AC 2011-2833: TEACHING ADVANCED ENGINEERING MATHEMATICS TO GRADUATE STUDENTS: LESSONS LEARNED

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Teaching Advanced Engineering Mathematics to Graduate Students: Lessons Learned

Abstract

Advanced engineering mathematics, sometimes known as advanced engineering analysis, is a required course in many graduate engineering programs as it provides coverage of topics of mathematics required to succeed in the graduate study of any engineering discipline. The author has been teaching a core engineering mathematics class to incoming first year graduate students in mechanical engineering since 2002.

Advances in mathematical soft ware and tools make teaching the class challenging as traditional methods of teaching engineering mathematics may not be appropriate when use of mathematics software is prevalent. In addition, teaching engineering students how to apply mathematical principles and models in relevant subjects of engineering is a necessity for this kind of class.

The class covers a wide range of subjects but the major emphasis is on partial differential equations and their solutions. Other topics include vector calculus, vector integral theorems, numerical solutions of ordinary and partial differential equations, statistics and probability. This paper discusses and summarizes the lessons learned from teaching the class for many years. The experience and lessons learned may be beneficial to other faculty members developing or teaching similar classes to graduate students.

Introduction

In many engineering graduate programs in US universities, the majority of students come from outside the United States. These students received their undergraduate engineering degree from different countries with varying degree requirements. As a result, many graduate programs offer one or more graduate mathematics courses to provide requisite math skills for the success of these students in subsequent courses and research works. For example**,** ENME 700 - Advanced mechanical engineering analysis I and ENME 702 - Partial differential equations for scientists and engineers at University of Maryland, AME 525 Engineering analysis and AME 526 Engineering analytical methods at University of Southern California, and 380Q Mathematical methods in engineering at University of Texas at Austin are such courses. At Lamar University, MEEN 5304 Advanced Engineering Analysis is such a course offered to incoming graduate students. It is a core course for graduate students in mechanical engineering providing coverage of a range of mathematics topics required to succeed in the graduate studies. The class is a required course for all mechanical engineering graduate students with the non-thesis option in their Masters' degree program and a required course for all Doctor of Engineering students. In addition, students from other engineering disciplines can also take the class to fulfill their core mathematics requirement.

Background and Motivations

College of Engineering at Lamar University offers Master of Science (MS, course work option), Master of Engineering Science (MSE, thesis option), and Doctor of Engineering (DE) programs. Starting from 2006, the department of chemical engineering offers Ph.D. and D.E. programs. There are 3 core areas in the graduate program at College of Engineering at Lamar University: Mathematics, Optimization, and Control. There are five engineering departments: Civil, Chemical, Electrical, Industrial, and Mechanical, and each department offer one course in each of the 3 core areas. MEEN 5304 Advanced Engineering Analysis course is the core mathematics course for the graduate program in mechanical engineering. The course is offered every fall semester and about 40 to 60 students typically enroll in the class.

Regardless of specific engineering discipline, the students in the masters program with course work option needs to complete at least three core courses with one in each core area. The students in Doctor of Engineering program need to complete all three core courses. As a result, the students in MEEN 5304 course typically consist of both master and doctoral students and they may be in different engineering disciplines. Every year, as many as 15 to 20 students from Civil engineering department take the class to replace their math core class.

Since 2002, the author has developed and taught the course, MEEN 5304 Advanced Engineering Analysis, to first year graduate students in mechanical engineering. One of the major considerations in conducting such as a class is to choose the areas of mathematic that will be needed the most in later engineering courses for students. Another important consideration is to demonstrate engineering students how to apply mathematical principles and models in relevant subjects of engineering. In addition, advances in mathematical soft ware and tools make teaching the class challenging as traditional methods of teaching engineering mathematics may not be appropriate when use of mathematics software packages is prevalent. The paper discusses various aspects of the course in details such as the text, delivery of content, exams and student grading, and results of the students' evaluations of the course. There are several papers dealing with teaching engineering mathematics but they typically apply to teaching mathematics to undergraduate engineering students [1-3]. The main objective of the paper is to provide some lessons learned in developing and conducting a graduate engineering mathematics course from the perspective of an instructor. These lessons discussed in the paper may be insightful and useful to other faculty members trying to develop such a course or teaching a similar course.

