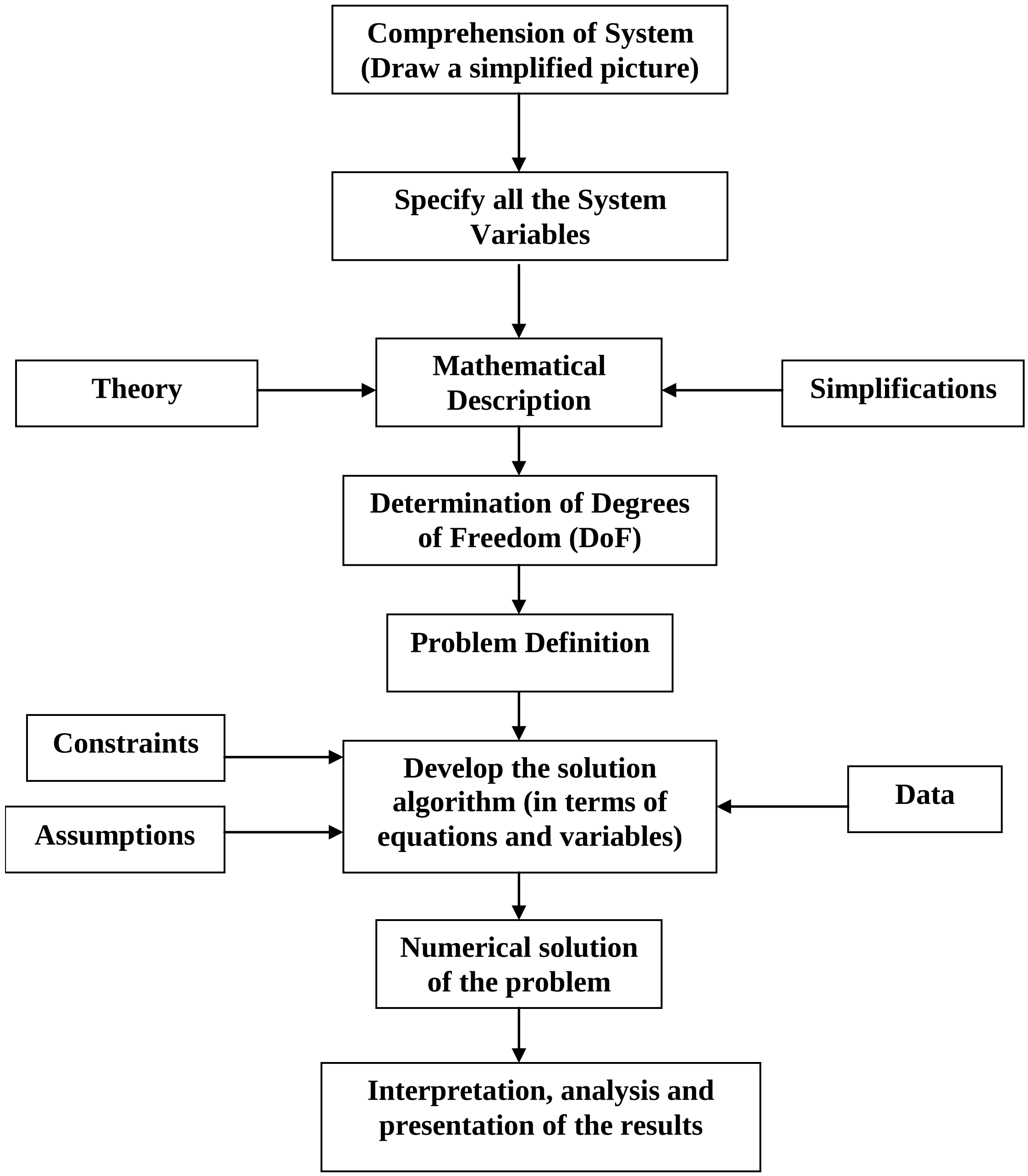



Figure 1: ‘System Analysis' Methodology as a Problem-Solving Strategy

problems. The problems were initiated by a short introduction to the underlying engineering principles for each subject. Subsequently, the teachers were presented with several word problems relevant to the design problem at hand, which they solved by practicing ‘system analysis’. As shown in Figure 1, the first step is drawing a simple diagram representing the system, where the art teachers worked with the algebra and technology teachers.

