Teaching Applied Mathematics in ET to Increase Student Engagement & Success in Engineering

Enrique Barbieri, Burak Basaran, University of Houston; Driss Benhaddou, University of Houston, Alfaisal University; Navdeep Singh, University of the Pacific; Vassilios Tzouanas, University of Houston – Downtown; Balan Venkatesh, Weihang Zhu, University of Houston

Abstract

The modern student-disengagement crisis is thought to be partly due to the COVID-19 pandemic of the last three years. However, many student disconnection reports date back several decades and recognize a lack-of-belonging as the key issue. University remedies range from social events and student organization activities to academic-related growth opportunities and curriculum improvements down to an individual course level. Mathematics skill building, particularly in engineering and engineering technology majors, is of paramount importance but often left to the standard mathematics course sequence. This article presents a course-level approach that engages students through solving engineering problems using mathematics in a more practical way. The approach is to (i) reveal common mathematical challenges arising in science and engineering problems from various fields; (ii) present the problem solution leading to a common mathematics formulation (e.g., a set of linear equations or a differential equation); (iii) review the relevant background that solves the specific mathematics question relating the solution back to the original problem and to upper-division courses; and (iv) use the MathWorks MATLAB & Simulink environment to simulate, verify and visualize the solution. The new course offered in 2023 will focus primarily on engineering technology sophomore and junior students and use applied mathematics as a universal interdisciplinary language that encourages a sense of belonging, increases students' confidence in their major, and prepares them for success in engineering careers.

1. Introduction

The return-to-normal after three years of lockdowns, travel bans, remote learning, and other mitigating actions to curb the spread of COVID-19, has re-opened a conversation in academia centered on a general observation of student disengagement in the learning process. The observation has been reported for decades in one way or another using terms such as apathy, or describing it as a student's feeling of emptiness, a lack of commitment, an interest in quick answers to questions, and a focus on a quick path to a degree showing little or no effort to fully participate in the learning process. The widely reported plagiarism crisis may very well be a consequence of the above-mentioned issues that students face. However, this disconnection with academic work is attributable to a lack of belonging to the University, or Department, or even an individual course mission [1].

Improving academic engagement is a top priority for many researchers [2]. Universities and their academic units tackle the issue in a variety of ways ranging from social events on campus, student organizations activities, and spirit-building via sports, to providing a freshman-year experience, study abroad opportunities usually in the junior year, undergraduate research during the summer, or a Senior Capstone integrative course particularly in engineering and engineering technology (ET) majors. Participating students likely experience a positive trajectory from freshman to graduation and career placement, adding to a strong alumni base. Many students do not participate because of many reasons that mainly infringe upon their time management for academic tasks, such as family and work-related constraints. In other cases, an inadequate preparation in lower-division courses prevents students' success in junior and senior-level classes. It is well known that at the course level in engineering and ET programs, inadequate mathematics knowledge is identified as one of the reasons for high failure rates and major attrition during the sophomore year. Nevertheless, none of the programs mentioned above focuses on the importance of mathematics for student success.

The trademark '*The degree is engineering technology, the career is engineering'* ™ guides students to a career that has grown increasingly interdisciplinary, complex, and math intensive. One challenge is overcoming a tendency to conclude that engineers in industry hardly ever need to use mathematics beyond calculus to perform functional engineering tasks. The evidence used to reach this conclusion may be based on feedback from engineers in industry. Moreover, the point is made that the profile of engineering technology faculty has morphed considerably to individuals who have a PhD in engineering, have not spent time in industry, and hence teach from a purely academic perspective. While these points may be true, the fact is that students must demonstrate abilities to use mathematics well beyond calculus in an educational path culminating in readiness for "engineer" titles in a competitive market with increasingly demanding technical tasks.

A Google search of the question "How much math is used by practicing engineers?" yields various interesting responses. Some respondents reason that the amount and level of mathematics usage depends on the field, the position, and the years of experience. Others point to the availability of specialized software packages that for example perform finite-element analysis, and that all is needed is setting up the geometry and initial and boundary conditions. However, a fair amount of differential equation knowledge is needed to understand what initial and boundary conditions are and what role they play in the solution of the problem at hand. In addition, without some knowledge of the mathematics behind the software analysis, a user may misinterpret the results.

Another example is in the field of Process Control. MATLAB has done an amazing job of creating a simulation environment that allows a designer to setup the configuration of the feedback system, the type of controller, the desired set of performance specifications, and even optimization parameters. One academic debate is whether to teach students to simply utilize a software package like MATLAB as a tool to design controllers, or to first teach the intricacies of the controller using differential equation language so that a true appreciation of the system performance can be acquired. Granted, one could use Excel or MATLAB to solve a differential equation as suggested by many practicing engineers. The educational question at the forefront is whether students can understand what a software package produces without some conceptual understanding of what is behind the calculations. Without that knowledge, one is blindly performing simulations, whereas solving math problems develops a thinking process that is found tremendously beneficial in engineering as a field of design and performance analysis under constraints.