Course Description

The author designed and developed the course, MEEN 5304 Advanced Engineering Analysis, in 2001 and started offering the course in fall 2002. The course contents were chosen according to the contents and requirements of other graduate courses offered in the mechanical engineering curriculum, some of which are optimization of thermal systems (core course in optimization, also taught by the author), control of mechanical systems (core course in control), finite element analysis, advanced materials science, failure theory, mechanical vibration, and advanced numerical analysis. The major topic of relevance to all these courses is the topic of partial differential equations. As a result, the emphasis of the course is based on partial differential equations and their solutions even though the course needs to address other areas of mathematics. The major topics covered in the course are listed below:

- Analytical and numerical solutions of differential equations (2 weeks)
- Fourier series (1 week)
- Analytical and numerical solutions of partial differential equations (4 weeks)
- Vector calculus and Vector Integral Theorems (3 weeks)
- Statistics and Probability (4 weeks)
- Introduction to optimization (1 week)

The rationale for choosing these topics is discussed here. The first two topics are discussed in the course because they are prerequisite to the major topic of the course, partial differential equations. However, their coverage is limited to four lecture sessions as it is intended to be an overview rather than an in-depth discussion of the differential equations (DEs). Laplace Transform method of solving DEs is not covered as it is covered in details in the other core course, Control of Mechanical Systems. Fourier series is covered with an emphasis on the series expansion of an arbitrary periodic function. As mentioned before, partial differential equations (PDEs) is the main topic of the course as PDEs are building blocks of many engineering courses and engineering models. However, the coverage of PDEs in the course is limited to linear PDEs. The topics of PDE covered in the course include classification of different PDEs, 1-Dimensional and 2-Dimensional Wave equations, Heat equation, and Laplace equation. The analytical solution methods as well as numerical solutions of both DEs and PDEs are covered.

The next topic is on vectors with primary emphasis on vector operations involving gradient, divergence, scalar and vector products, and curl. The three Vector Integral Theorems: Green's Theorem, Gauss's Divergence Theorem and Stokes' Theorem, are covered next with their applications in fluid mechanics, engineering mechanics, and electrical engineering. The critical importance and applications of probability and statistics in all engineering disciplines and industry sectors are well known. Thus, probability and statistics are covered next with an emphasis on probability distributions (Normal, Binomial, and Poisson distributions) and their applications, linear regression, the correlation coefficient, sampling, and quality control using both X-bar and R-charts. The last topic is a brief overview of optimization with some discussions on steepest descent and linear programming methods. Since optimization of thermal systems course is the next core class that most students take, an introduction to optimization is included as a preview to those students.

Based on the course contents, a number of different textbooks [4-7] were explored for adoption as a course textbook. These are Advanced Engineering Mathematics by Irwin Kreyszig[4], Partial Differential Equations of Applied Mathematics by Erich Zauderer[5], Advanced Engineering Mathematics by Dennis Zill and Michael Cullen[6], Advanced Engineering Mathematics by Michael Greenberg. The books are compared based on the following criteria: coverage of relevant mathematic topics, suitability of examples and problems for engineering students, pedagogy and readability for students' learning, availability of many problems and projects, and resources for both students and instructors. Among these books, the book by Kreyszig[4] is the only book that includes coverage of statistics, probability and optimization. All books considered have many examples and problems available but the examples, problems and design projects in Kreyszig's book are more tailored to graduate engineering students. Kreyszig's book also provides student with student solution manuals available in either Mathematica or Maple software packages. Based on the comparison, Advanced Engineering Mathematics by Irwin Kreyszig[4] was chosen as the text for the course, MEEN 5304 Advanced Engineering Analysis.

Since the adoption of the text in 2002, two new editions of the book (9th edition in 2006 and 10th edition in 2010) have been published so the textbook seems appropriate and well suited for the course as new problems and materials are regularly updated placing fewer burdens on the instructor. The text provides many problems so assigning weekly homework problems to students is made easier for the instructor. The following figure shows the rating of the text and the homework problems from the text by the students for the last four years based on students' evaluation of the course at the end of the semester. The ratings are based on the scale of 1 to 5 with 1 being the lowest and 5 being the highest. Both the book and the homework problems received high rating (above 4 for all four years) indicating that the students believe the book is better suited to the course materials and useful for their learning. The rating drops in 2009 may be attributed to replacing the old edition of the text (8th edition) with the new edition of the text (9th edition) that year.