### *A. Bridge Design Problem*

The application of the system analysis method to problem solving was introduced to the teachers by a truss bridge design problem. An analysis of the bridge design problem in terms of the pertinent variables and equations established from the balance of force equations allowed for an algebraic analysis. The development of the bridge problem considers the introduction of background information on bridge types (e.g. arch, truss, suspension, etc), but focuses on the truss bridges, since the modeling of such structures are considered to be more convenient and manageable for secondary school students who will eventually receive instruction from teachers under training. The scope of the bridge design encompasses the determination of the bridge type, bridge span (S) and width (w), truss height (h), the number of beams (n), number of joints (j), length of diagonal beams (a), length of lower chords (L), shape and dimension of beam cross-section, and the material of construction as illustrated in Figure 2 using the Warren truss bridge type. The constraints on such a system are traffic load, weather, environmental, and geological conditions; and the objectives of the design may be to achieve the best safety conditions and the lowest cost. The safety factor is established by the maximum weight/force the bridge structure can support at failure condition.

The system analysis methodology establishes relevant equations from mathematics, physics, and material science. The relationship of the bridge dimensions in Figure 2 is established from trigonometry and the Pythagorean theorem such that:
$sin\theta = \frac{h}{a}$; $cos\theta = \frac{L}{a}$; $tan\theta = \frac{h}{L}$; and $a^2 = h^2 + L^2$
The mass (M) and weight (W) properties are established as M = ρV and W = mg respectively; where ρ is density, V is volume, g is gravitational acceleration ($9.8 m/s^2$). In consideration that the truss is static and 2-dimensional, free body diagrams help in developing relevant force balance equations at the joints.

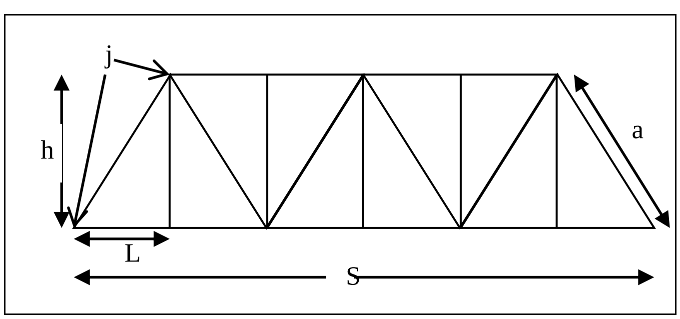



Figure 2: The Warren Truss Bridge

Based on Newton's laws, the sum of forces is zero and the truss sample in Figure 3 yields force balance equations at the four joints (illustrated in Figure 4) in addition to the overall truss force balance such that:

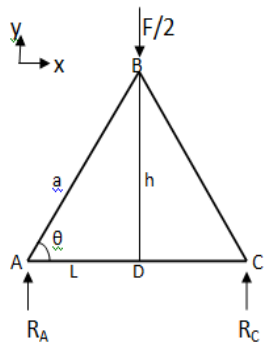
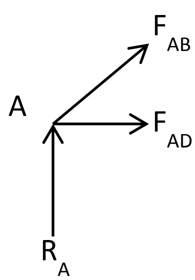


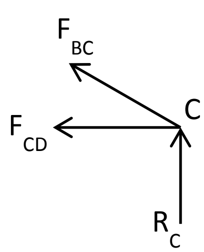

Figure 3: Truss Free Body Diagram

The overall force balance is: $R_A + R_C - \frac{F}{2} = 0$

Based on the symmetry of the truss: $R_A = R_C = \frac{F}{4}$

An independent equation for the determinate truss structure example is given by: 2j = n + 3 [13].