It is important that faculty continuously modernize their teaching methodology to engage students, update degree plans to enhance the students' appreciation of the goals of the chosen major, and revise course content to keep them relevant for students to achieve those goals toward a solid career placement. The COVID-19 pandemic created a uniquely stressful situation whereby faculty and students had to adapt literally overnight to blended technologies to facilitate engineering students' achievement of competencies [3]. Assessment of competency achievement is equally challenging and can be approached by treating learning as a complex dynamical system with inputs and outputs in a dynamical continuous discrete assessment methodology [4].

For students interested in an engineering career, degree updates in the US follow ABET, which in 2011 renamed the Technology Accreditation Commission (TAC) to the Engineering Technology Accreditation Commission (ETAC). ET students are expected to develop a mathematics proficiency level adequate to achieve the career objectives set by ABET and state requirements. Then, in 2015, the ET Council of ASEE obtained approval to trademark the statement '*The degree is engineering technology, the career is engineering'* ™ furthering the need for mathematics proficiency in an ET degree.

An effective way to situate the ET degree in the engineering spectrum was suggested in 2009 [5] via the "Conceive, Design, Implement, and Operate" (CDIO) framework that visualizes how the majority of graduates gravitate toward the "Design-Implement" area where functional engineering tasks are performed. While it may be true that most functional engineering tasks do not rely on mathematics beyond calculus, it is an error in judgement to conclude that the educational path to an "engineer" title does not need more advanced mathematics. Furthering such conclusions tends to contribute to students' misguided expectations and poor performance due to a so-called "Temporal Relevance" perception [6].

Therefore, the key hypothesis throughout this article is that mathematics is critically important along the educational path of an ET degree as one of the two main university degrees that leads to an engineering career, and that there is a way to engage ET students through application-oriented courses using computational tools. Many corroborating studies can be found in the literature [7-10], but primarily in the context of engineering curricula. In ET, the typical ETAC/ABET Accredited B.S. degree includes core mathematics requirements covering standard Calculus I and II material. Then, a third mathematics course may be needed to satisfy university requirements with options such as Engineering Math, Probability & Statistics, or others. Some students opt to earn a minor in mathematics requiring other courses such as Linear Algebra and possibly additional electives. An anecdotal observation by engineering and ET faculty across different programs and universities is that many math courses tend to focus on the marvel of mathematical intricacies and therefore lack the application side that engineering-bound students seek. Another anecdotal observation is that engineering and ET faculty must take time away from upper-division courses to cover math background resulting in less time for engineering topics to be covered and a redundant effort by several faculty members across many programs.

The Mathematical Association of America (MAA), via its Committee on the Undergraduate Program in Mathematics, conducts periodic studies on 'where undergraduate students should be' in each of the four years leading to science, mathematics, and engineering degrees regarding foundations of mathematics and its applications in various disciplines, pedagogy, and the use of technology in mathematics [11]. The MAA recommends five cognitive learning goals in mathematics programs for students interested in engineering:

1. *Critical thinking*. Students will be able to think creatively and analytically.
2. *Problem solving*. Students will be able to apply the necessary mathematical tools and appropriate computational technology to solve complex problems.
3. *Modeling*. Students will be able to construct valid mathematical models for engineering applications.
4. *Visualization*. Students will be able to use visualization skills to assist with problem-solving and modeling.
5. *Communication*. Students will be able to clearly articulate ideas orally and in writing.

This article seeks to help address several of the issues above with a new course “Applied Math in ET” offered by the ET Department at the University of Houston in 2023. Applied Math is not simply the application of mathematics to solve “real world” problems. A more effective pedagogical perspective lies in the presentation of the material starting with a science or engineering problem definition followed by mathematical solutions and their visualization. Two key components guiding the design of this course cover several MAA recommended cognitive goals as described below:

1) *Applied and connected presentation*. The content should be drawn from science and engineering problem definitions rather than mathematical definitions so that modeling is included. The mathematics should focus on the utilization of tools rather than on the intricacies of how they came about. Finally, solutions should be connected as much as possible to upper-division ET courses to avoid the Temporal Relevance perception [3].
2) *Computational thinking*. A software environment should be chosen as the primary venue to decompose the problem, simulate, verify, and visualize the solution, thus promoting computational thinking as a process applied in real-world applications. Many studies report on the measurable advantages that a computing environment such as the MathWorks MATLAB/Simulink provide in engineering [12].

