AC 2009-158: TEACHING COURSES ON PROBABILITY AND STATISTICS FOR ENGINEERS: CLASSICAL TOPICS IN THE MODERN TECHNOLOGICAL ERA

Natarajan Gautam, Texas A&M University

Dr. Gautam is an Associate Professor in the Department of Industrial and Systems Engineering at Texas A&M University since Fall 2005. Prior to that he was on the faculty at the Department of Industrial and Manufacturing Engineering at Penn State University for eight years. He teaches courses in applied probability and stochastic processes. Dr. Gautam has taught eight different courses in each of the universities he has worked in and has won several teaching awards. His research is in design, control and performance evaluation of stochastic systems with emphasis on computer-communication networks and transportation. Dr. Gautam is a member of ASEE, INFORMS and IIE.

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Abstract

Most Industrial Engineering departments offer courses on applied probability and/or statistics to engineering students. These courses often tend to be perceived as dry and far removed from engineering. This poses a significant challenge for instructors, especially junior faculty members that have been assigned to teach such courses. Not only do they have to spend significant amount of time away from research to make interesting classroom material, but they also have to teach material that is not even remotely close to what they do for research. To make matters worse, since the High School curriculum in the United States does not mandate a basic foundation in probability and statistics, most students are extremely unprepared and hence the instructors have to start at a phenomenally fundamental level.

The objective of this paper is to describe some strategies to overcome the concerns mentioned above and effectively educate engineering students on topics in applied probability and statistics. The first aspect is to teach a predominantly chalk-and-talk type of class by carefully using technology in strategic places and avoiding technology in certain other places. We quantitatively evaluate the effectiveness of our strategies and provide insights. Next, a good portion of this paper is devoted to one specific use of technology which is in laboratory-like exercises. These exercises were developed to teach more difficult concepts such as Central Limit Theorem and show how it applies to project evaluation and review technique (PERT). As a result, not only did the student understanding of complex material improve, but also the material was covered in a much shorter time. Finally the paper concludes with a qualitative discussion of issues where it is unclear whether technology boosts or hinders understanding of concepts in applied probability and statistics.

Introduction

Courses on applied probability and statistics are usually part of almost all Industrial Engineering undergraduate curricula. A large subset of these courses is usually offered by Industrial/Systems Engineering departments itself and the rest are perhaps offered by statistics departments. In fact many Industrial/Systems Engineering departments even offer service courses on these topics to other engineering students as well. Typically these courses involve basic probability, elementary statistics, quality control and sometimes even stochastic models for operations research. Based on the author's dozen years of teaching such courses as well as reputed colleagues that have taught such courses for over 25 years in Research-I universities, the remaining observations in this section of the paper are made. These courses, especially the probability parts, often tend to be perceived (by students especially) as dry and far removed from engineering because the material is rather abstract and the only skill needed to be successful is a strong foundation in mathematics.

However, the lack of motivation is not the only issue. The students typically do not have the right background either to get a full appreciation of the materials taught in these courses. This is because High School curriculum in the United States does not mandate a basic foundation in probability and statistics. Hence most students are extremely unprepared to take such a course, although everyone agrees that engineers (especially industrial engineers) ought to know not only how to operate tools based on probability and statistics but also know how they work. In that light, this paper seeks to address the question: *Can technology help to efficiently overcome the lack of motivation as well as lack of adequate background*? The term "efficiently" is used in the above sentence for instructors to buy in. From an instructor's standpoint this material is extremely fundamental compared to what they do for research. In addition, making course material *interesting* is perceived to be both time-consuming as well as a missed opportunity to teach other important course material.

Using technology to teach courses in engineering is a well-explored area[1]. This paper does not attempt to duplicate or undermine those efforts, but to propose strategies to complement them. In particular we begin by discussing appropriate places to use as well as avoid technology especially for teaching courses that include concepts in probability. Then we describe an example of a laboratory exercise that can be performed in class, as homework, in a computer lab or just by the instructor, whichever method is suitable. We evaluate the findings based on incorporating such an exercise in various courses. We conclude by presenting a qualitative discussion on whether certain specific instances of technology enhances or hinders understanding.