**Joint A**: $\Sigma F_X=0$ $F_{AD}+F_{AB}\cos\Theta=0$

$\Sigma F_Y=0$ $R_A+F_{AB}\sin\Theta=0$

**Joint B**: $\Sigma F_X=0$ $-F_{AB}\cos\Theta + F_{BC}\cos\Theta=0$

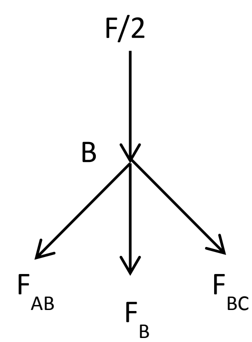
$\Sigma F_Y=0$ $-F/2-F_{AB}\sin\Theta-F_{BC}\sin\Theta-F_{BD}=0$

**Joint C**: $\Sigma F_X=0$ $-F_{CD}-F_{BC}\cos\Theta=0$

$\Sigma F_Y=0$ $R_C+F_{BC}\sin\Theta=0$

**Joint D:** $\Sigma F_X=0$ $F_{CD}-F_{AD}=0$

$\Sigma F_Y=0$ $F_{BD}=0$



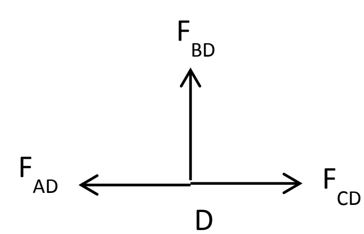




Figure 4: Force Balance Equations Using the Method of Joints

and completing the calculations at different values of the former two variables subject to the two constraints mentioned above. Using the results from the method of joints and the critical force equation, the buckling force ($F_{buckling}$) can be established. The buckling force is the background information on compression/tension forces and material properties that leads to an understanding of the beam safety conditions necessary for considering material variables. A set of equations are established for the beam critical force where buckling/failure occurs such that[15]:

$$F_{critical} = \frac{\pi^2 EI}{l^2}$$

where $E$ is Young's Modulus of elasticity
$I$ is the area moment of inertia to test buckling

Therefore, performing system analysis on a simple truss as in this example, yields 13 variables. Since trigonometry and the Pythagorean Theorem produced two equations, the DoF is reduced to 11. Variables such as the span (S), bridge width (w), F(maximum load that the bridge should carry), and a standard length bottom chord (L) will be imposed by other considerations and constraints, such as the traffic load, geographical conditions, market availability, etc., reducing the DoF to 7. When a relationship between the span to bottom chord ratio and number of beams is derived and the condition for a determinate truss structure is used, the DoF reduces to 5. Thus, it follows from the system analysis that there are 5 design variables, which implies that objective functions can be introduced to select between alternative designs. This point clearly illustrates to the teachers the implication of an 'open-ended' problem. The most plausible objective functions for this design are the strength and cost of the bridge. The acceptable strength is measured by the force on the beam that should be always less than the buckling force, while the cost can be determined directly from the mass of the truss. The 5 design variables can be selected to be truss height, size of beam cross-section, type of bridge, material of construction and shape of beam cross-section. The latter three variables are discrete, so alternative designs can be obtained by selecting them[15]:

$$F_{buckling} = \frac{4h\pi^2 EI}{a^3}$$

This approach teaches the teachers to formulate and define their own word and design problems. In demonstrating this design problem to teachers, a PASCO™ bridge set of ABS plastic material was used to design an I-beam truss bridge with a set span (0.34m), width (0.17m), and lower chord (0.17m). For the I-beam (Figure 5)[16]:

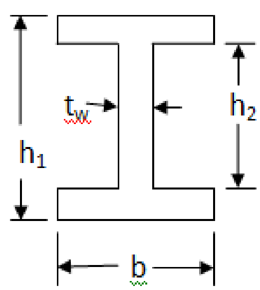



Figure 5: The I-Beam

$$I = \frac{bh_1^3 - (b - t_w)h_2^3}{12} = 6.9\text{x}10^{-10}\ \text{m}^4$$

Based on PASCO set: $h_1$=0.4" = 0.01016 m
$h_2$= 0.27" = 0.0068 m

$$t_w = 0.1\text{"} = 0.00254 \text{ m}$$
$$b = 0.4\text{"} = 0.01016 \text{ m}$$

The resultant $F_{buckling}$ is ~704 N before applying a safety factor and selecting L=0.17m. The design variables can be varied to achieve alternative designs or new word problems could be formulated based on the design problem.