Therefore, the approach taken to organize this new course comprised the following four steps:

Step 1. Reveal how science and engineering problems from various fields exhibit overlapping and often equal mathematics challenges and that complex problems can be decomposed into more basic problems (computational thinking).

Step 2. Present the science or engineering problem solution that can be decomposed to a common mathematics formulation, for example, a set of linear equations or a differential equation.

Step 3. Review the relevant background that solves the specific mathematics question and relate the solution back to the original problem.

Step 4. Use a computation tool such as the MathWorks MATLAB/Simulink environment to simulate, verify and visualize the solution. Connect the problem and solution to upper-division courses.

The remainder of the article is organized as follows: Section 2 describes the content of the course; Section 3 presents various examples; and Section 4 concludes and offers recommendations for further studies.

2. Course Content

Table A in the Appendix provides a content summary organized into six major units for a semester-long course. The starting expectations are that students have a working knowledge of differentiation and integration as covered in the sequence Calculus I and II. The first assignment is to complete a two-hour self-paced MATLAB OnRamp course [13]. Students upload a Completion Certificate for credit. In addition, students view a computational thinking video and participate in a discussion board on how a computation tool such as MATLAB can be used to solve problems. The instructor reviews the concept in the class and follows the computational thinking process to solve the problem.

Student performance is assessed in a variety of ways each following the computational thinking process:

1. *In-Class Problem (ICP)*. This is a team-solved problem in class using all available resources including MATLAB. Sometimes, more complex or extensive portions of an ICP may be converted into a Homework assignment.
2. *Homework Assignments*. Students are required to present solutions in a professional engineering manner leading with problem definition, assumptions, and logical analysis. The advice is to avoid lengthy derivations but show intermediate steps, label figures, include relevant software code e.g., MATLAB, and provide explanations for equations and figures.
3. *Tests*. Two in-person tests gauge individual knowledge acquisition.
4. *Group Project*. Groups of 2-3 members present the solution to a science or engineering problem of their choosing from other courses in their respective majors.

3. Typical Illustrative Examples

The following examples illustrate the teaching methodology using computational thinking and computation tools to solve mathematical problems. Note that the process will help students take the systematic steps to solve ‘complicated' problems. Computational thinking takes the following steps: first describe the problem in words, second decompose the problem into simpler sub-problems, then solve the sub-problems, and finally build the solutions up to complete the whole solution.

Example I. A word problem leading to solving an algebraic equation

Step 1 – problem statement. Car #1 drives north at a constant speed v_1, while Car #2 starts to drive west a time T_d later. How long does it take for the cars to be a bird-fly distance D apart?

Step 2 – problem decomposition and modeling.

Find the relation between the distance traveled by each car and the time it takes: $d = vt$; then, use the Pythagorean theorem to relate the distance travelled by each car and the bird-fly distance, (D). This relation is illustrated in Figure 1.

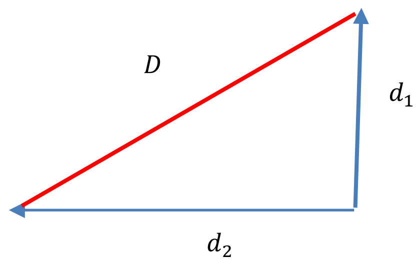

Figure 1. Problem illustration

Step 3 – write the appropriate mathematical equations that solve the built-up problem.

$$d_1^2 + d_2^2 = D^2 \Rightarrow (v_1 T)^2 + \left(v_2(T - T_d)\right)^2 = D^2$$

Given v_1, v_2, T_d, D, this is a quadratic equation in the unknown T. Suppose further that Car #2 is faster than Car #1, $v_2 = \beta v_1$, $\beta > 1$. Then,

$$(v_1 T)^2 + \left(v_2(T - T_d)\right)^2 = D^2 \Rightarrow \; v_1^2 T^2 + \beta^2 v_1^2 (T^2 - 6T + 9) = D^2$$

$$(1 + \beta^2)T^2 - 2T_d \beta^2 T + T_d^2 \beta^2 = \left(\frac{D}{v_1}\right)^2 \Rightarrow aT^2 + bT + c = 0 \Rightarrow T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 4 – use MATLAB to solve the above equation. A numerical case is summarized in Table 1.

Table 1. Numerical case for Example I.

	MATLAB and Comments

$v_1 = 40 \frac{Km}{h}$; $T_d = 3\,h$
$\beta = 1.5$; $v_2 = 60 \frac{Km}{h}$; $D = 500\,Km$
	`>> beta=1.5; D=500; v1=40;`
	`>> roots([1+beta^2 -6*beta^2 9*beta^2-(D/v1)^2])`
	`ans =` **`8.8710`** `-4.7172`
	The negative root is not admissible

	$\Rightarrow T = 8.87\,h$ which means Car #1 travels 8.87 h (8 h, 52 m, 12 s) and Car #2 travels 3 hours less.