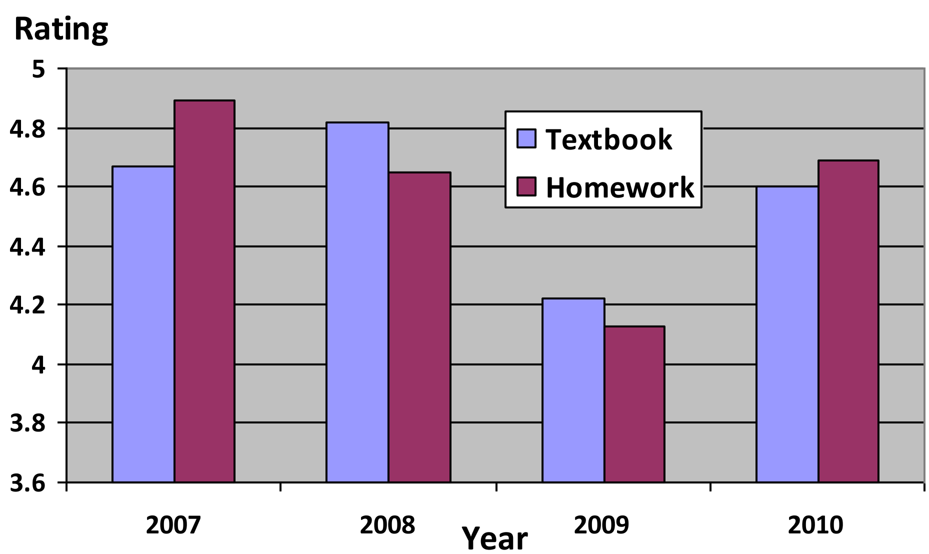

Figure 1 Results of Students' Evaluation on Textbook and Homework

Lesson 1: Course contents must serve requirements of the graduate program and relevant graduate courses. Discussion with the graduate program advisor and instructors of other graduate courses is a necessity.

Lesson 2: Choosing an appropriate textbook based on different criteria such as contents, pedagogy, problem sets, and resources will help improve student learning significantly.

Delivery and evaluation of the course

The course contents are delivered in sequence in a traditional classroom setting with lecture materials provided by the book and additional materials supplemented by the instructor. The class meets 2 times a week for 2 hours and 40 minutes. With 16 weeks in the semester, the total classroom time is about 45 hours. The instructor delivers the lectures by writing on the blackboard as well as PowerPoint presentations. Even though the book provides PowerPoint presentations, they are simply copies of the text. In addition, the publisher does not provide all the figures of the text separately so preparing one's own PowerPoint requires more efforts than other textbooks. The instructor resource includes an instructor solution manual but its use is limited as solutions to only even-number problems are included. For students, there are student solution manuals in mathematical software packages such as Maple, and Mathematica.

One of the most critical issues in delivery of this kind of course is making decisions about balancing between mathematical knowledge and application of mathematics regarding each topic. For example, in dealing with the wave equation in PDE, the instructor must make a decision on his/her emphasis regarding modeling aspects of the physical problem, solution methodology, the analytical solution to the wave equation, and interpretation of the analytical results in terms of the physical problem in question. An example would provide some insights to these issues. In many texts, the 1-diemnsional wave equation will be given as

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad \text{Equation (1)}$$

where the value of c^2 would be taken as unity for convenience. For engineering students, however, the meaning and the numerical value of c^2 are an essential and important part of modeling of the problem as well as interpretation of the results. In addition, the value of c^2 will rarely be equal to unity in most of the physical wave problems. Therefore, it is ultimately the instructor's responsibility to decide on how to deliver and present these aspects of engineering mathematics to enhance learning of the engineering students. In addition, the value of the course to the engineering students will also depend a lot on the assessment process whether it is the knowledge of the material or the ability of students to apply the knowledge to engineering problems. In this respect, the text by Kreyszig provides more engineering-related examples as well as problems than other comparable textbooks.