Some determinate word problems were prepared based on the presentation on the bridge design, and were solved by the teachers, so that they experienced the application of algebra in a real technology problem. An open-ended word problem was also introduced, and was solved by the teachers as the final problem on the topic. This exercise allowed them to solve simultaneous algebraic equations in real variables, and observe the emergence of different solutions by varying the values of the input variables. Based on this design problem, the teachers constructed a PASCO bridge structure and made force measurements on it, comparing the results with their calculations. They discussed other possible solutions if they had beams of different size and cross-section in the PASCO set. They also prepared new word problems by employing the introduced 'system analysis' methodology to be used in their classrooms.

*B. Shipping Terminal Design Problem*

Robotics is considered as one of the great motivators for future STEM majors especially engineering[17]. However, the mathematical modeling fundamentals do not always resonate with many participants of robotics programs because they tend to rely a lot on trial and error in designing their robots. An idea for the application of robotics was sought out and the shipping terminal design problem was developed as a demonstration platform for robotics. Since the robotics applications have a lot of trigonometry, the design problem focused on algebra. The shipping terminal design problem was developed as a task for teachers to determine the maximum capacity of storage for a shipping terminal design where robots will be used. They related equations and variables based on areas and dimensions for different components of the terminal design. The goal of the project was to reiterate the application of 'system analysis' approach to solving design problems as identified in the bridge design. The shipping terminal design was defined as a maze problem. A maze is considered as a form of complex branching passages in which a correct navigational route must be found from a particular starting point to the end point, and as a result they help in testing robotics spatial and navigational control.

The design of a maze is considered to be an art form, but for navigational efficiency, it is important to gain knowledge of the types of mazes (e.g. arrow, block, logic, etc.[18]). The problem definition of the shipping terminal design is first introduced in the context of a maze design by first defining the type of maze, area and perimeter, size of maze blocks, number of blocks, pathway size (length and width), start points and end points as variables. A discussion of constraints such as maze traffic, the environment, geology, and complexity ensued as well as the objectives of maze problems such as capacity, cost, distance, time, etc. An example configuration[19] of a maritime shipping container was introduced to put in context variables and variable relationships as summarized in Table 1 below. The analogy to the maze is also illustrated in Table 2.

Table 1: Components and Variables of a Maritime Terminal Design Problem Example

| **Components of Design** | **Summary Variables & Symbols** |
|---|---|
| Terminal Area | Dimensions of the shipping terminal- {TA, L, W}; |
| Loading and Unloading area | Dimensions of loading/unloading area- {LUA} |
| Administration Area | Dimensions of administration area- {AA} |
| Container Storage Area | Dimensions of containers-{CA, $L_C$, $W_C$}<br>Dimensions of container storage slots-{CSS, $L_S$, $W_S$};<br>Capacity of storage- {CoS};<br>Dimensions of terminal pathways -{PA, PL, PW};<br>Type/Size of traffic- {TW, TL, TH} |
| Gate(s) Area | Dimensions of Gate(s) – {GA, GW} |
| Chassis Storage Area | Dimensions of chassis storage- {CSA} |
| Truck Loading and Unloading Area | Dimensions of truck loading/unloading area- {TLA} |
| Repair/Maintenance Area | Dimensions of repair/maintenance- {RMA} |
| Rail Terminal Area | Dimensions of rail terminal area- {RTA} |

Table 2: Maze and Terminal Design Analogy

| **Maze Design Consideration** | **Terminal Design Configuration** |
|---|---|
| Shape and type | Terminal area and perimeter |
| Starting points | Loading/Unloading zones |
| End points | Truck Loading/Unloading; Rail Terminal; Gates |
| Size and type of building blocks | Shipping containers |
| Pathways | Pathway width; Size of traffic (robot) |