Where to use and where to avoid technology: some strategies

We begin by addressing the most common use of technology which is slide presentation. For a course on applied probability or statistics, this technology should be avoided. Anyone that has taught a course involving probability would agree that a predominantly chalk-and-talk type of class is most appropriate for such courses. The main reason is that one tends to go very fast, not explaining the details when slides are used. If the students do not have the adequate background, rushing through the material can only make matters worse. However there is a temptation for instructors to consider slides as it is a one-time preparation for potentially a large number of offerings. Using a chalk-and-talk approach one tends to prepare for every single lecture every time the course is offered. One way to avoid multiple preparations is to use a smart board type of technology. Essentially the instructor would write on something similar to a Tablet-PC which would in real time be projected on a large screen. This technology allows the instructor to use chalk-and-talk as well as only a single preparation. It also allows the instructor to save the lecture notes and post it on the course web page. Doing that as well as possibly audio-taping lectures would be extremely useful especially when students do not have the right background.

Another strategy to help students without the right background is to do away with office hours. This appears counter-intuitive because office hours are supposed to be where the students without the background can ask questions. What actually happens is that students instead of putting the efforts to learn the material, take the easy way out of stopping by office hours. In fact many times instructors almost give away solutions to homework exercises during office hours and the whole purpose of homework to catch up remedial material is lost. In addition, not all students show up at office hours and hence any knowledge that is imparted during office hours is not received by the rest of the students. Instead of office hours the students should be encouraged to ask questions in class or via email. Email responses can be copied to the entire class making it an excellent mode of communication outside class. In this era students are expected to know to effectively communicate technical material via email and such opportunities are excellent to foster these skills.

Doing away with office hours appears to be a strategy that deserves some testing for obvious reasons. The only course we experimented with having office hours and doing away with it was a graduate course on stochastic models for queues. For this course, in the first offering there were office hours regularly scheduled and the second offering there were none. Table 1 summarizes the mean and standard deviation of the student evaluation scores on a scale of 0 (worst) to 5 (best) for the aspect of *instructor availability*. Each row corresponds to different years the course was offered. All the other measures in the student evaluations were highly insignificant (across the two offerings the numbers coincided to the first decimal place, they were that close!). From Table 1, for instructor availability, although the mean availability score appears to have reduced, the reduction is not significant at a 95% level. This is using a paired t-test[2]. In terms of student performance, the only aspect that remained the same in the two offering is the homework. Interestingly, the homework performance improved when office hours were removed. The exams did not provide any significant difference in scores. But it should be noted that the exams were not tested on the same material (in the version with office hours there was one exam and in the version with two office hours there were two exams).

Method	Mean	Standard deviation	Number of students
Office Hours	4.78	0.41	23
No Office Hours	4.46	0.63	13

Table 1: Student evaluations on the topic of "instructor availability".

Another strategy to help students come up to speed with background is to encourage them to learn pre-requisites via the web. Students have been trained well to search the web for information since their high school (if not earlier). This is a skill they have mastered. It is crucial however to point out that not all web sites have authentic and accurate information. Further, a strategy to help in the area of motivation is to allow students to select data they would like to analyze. Having offered such courses in the fall semester we have observed that students are usually interested in analyzing NFL data or college football statistics.

The most important area where technology can be an asset when it comes to teaching courses in probability and/or statistics for engineers, is in the development of laboratory-like exercises. Concepts in probability are easier understood via simulations. In particular we have developed and tested laboratory-like exercises in various courses for concepts such as probability of an event, conditional probability, Bayes' rule, discrete random variables, continuous random variables, central limit theorem, hypothesis testing, confidence intervals, and quality control. These laboratory-like exercises can be offered in the following formats: in class (if all the students have access to a computer), in a laboratory session (in a computer laboratory), as a homework exercise, or just for the instructor to demonstrate in class. As an example to illustrate these laboratory-like exercises as well as to test their effectiveness, we consider the exercise on central limit theorem (CLT) and its application to project evaluation and review technique (PERT) [3]. We next present the exact exercise that was used in our courses.

Example: central limit theorem and application to PERT

This lab consists of two computer simulations.

Simulation set 1. Fill the blanks in the following statement:
Using Central Limit Theorem results, if N samples are generated from a distribution with mean μ and standard deviation σ, then the sample mean will be distributed according to __________ distribution with mean ____________ and variance ___________ when N is extremely large.