Another example to demonstrate the issues discussed in the previous paragraph would be the differences in characteristics of PDEs representing the wave equation, the heat equation, and Laplace equation. For example, Laplace equation is an elliptic type of problem while heat and wave equations are parabolic and hyperbolic in nature. In all three types of problems, boundary and/or initial conditions are critical to obtain physically meaning results. While the solution of Laplace equation is completely dependent on the boundary conditions, the solutions to the other two types of equations differ as time plays a critical role in their solution. Therefore, detailed discussions of domain of dependence, boundary and initial conditions, and their influence on the type of solution obtained analytically will be very important for students' learning and their ability to apply these PDEs and interpret the results with regard to engineering problems.

The evaluation of students contains three parts: homework (10%), quizzes (40%), and exams (50%). Homework problems are assigned weekly and graded by the grader. At the end of each topic, an in-class quiz is given and graded by the instructor. There are two long exams, one mid-term and one final, that are also graded by the instructor. The book provides both conceptual and numerical problems but the solutions of many problems require longer time than the duration of the class session which is 1 hr and 20 minutes. Therefore, the instructor generally divides up the problems into smaller parts and uses them in in-class quizzes. An example problem in a quiz is given below.

A sample problem
The temperature T is maintained at 0°C along three edges of a square plate of length 5 cm, and the fourth edge is maintained at 120°C until steady state conditions prevail.

(a) Write down the governing PDE equation.

(b) Write down the boundary conditions.

(c) Find the solution for the temperature T at any point (x, y) in the plate using the analytical solution in the textbook.

(d) Calculate the value of the temperature at the center of the plate using the expression in part (c).

A very important consideration in developing exam questions is whether the problem tests the students' understanding of mathematics or their ability to apply the mathematical knowledge to solving engineering problems. Both skills are essential but in the author's opinion the latter skill may be more essential for engineering students to master as increasing use of mathematical software packages makes the former skill less demanding to students. As an example, a kinematic problem from an engineering mechanics textbook can be used to test the numerical solution method of a differential equation. This kind of question is better suited to engineering students than simply giving them a differential equation and asking them to solve it numerically. An example problem is given below.

A sample problem
The following equation describes the velocity of a car. Determine the positions of the car, x in meters, at t = π, 2π, 3π, 4π, 5π s using Euler method. Compare the numerical solutions with the exact solution at t = 2π and 5π s.

$$\frac{dx}{dt} + 9\cos\left(\frac{2}{5}t\right) = 9, \quad x(t=0) = 0 \quad \text{Equation (2)}$$

In addition, plotting the solutions and asking the students to interpret the graphical results will provide additional insights to the physical nature of the problems. Availability and expanding use of software packages such as Matlab and Maple greatly facilitates obtaining the solution and plotting the results much easier for the students.

In summary, the success of the course and appreciation of students on the usefulness of the course depends on how the instructor addresses the balance between mathematical knowledge and application of mathematics. The following figure shows the students' ratings on the level of the materials and the value of the course for the last four years. The level of the course is related to how appropriate the course materials are for graduate studies, and the value of the course is how valuable the students believe the course materials are for their graduate studies.