The main theoretical considerations for such a problem are established from the area and perimeter mathematical formulations. The ‘system analysis’ is also based on the constraining relationship of storage containers and the area to store them (container storage slots) such that: $\frac{L_S}{L_C} = \frac{65}{61}$ and $\frac{W_S}{W_C} = \frac{71}{61}$; therefore, the equation to determine the capacity of storage can be derived from geometry by algebraically relating the terminal area to the components of the design (illustrated in Figure 6) as follows:

$$TA = L \times W$$

$$TA = n_S(L_S + PW)(W_S + PW) + (L \times PW) + (W - PW)PW$$

$$PA = TA - n_S(L_S \times W_S) = TA - n_S \times \left(\frac{65L_C}{61} \times \frac{71W_C}{61}\right)$$

$$CoS = TA - \sum\{LUA, AA, RMA, TLA, CSA, RTA, PA\}$$

where $n_s$ is total terminal storage slots, PA is pathway area, and CoS is capacity of storage.

Additional relevant equations can be established using information that defines other design components as fractions of other components according to design standards for the type and size of the terminal. These relationships lead to environmentally imposed variables for 'system analysis'. The shipping container's fixed width and standard length also imposes fixed and standard variables for system analysis. The design variables for this problem depend on the relationships established or defined for the different shipping terminal components.