Example II. Analysis of inequalities arising in a feedback control problem

Step 1 – problem statement. The bounded-input, bounded-output stability of a linear model of an unmanned submersible vehicle under pitch angle feedback control is determined by analyzing the associated Routh-Hurwitz entries in Table 2,

Table 2. A Routh-Hurwitz Stability Table for Example II.

s^4	1	3.457	$0.0416 + 0.109K$
s^3	3.456	$0.719 + 0.25K$	0
s^2	$11.228 - 0.25K$	$0.144 + 0.377K$	0
s^1	$\frac{-0.0625K^2 + 1.324K + 7.575}{11.228 - 0.25K}$	0	0
s^0	$0.144 + 0.377K$	0	0

where K is an amplifier pitch gain to be designed. It can be shown that the system remains in a stable operating mode if there are no sign changes in the first column.
Step 2 – problem decomposition and modeling. Analyze each entry in the first column and provide conditions that avoid sign changes.
Step 3 – write the corresponding mathematical relations and analyze each one. Since the first two entries in the first column are positive, the following sequence of conclusions are made:

a. From the s^2-Row: $11.228 - 0.25K > 0 \Rightarrow -\infty < K < 44.91$
b. From the s^1-Row: since the denominator $11.228 - 0.25K > 0$ is kept positive, then the numerator remains positive if the following condition is satisfied:

$$-0.0625K^2 + 1.324K + 7.575 > 0 \Rightarrow K^2 - 21.184K - 121.2 < 0 \Rightarrow (K - 25.869)(K + 4.685) < 0$$

Use MATLAB to obtain a graph of the quadratic function shown in Figure 2 to assist with analysis visualization. This is done with three basic commands shown next.

```
>> K = -10:0.01:30;              % Set up K interval
>> F = K.^2 - 21.184*K - 121.2;  % Compute F
>> plot(K,F), grid               % Plot
```

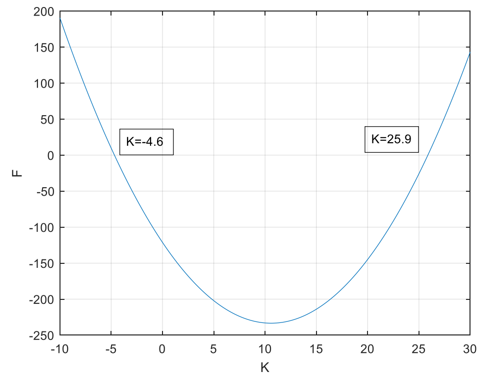

Fig. 2. Graph of the function $F = K^2 - 21.184K - 121.2$.

The visualization of the inequality $(K - 25.869)(K + 4.685) < 0$ yields the revised constraint

$$-4.685 < K < 25.87$$

c. Lastly, from the s^0-Row: $0.144 + 0.377K > 0 \Rightarrow -0.382 < K < \infty$

Step 4 – provide the built-up solution. Combining the partial results, yields the overall conclusion that the system remains in a stable operating mode if the pitch gain value is kept in the range $\boldsymbol{-0.382 < K < 25.87}$

Example III. A system of linear equations

Step 1 – problem statement. Many flow control problems can be visualized as in Figure 3 showing a 5-node network, where the arrows from one node to another represent known values or unknown flows $x_i,\ i = 1,2,\cdots 5$, and fundamental laws require that at each node, the net flow be zero. Examples include electric circuits; computer networks, traffic flow at intersections; water irrigation systems; mechanical trusses; and others.

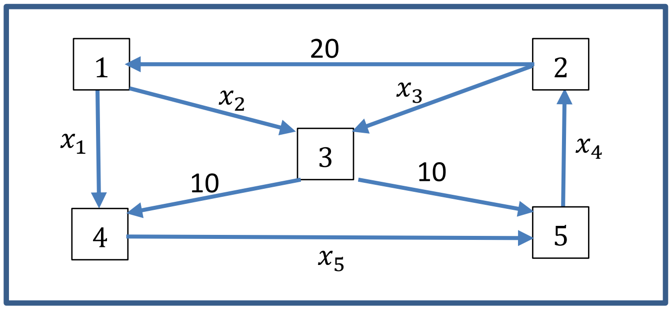

Fig. 3. A typical flow control network for Example III.