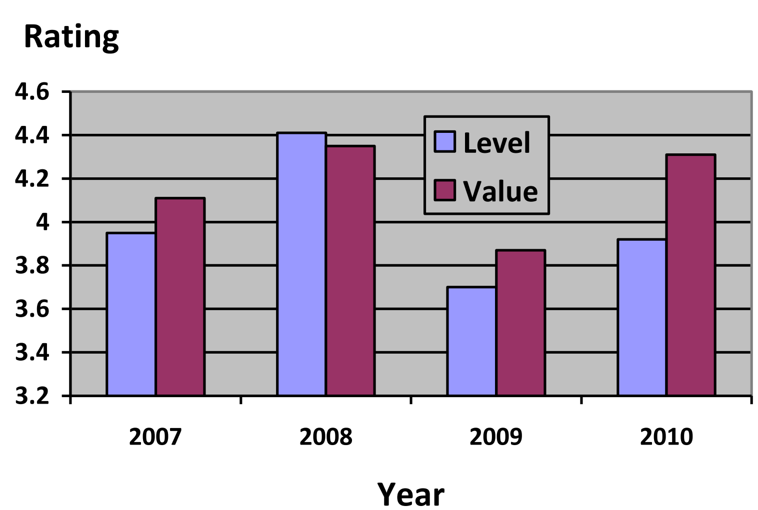

Figure 2 Students' Rating on Level and Value of the Course

As shown above, the students give higher rating for the value of the course compared to level of the course. Based on these results, it may be concluded that the majority of students believes the course materials are suitable and the course is valuable for their graduate studies. There is a drop in rating in 2009 but there are no significant differences in the course content as well as how the course is conducted so there is no simple explanation to the rating differences.

Lesson 3: It is imperative for the instructor to balance mathematical knowledge and application of mathematics in all aspects of the course delivery.

Lesson 4: The tests and exams should reflect the importance of mathematical knowledge and application of the knowledge in engineering-related problems.

Role of Mathematics Software Packages

Advances in mathematical soft ware and tools make teaching the class challenging as traditional methods of teaching engineering mathematics may not be appropriate when use of mathematics software is prevalent. As an example, Wolfram Corporation, the maker of Mathematica software, provides many demonstration projects on different disciplines including engineering. One of them is the problem of vibrating string[8] that is the solution to the 1-dimensional wave equation. The program allows variation of variables in a slider form and provides instant solutions (displacement of the string, and mode diagrams) in a graphical format. The program interface is shown in Figure 3.