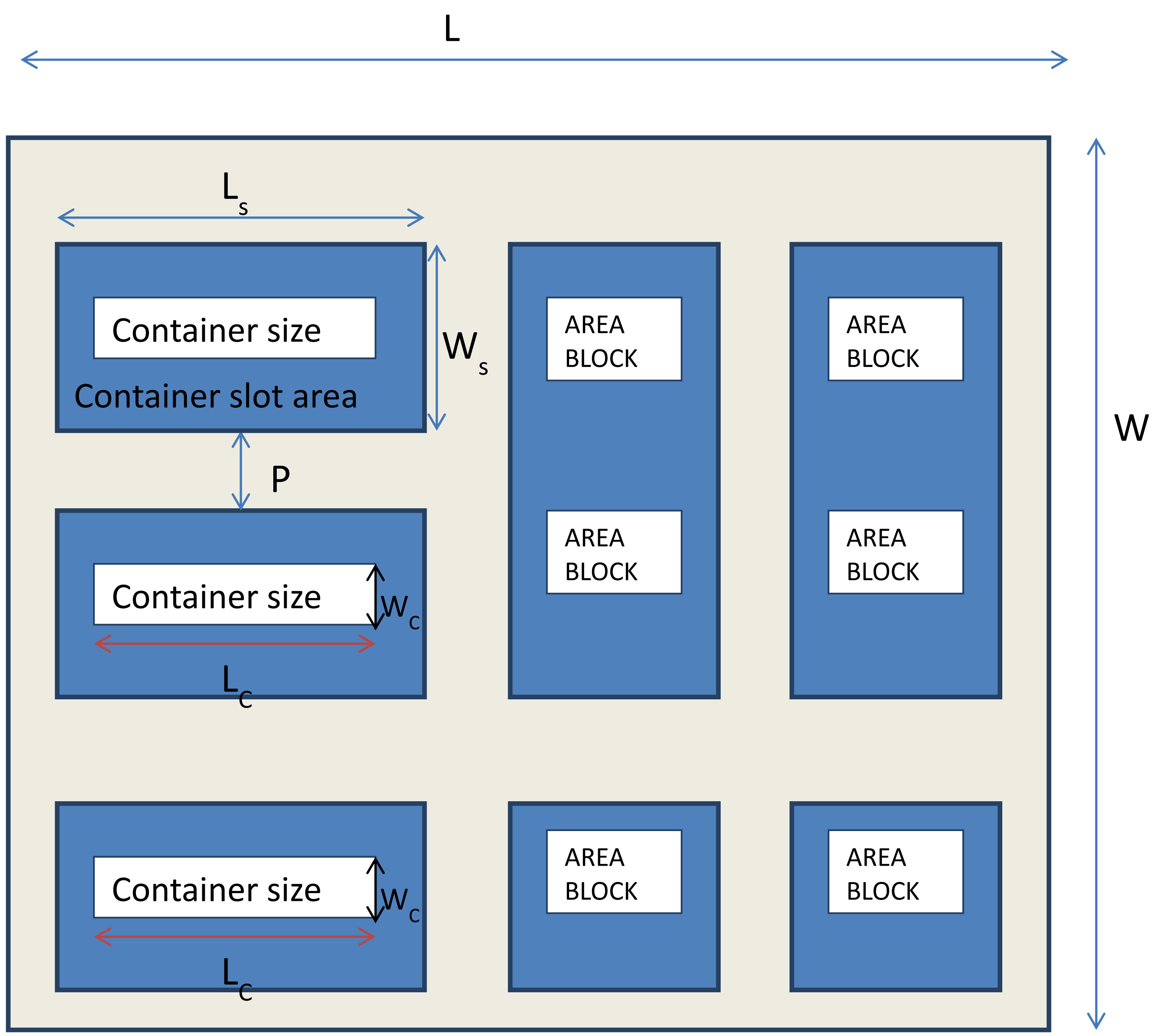



Figure 6: Illustration of Variable Relationships (Area Block represents other design components)

**Impact of the System Analysis Approach**

A total of 82 teachers from Hampton, Chesapeake, Norfolk, and Newport News school divisions participated in the Teachers' Circles along with faculty from Paul D. Camp and Thomas Nelson Community Colleges. The teachers associated with the circles were recommended by the superintendents' representatives in the partner school divisions (Hampton, Chesapeake, Newport News and Norfolk), who were also members of the leadership team. 39% of the participants were from Hampton, 17% Newport News, 30% Chesapeake, and the rest were from Norfolk and community colleges. The majority of the participants were math teachers (57%) complemented by 31% and 12% technology and art teachers, respectively. Three Teachers' Circles per academic year, of 2.5 hours duration, were held in an active learning environment. A total of 44 teachers from Hampton, Chesapeake, and Newport News school divisions participated in the Summer Institutes, including 6 faculty members from Thomas Nelson and Paul D. Camp Community Colleges. The information gained from the Teachers' Circles formed the basis for the agenda of the **Summer Institutes.** The participants were selected from among the participants of the Teachers' Circles by the respective supervisors.

The teachers were introduced to word problems and Algebra-in-Action projects as tools for problem solving so that:

- the variables that define the performance of a system are determined and analyzed;
- the systematic approach to solving algebra word problems in secondary education is incorporated in the integration of mathematics, technology and art teaching;
- the preparation of secondary school math teachers to solve word problems is enhanced through utilization of system analysis in teachers' circles and summer institutes;
- the activities of the teachers' circles and summer institutes help facilitate the development of algorithmic thinking skills;
- the teachers' understanding of the visualization of ideas and data is enhanced;
- the ability of art and technology teachers to incorporate algebraic foundations and 'system analysis' with visual representations into their courses is enhanced.

The art teachers were involved in the project to stress the importance of drawing a diagram to represent the system in a word problem. However, at this conjecture, after the implementation of six Teachers' Circles and two Summer Institutes, it is still in contention whether to continue to involve art teachers in these programs or not.

Several assessment tools were administered for the evaluation of the summer institutes and the teachers' circles, namely, Questionnaire for Evaluation of Teachers' Circles before the start of the Summer Institute, Pre- and Post- Surveys for the Evaluation of the Summer Institutes, and Daily Content and Inquiry reflections. The t-test results (Table 3) from pre/post surveys revealed that the teachers showed significant changes after participating in the Summer Institute in the use of a logical approach to explain algebra, for example;

- in demonstrating to students how to check work,
- in analyzing students' mathematical thinking skills,
- in requiring students to draw representation of word problems and encouraging students to make connections between the math curriculum and "real life",
- in encouraging students to generate alternative solution strategies and different ways of determining evidence.

There were also several items on the survey to assess teachers' perceived confidence in applying some of the problem-solving methods. The t-test results on these items indicated that the teachers exhibited significant changes in their confidence in demonstrating examples on the use of critical thinking skills, and actually providing opportunities to solve problems relating to real life situations.