Step 2 – problem decomposition and modeling. There are five nodes leading to five constraint equations obtained using the convention $\sum(Flow\ OUT) = \sum(Flow\ IN)$.
Step 3 – write the corresponding mathematical relations and reason the solution. The five node equations that need to be solved simultaneously are:

Node 1	$x_1 + x_2 = 20$
Node 2	$x_3 + 20 = x_4 \Rightarrow x_3 - x_4 = -20$
Node 3	$10 + 10 = x_2 + x_3 \Rightarrow x_2 + x_3 = 20$
Node 4	$x_5 = x_1 + 10 \Rightarrow x_1 - x_5 = -10$
Node 5	$x_4 = x_5 + 10 \Rightarrow x_4 - x_5 = 10$

In matrix form, $AX = B \Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 20 \\ -20 \\ 20 \\ -10 \\ 10 \end{bmatrix}$ or in augmented form

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \end{bmatrix}$$

which is reducible by hand using elementary row and column operations to

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 1 & 30 \\ 0 & 0 & 1 & 0 & -1 & -10 \\ 0 & 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using MATLAB,

```
>> A = [1 1 0 0 0 20; 0 0 1 -1 0 -20;
0 1 1 0 0 20; 1 0 0 0 -1 -10; 0 0 0 1 -1 10]
>> Ared = rref(A)   % Reference: Row Reduced Echelon Form
Ared =
     1     0     0     0    -1   -10
     0     1     0     0     1    30
     0     0     1     0    -1   -10
     0     0     0     1    -1    10
     0     0     0     0     0     0
```

Step 4 – provide the built-up solution. The last equation reads $0 = 0$ indicating there are an infinite number of solutions that can be parametrized by $x_5 = k$, a given constant. Then,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} k - 10 \\ -k + 30 \\ k - 10 \\ k + 10 \\ k \end{bmatrix}.$$

For instance, suppose one could control the flow x_5 from node 4 to node 5, $x_5 = k = 10$. Then the unique solution to this network flow control problem is given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \\ 0 \\ 20 \\ 10 \end{bmatrix}$$

Example IV. A second order differential equation

Step 1 – problem statement. The switched electric circuit in Figure 4 leads to two distinct time segments separated by the instant when the switch is closed at $t = 0$. Determine the voltage in the capacitor $v_c(t)$ during each of these time segments.

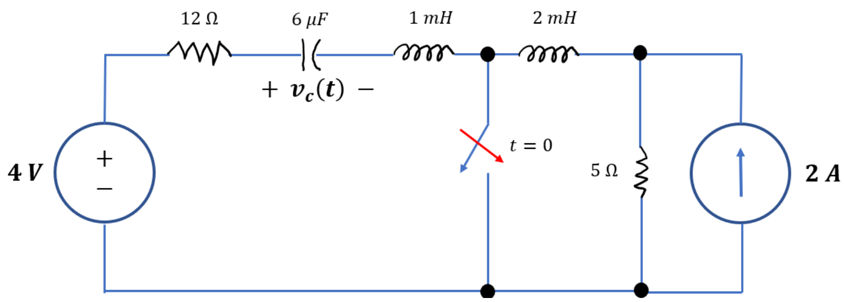

Fig. 4. A switched electric circuit in Example IV.

Step 2 – problem decomposition and modeling. Prior to closing the switch, that is, for $t < 0$, the circuit is a DC circuit governed by fundamental laws. It can be shown that the resulting energy balance in the elements forces the so-called initial conditions on the capacitor voltage and its first derivative to be $v_c(0) = -6V$ and $\frac{d}{dt}v_c(0) = 0$. Then, closing the switch at $t = 0$ changes the electric energy balance in such a manner that the capacitor voltage is the unique function $v_c(t),\ t > 0$ that satisfies the forced, second-order, ordinary differential equation given by

$$\boldsymbol{\frac{d^2}{dt^2}v_c(t) + 12 \times 10^3 \frac{d}{dt}v_c(t) + \frac{1}{6} \times 10^9 v_c(t) = \frac{4}{6} \times 10^9}$$

Step 3 – write the corresponding mathematical relations to build-up the complete solution. The homogeneous or natural component of the total solution of the differential equation is found from the associated pair of complex-conjugate roots of the characteristic equation

$$s^2 + 12 \times 10^3\, s + \frac{1}{6} \times 10^9 = 0 \qquad \text{or} \qquad s_{1,2} = -6 \pm j11.431 \times 10^3 = -\sigma \pm j\omega_d$$

easily verified with MATLAB

```
>> roots([1  12000  (1/6)*10^9])
ans =   1.0e+04 *
  -0.6000 + 1.1431i      -0.6000 - 1.1431i
```

The homogeneous solution of the differential equation therefore has the functional form

$$v_{ch}(t) = K_1 e^{-\sigma t}\cos(\omega_d t - \phi)$$

written in terms of the two arbitrary constants K_1 and ϕ. Using trigonometric identities, the solution can also be written as follows:

$$v_{ch}(t) = K_1 e^{-\sigma t}\left(\cos(\omega_d t)\cos(\phi) + \sin(\omega_d t)\sin(\phi)\right) = e^{-\sigma t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$$

where the new arbitrary constants are $A_1 = K_1\cos(\phi)$ and $A_2 = K_1\sin(\phi)$.