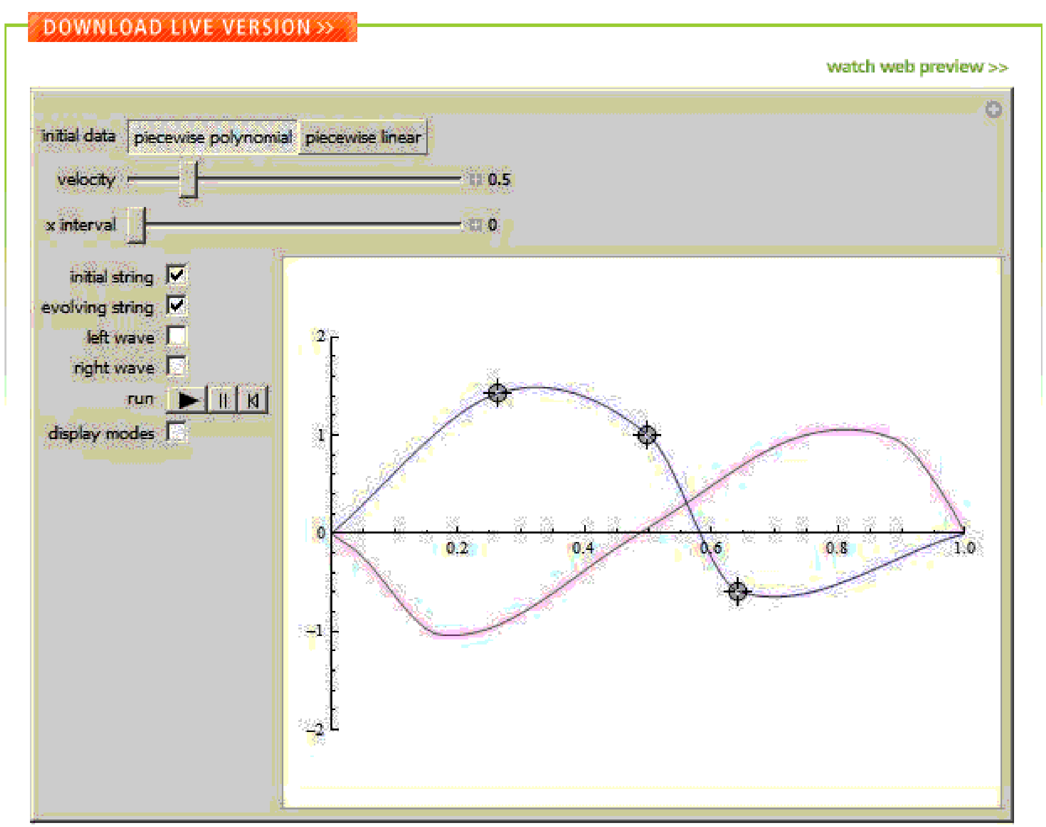

Figure 3 GUI Interface of Vibrating String Demonstration from Wolfram Research

These kinds of programs provide many benefits: improve the learning process of students, facilitate the students in completing homework assignments and projects, enable instructors to provide better learning environments, improve course delivery through more emphasis on modeling and interpretation of engineering problems, demonstrate key principles and solutions interactively and visually, and many others. The author has also started implementing these interactive tools and programs in the course delivery. The future plan for the course include increasing usage of these tools and programs in demonstrating classroom examples and assigning open-ended problems to students as group projects.

In addition to these tools, physical modeling capabilities of these software packages have been greatly improved. Maple has released a physical modeling program called MapleSim. According to Maple website[9], MapleSim is a physical modeling tool unlike any other. It is built on a foundation of symbolic computation technology, which efficiently handles all of the complex mathematics involved in the development of engineering models, including multi-domain systems and plant models for control applications. Because MapleSim is based on Maple, researchers and engineers working on advanced projects can also take advantage of an extensive range of analytical tools that provide greater insight into their systems. MapleSim reduces model development time from months to days while producing high-fidelity, high-performance models. Similar modeling tools have also been incorporated into Mathematica. Matlab, another mathematic software package, has Simulink program that greatly enhances modeling and simulation of dynamic systems. Therefore, integration of modern mathematical packages into engineering mathematic courses is inevitable and the instructors must be better prepared for adapting the course delivery, materials, and evaluations to incorporate these mathematical tools.

Lesson 5: Integrate and incorporate modern computing and mathematical tools in engineering mathematic courses to improve and facilitate the learning process.

Aspects of student management

Most of the graduate students in engineering come from foreign countries as mentioned before in this paper. At Lamar University, more than 90% of engineering graduate students come from India and China. As a result, one of the issues facing instructors is the communication. Language barriers may prevent some students from following the lecture presentations and understanding the materials. However, providing additional reading materials and more discussion times in class may alleviate or eliminate these problems completely. Most of the course materials are now distributed via online course management programs such as Blackboard or similar programs. Thus, providing materials one week before lectures and giving reading assignments can help solve these communication issues. Another aspect of foreign students that needs to be dealt with is course expectations and grading. Many countries have different grading systems (100-point, 1 to 5 grades, A to F grades) and passing scores. It is important to explain the grading procedures and passing scores and include them explicitly in the syllabus to eliminate confusion or misunderstandings. Class policy regarding attendance, textbook, missed quizzes and exams, plagiarism policy, use of cell phones, in-class behavior, must also be explicitly mentioned in the syllabus. Expectations and implications regarding open- and closed-book exams should also be discussed in class.

Lesson 6: Course syllabus must include clear explanations of course expectations, policies, and grading system.

Conclusions

Advanced engineering mathematics is a core course in the College of Engineering at Lamar University. The author has taught the course to incoming first year graduate students in mechanical engineering since 2002. The class covers a wide range of subjects but the major emphasis is on partial differential equations and their solutions. Other topics include vector calculus, vector integral theorems, numerical solutions of ordinary and partial differential equations, statistics and probability. The paper discusses various aspects of the course in details such as the text, delivery of content, exams and student grading, and results of students' evaluation of the course.

The lessons learned from teaching the class for many years are summarized below.

- ***Lesson 1:*** Course contents must serve requirements of the graduate program and relevant graduate courses. Discussion with the graduate program advisor and instructors of other graduate courses is a necessity.

- ***Lesson 2:*** Choosing an appropriate textbook based on different criteria such as contents, pedagogy, problem sets, and resources will help improve student learning significantly.

- ***Lesson 3:*** It is imperative for the instructor to balance mathematical knowledge and application of mathematics in all aspects of the course delivery.

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- ***Lesson 5:*** Integrate and incorporate modern computing and mathematical tools in engineering mathematic courses to improve and facilitate the learning process.

- ***Lesson 6:*** Course syllabus must include clear explanations of course expectations, policies, and grading system.

These lessons discussed in the paper may be insightful and useful to other faculty members trying to develop such a course or teaching similar courses.

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