**Table 3:** t-Test results for teachers' understanding and use of problem-solving methods with students (n=20)

| **Items Used to Assess Teachers' Reported Changes in the Use of Problem Solving Methods as a Result of Summer Institute Participation (on a scale of 1 (very often), 2 (sometimes), 3 (never)** | **Difference in Mean** | **t** | **df** | **Significant at .05 (in bold and italics)** |
|---|---|---|---|---|
| 1. I use a logical approach to explain algebra | .300 | 2.349 | 19 | ***.030**** |
| 2. I demonstrate to students how to check work | .000 | .000 | 19 | 1.000 |
| 3. I analyze students' mathematical thinking skills | .250 | 2.517 | 19 | ***.021**** |
| 4. I require students to draw representations of word problems | .300 | 2.349 | 19 | ***.030**** |
| 5. I encourage students to make connections between curriculum and real life | .550 | 4.067 | 19 | ***.001**** |
| 6. I discuss mathematical problems and solutions so that students can develop an analysis vocabulary | .100 | .525 | 19 | .606 |
| 7. I ask students to demonstrate more than one way to solve a problem | .200 | 1.165 | 19 | .258 |
| 8. I encourage students to generate alternative solution strategies and different ways of interpreting evidence | .350 | 2.333 | 19 | ***.031**** |
| 9. I provide opportunities for students to make predictions estimations and hypothesis | .200 | 1.710 | 19 | .104 |
| 10. I provide opportunities for students to devise means for testing their estimations | .150 | 1.371 | 19 | .186 |

Sustainability was also evident from the products developed by the teachers. At the end of the summer institute, the teachers developed word problems that would be ideal for their classes. The word problems were formulated using the problem solving knowledge and skills attained during the summer institute and were designed to be adaptable to multiple grade levels of mathematics and technology education. The teachers also had the understanding of presenting the problems as short word problems (closed solution problems) or as challenging design problems (open-ended problems). At the end of the summer institute, the participating teachers presented their formulated problems with examples such as the capacity of storage of shoe boxes in a warehouse and designing a garden based on the shipping terminal template. Other participants used the bridge design as a template to create new word problems focused on multiple-step variable manipulations to determine the values of the design variables based on different constraining factors.

The pilot efforts of both the Teachers' Circles and the Summer Institutes also resulted in teachers applying 'system analysis' in the classroom when teaching students to explore an algebra problem. The teachers communicated the connections between math and other subjects, as well as the connections between math and real life situations. The students were also taught the differences between true problem-solving and trial-and-error, and the importance of the problem-solving process in mathematics. More teachers now encourage students to work in teams; and use "open-ended" problems with multiple answers, having students design their own questions by applying the 'system analysis' approach.

**Conclusion**

A 'system analysis' approach for solving real-world problems has been introduced as a template to help teachers deepen their algebra content and problem solving skills to ultimately teach Algebra using the same approach. The approach is designed as a step-by-step 'system analysis' process for solving multistep problems as well as to aid in formulating and defining new word problems. The approach was taught to a group of algebra, technology, and art teachers in solving bridge design and shipping terminal design problems over a 2-week summer institute. Before the Summer Institute, three Teachers' Circles (2.5-hour professional learning communities) were held during the academic year, where the teachers solved short word problems and were introduced to the methodology of 'system analysis'.

The goal of this project is to change the teachers' way of thinking in solving multistep problems, so that they can carry this new skill to their algebra classes and ultimately train future engineering student majors. Several assessment tools were administered during the summer institute, incorporating questions like how and when the teachers can implement this method in their classrooms, what major problems are foreseen during implementation, etc. The answers to these questions were discussed during a session in the summer institute. A symposium was held after nine months after the first Summer Institute in 2012 to discuss how the teachers used 'system analysis' in their classrooms; how they shared this knowledge with their colleagues who did not attend the program; and the challenges they faced from the students. In summary, most of the teachers said they can use this approach in the classroom. Teachers are already working on training other teachers in understanding the 'system analysis' approach.

It is believed that this approach will impact the success of the attainment of the Next Generation Science Standards[4,5] if implemented in the algebra classrooms in parallel to the application of NGSS in science classrooms, because mathematics is the language of science and engineering. Therefore, if new Algebra-in-Action projects are prepared in the subjects covered in the science courses and are introduced as projects in the algebra courses, the goal of the integration of engineering design practices into science education[4] will be realized in a more meaningful and productive manner.

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