The particular or forced component of the solution is a constant $v_{cp}(t) = K_2$, which when substituted into the differential equation yields

$$\frac{1}{6}10^9\, K_2 = \frac{4}{6}10^9 \;\Rightarrow\; K_2 = 4.$$

Step 4 – provide the built-up solution. The total solution is then

$$v_c(t) = 4 + K_1 e^{-\sigma t}\cos(\omega_d t - \phi) \quad \text{or} \quad v_c(t) = 4 + e^{-\sigma t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$$

Finally, use the two initial conditions to find the constants K_1 and ϕ, or A_1 and A_2.

First, using $v_c(t) = 4 + K_1 e^{-\sigma t}\cos(\omega_d t - \phi)$ and

$$pv_c(t) = -K_1\sigma e^{-\sigma t}\cos(\omega_d t - \phi) - K_1\omega_d e^{-\sigma t}\sin(\omega_d t - \phi)$$

leads to the two equations $v_c(0) = -6 = 4 + K_1\cos(\phi) \Rightarrow \boldsymbol{K_1\cos(\phi) = -10}$

$$pv_c(0) = \boldsymbol{-K_1\sigma\cos(\phi) + K_1\omega_d\sin(\phi) = 0}$$

with solution $\boldsymbol{\phi = 0.4834}$ and $\boldsymbol{K_1 = -\frac{10}{\cos(\phi)} = -11.29}$

Finally, $\boldsymbol{v_c(t) = 4 - 11.29e^{-6000t}\cos\left(11.431\times 10^3 t - 0.4834\right)\ (V),\quad t > 0}$

Alternatively, using $v_c(t) = 4 + e^{-\sigma t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$ and

$$pv_c = -\sigma e^{-\sigma t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right) + e^{-\sigma t}\left(-A_1\omega_d\sin(\omega_d t) + A_2\,\omega_d\cos(\omega_d t)\right)$$

the two simultaneous equations and their solution are

$v_c(0) = -6 = 4 + A_1 \Rightarrow \boldsymbol{A_1 = -10}$ and $pv_c(0) = 0 = -\sigma A_1 + A_2\omega_d \Rightarrow \boldsymbol{A_2 = -5.24}$

Finally, $\boldsymbol{v_c(t) = 4 - e^{-6000t}\left(10\cos\left(11.431\times 10^3 t\right) + 5.24\sin\left(11.431\times 10^3 t\right)\right)\ (V),\quad t > 0}$

The capacitor voltage is of the underdamped type with a time constant $\tau = \frac{1}{6}\ msec$. The signal starts at $v_c(0) = -6\ V$, exhibits a peak overshoot, some oscillations lasting for about 5 time constants or $\frac{5}{6}\ msec$, and reaches a steady-state value of $v_c(\infty) = 4\ V$. MATLAB is used to visualize the solution. First, rescale time t to msec ($6000t = 6t_1$). Then, the following commands create a plot of the capacitor voltage $v_c(t), t > 0$:

```
>> t1=0:0.01:10/6;  % Create a time vector of 10 time constants
```

```
>> Vc=4-11.2938*exp(-6*t1).*cos(11.431*t1-0.4843);
% Note the use of the .* command
>> plot(t1,Vc),grid
% Use X Label, Y Label, Title, and Textbox tools under the Insert menu
```

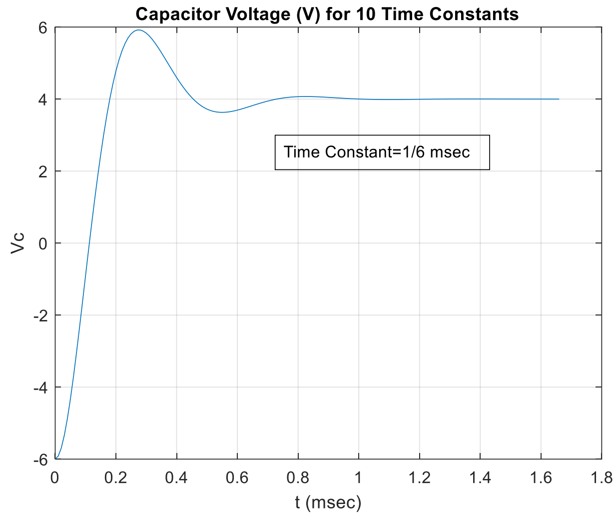

Fig. 5. Capacitor voltage in Example IV.

5. Conclusions and Further Work

A new course, Applied Math in Engineering Technology, is created to increase students' analytical confidence and prepare graduates for success in engineering careers. The course is envisioned to further instill in students a sense of belonging to a course, a major, and by expansion to an engineering career. Six modules comprise the full content and assessment is accomplished with homework assignments, in-class problem exercises, tests and a project. Four examples were provided that illustrate the general course structure to reveal common mathematical challenges arising in science and engineering problems from various fields; to describe the problem solution leading to a common mathematics formulation (e.g., a set of linear equations or a differential equation); to review the relevant background that solves the specific mathematics question relating the solution back to the original problem and to upper-division courses; and finally, to use the MathWorks MATLAB & Simulink environment to simulate, verify and visualize the solution. The authors are preparing a website that grows to compile relevant examples available to the public. A longitudinal assessment is also planned to demonstrate the hypothesized educational value of this effort.

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Biographies

ENRIQUE BARBIERI received the PhD in Electrical Engineering from The Ohio State University and was on the Electrical Engineering faculty of the School of Engineering at Tulane University as an assistant professor (1988-94), associate professor (1994-96), associate professor of Electrical Engineering & Computer Science (1996-2002), and chair (1996-98). During 2002-11, he was on the faculty of the College of Technology at the University of Houston as professor and chair of Engineering Technology (2002-09), associate dean for research and graduate studies (2009-10), member of the Executive Council of the TX Manufacturing Assistance Center (2006-11), chair of the Council (2007-09), and director of the Center for Technology Literacy (2006-10). During 2012-18, he was professor and chair of Engineering Technology, College of Engineering at the University of North Texas. In 2018, he rejoined the University of Houston's College of Technology and served as chair of Information & Logistics Technology (2012-2020). Currently, he is a professor of Engineering Technology. He is a Senior Member of IEEE – Control

Systems Society and was an active contributor to the ASEE Engineering Technology Council and the Engineering Technology Leaders Institute (2003-2018).

BURAK BASARAN received his BS (1996) and first MS (2001) degrees in Mechanical Engineering from his native country, Turkey. After coming to US on a Fulbright Scholarship, he received a second MS (2003) degree in Mechanical Engineering and his Ph.D. (2009) degree in Materials Sci. & Engineering from Texas A&M University, College Station, TX. He served as a postdoc research associate (2009-2011) in Mechanical & Aerospace Engr. Dept. at University of Kentucky. He joined University Houston, Dept. of Engr. Technology, Mechanical Engineering Technology program as a visiting assistant professor (2011-2012). After working as a tenure track assistant professor and interim department head at the Mechatronics Dept. of University of Turkish Aeronautical Association, Ankara, Turkey (2012-2014) he returned to US and has been with the University of Houston, Dept. of Engr. Technology, Mechanical Engineering Technology program since. He currently is an Instructional Associate Professor teaching solid mechanics, dynamics, materials science, engineering graphics and machine design. He is a registered member of ASEE, SAE & ASME. He focuses on engineering education research.

DRISS BENHADDOU holds two PhD degrees, one in optoelectronics from the University of Montpellier II, France (1995), and the second from the University of Missouri-Kansas City in computer networks and telecommunications (2002). Dr. Benhaddou is a professor at the University of Houston, where he is actively involved in a broad range of research activities in the application of computation and communication technologies in smart grid and smart cities applications. He is currently a visiting Electrical Engineering professor at Alfaisal University. Dr. Benhaddou’s areas of expertise include simulation tool modeling, software development, smart grid, networking, switching system design, and routing protocols development.

NAVDEEP SINGH received his BS (2001) and MS (2003) degrees from his native country, India, and PhD (2010) from Texas A&M University, College Station, TX, in Mechanical Engineering. He later served as a postdoc research associate (2011-2013) in the Department of Mechanical Engineering at Texas A&M University. He joined the University Houston, Department of Engineering Technology, Mechanical Engineering Technology program as a visiting assistant professor and later rose to the rank of instructional associate professor. He also served as program coordinator for the MET program from 2019 to 2022. In 2022, he joined University of the Pacific, Stockton, CA as an associate professor of Practice in the Department of Mechanical Engineering. He is a registered member of ASEE. His research interests are in the area of engineering education and computational design of materials.

VASSILIOS TZOUANAS is a professor of Computer Science and Engineering Technology at the University of Houston–Downtown, in Houston, Texas. He also serves as department chairman. He received all his degrees in chemical engineering and obtained his PhD from Lehigh University. He has worked in the industry for 19 years where he held technical and management positions with major operating companies as well as process control technology development companies. Since 2010, he has been with UHD. Dr. Tzouanas' research interests include process modeling, simulation and design, process control, and renewable energy systems. Dr. Tzouanas is an ABET program evaluator for Engineering and Engineering Technology programs. He is also member of AIChE and ASEE. He has served as director for ASEE's Engineering Technology Council for the period 2017-2021. Currently, he is co-chair of ETC’s Engineering Technology National Forum.

BALAN VENKATESH received his BSc (1989) and MSc (1991) in Chemistry from Madras University and PhD at Indian Institute of Technology, Chennai, India (1999). He has experience working in Japan for five years (1999-2004) including Osaka University, Osaka; Oji paper company, Tokyo and National Institute of Advances Science and Technology, Ikeda. He served in various positions at Michigan State University at the Department of Chemical Engineering and Material science for twelve years for the Great Lakes Bioenergy Center funded by the U.S. Department of Energy (2005-2017). Currently he serves as an Associate Professor and Graduate coordinator for Biotechnology program in the Department of Engineering Technology with a joint appointment in Department of biomedical engineering, University of Houston, Houston, Texas since 2017. His research expertise includes processing lignocellulose and algal biomass to fuels and chemicals, engineering, and processing substrates to reduce the growth cycle of edible fungus, Engineer chitin and chitosan extracted from fungus to produce biomaterial.

WEIHANG ZHU received his BS and MS in Mechanical and Energy Engineering from Zhejiang University and Ph.D. in Industrial and Systems Engineering from North Carolina State University. Currently he serves as professor, program coordinator, and graduate program coordinator for Mechanical Engineering Technology program in the

Department of Engineering Technology, with a joint appointment in the Department of Mechanical Engineering, University of Houston, Houston, Texas. His research expertise areas include automation/robotics/ haptics, design and manufacturing, machine learning, computational optimization, mariner and offshore safety, and engineering education.

Appendix Table A. Applied Math in ET Content Summary and Semester Schedule

1	**Introduction.** Engineering problems with common mathematics challenges. Computational Thinking – the need for computation, modeling and simulation https://serc.carleton.edu/teaching_computation/why_computation.html A review of units, complex numbers & algebra, trig functions, common trig identities, and engineering functions (impulse, step, ramp, exponential, sinusoids, and other). MATLAB tools (e.g., plot. Complex number algebra). Engineering applications.	Weeks 1 – 3	**Review Calculus I and II Make sure you have Matlab access.** Visit https://matlabacademy.mathworks.com/ for Matlab tutorials.
2	**Engineering Data Analysis & Manipulation**: Solution of linear equations, overdetermined and underdetermined cases, basic linear algebra (matrices, eigenvalues, and eigenvectors), mean and variance, curve fitting, least-squares minimization. Calculus operations, Zeros of functions, local minima/maxima; calculating area via Simulink. Numerical solution of nonlinear equations. MATLAB tools (e.g., polyfit, fzero, spline, fsolve; symbolic manipulation)	Weeks 3 – 4	
3	**Linear Constant Coefficient Ordinary Differential Equations (DFQ)**: Classical solution of n^{th}-order DFQ: decomposition into natural (homogeneous) and forced (particular) components. Decomposition into its equivalent matrix representation using n first order DFQs of the form $\frac{d}{dt}w(t) = Aw(t)$, and finding e^{At} ($A \in \Re^{nxn}$ a square matrix). Lab simulation. MATLAB tools (e.g., *lsim, ilaplace, residue).* Block diagrams and numerical Simulation in Simulink.	Weeks 4 – 7	**2 ICP and 3 HW (Weeks #2-#6)** **Test T1**
4	**Transformations**: introduce the notion of transformations as a means of easing mathematical operations from logarithms to phasor analysis, Fourier Series/Transforms (vibrations), Laplace Transforms (solving DFQ), and Z-Transforms (sampling).	Weeks 7 – 10	**3 ICP and 3 HW (Weeks #8-#13)**
5	**Partial Differential Equations (PDE)**: classical heat and wave equations, method of separation of variables $y(x,t) = X(x)T(t)$, finite mode expansions $y(x,t) = \sum_{k=1}^{N} \phi_k(x) q_k(t)$, and matrix representation $\frac{d}{dt}w(t) = Aw(t)$. Engineering applications.	Weeks 10 – 13	**Test T2**
6	**Further Topics:** Introduction to Discrete-Time Signals and Systems, sampling, difference equations. Nonlinear differential equations and linearization.	Weeks 14 – 15	**Group Project Due (Week 15)**