Now let us perform a few experiments to verify the above statement. Generate N columns of 1000 random numbers that are according to a uniform distribution with $A = 1$ and $B = 7$. In the $(N + 1)$st column compute the sample mean of the N columns for each of the 1000 rows.
(a) For the case $N = 25$ compute the mean and variance of the sample mean using the $(N+1)$st column statistics. Compare with the theoretical results. Note that you only have 1000 data points and hence the observed value would only be close to the theoretical but not exact. Attach the density plot and verify the normal distribution property of the Central Limit Theorem.
(b) For the case $N = 100$ compute the mean and variance of the sample mean using the $(N+1)$st column statistics. Compare with the theoretical results. Note that you only have 1000 data points and hence the observed value would only be close to the theoretical but not exact. Attach the density plot and verify the normal distribution property of the Central Limit Theorem.

Based on the above 2 experiments, comment on what happens to the sample mean when the number of samples N is increased.

Repeat the experiment with (a) exponential distribution with population mean 2 and $N = 25$; and (b) Bernoulli distribution with $p = 0.4$ and $N = 100$ (note that the population is discrete and not continuous).

Simulation set 2. A new manufacturing project is to be undertaken. It consists of 15 manufacturing operations on an assembly line (for example turning, milling, boring, welding, finishing, grinding, assembling, painting, material handling, . . .). The processing times of each of the 15 operations are random. The following table summarizes the random variables (with parameters in brackets) associated with the processing times of the 15 operations (all parameters are of appropriate units).

Operation #	1	2	3	4	5
Type	Turning	Mat. Handl.	Milling	Mat. Handl.	Boring
Distribution	exp(1)	norm(2,0.1)	gam(1.2,1.3)	unif(0.5,2)	norm(4.2,0.4)
Mean					
Variance					
Operation #	6	7	8	9	10
Type	Mat. Handl.	Welding	Mat. Handl.	Finishing	Mat. Handl.
Distribution	gam(1.4,1.1)	norm(3,0.5)	exp(1.3)	unif(0.2,2.2)	norm(6.2,2)
Mean					
Variance					

Operation #	11	12	13	14	15
Type	Grinding	Mat. Handl.	Assembling	Mat. Handl.	Painting
Distribution	gam(2,3)	unif(0,1)	norm(2.1,0.3)	exp(4)	gam(4,2.5)
Mean					
Variance					

Using the parameters of the distribution, fill up the above table with means and variances.

The main task is to be able to predict the distribution of the total project completion time (which is the sum of the 15 individual operation times). Assuming that we have a large enough number of operations (note that strictly speaking, 15 is NOT large enough), using Central Limit Theorem we can predict that the total project completion time will be distributed according to a _______ distribution with mean ____________ and variance _____________.

To verify the above results, simulate 100 project completions. To do this, use a fresh MINITAB worksheet (or any other software) to generate 100 rows and 15 columns of data. For example, column C1 corresponds to 100 sample realizations of an exponential random variable with parameter 1 (turning operation). Similarly generate columns C2, C3, . . ., C15 corresponding to operation # 2,3, . . ., 15 respectively. Add columns C1 to C15 and store in column C16. This C16 column represents 100 simulated realizations of total project completion times. Graph the density plot using the histogram option. Comment on the shape of the density plot, the sample mean and the sample variance with respect to the theoretical population values.

Output, outcomes and discussion

The exercise on central limit theorem was tried in different formats in three different undergraduate courses in two different universities. For sake of confidentiality let us call them courses A, B and C. In course A only the instructor demonstrated the lab exercise, however students just observed it. In course B, the students performed the exercise as homework after partially completing in class the portions that do not require a computer. Course C had a laboratory component and the students had a computer on their desks in class where they performed the experiment. The objective of this experiment is *not* to compare the three methods of delivering the laboratory exercise. However, it is to show that there are many ways to present this material depending on time and resource availability. The courses were offered over a span of 8 semesters and once again for confidentiality reasons they will be numbered as semesters 1, 2, …, 8. No two courses were offered the same semester.

Let us now consider one course at a time and explain the findings. In Table 2 we summarize the mean and standard deviation of the scores students obtained in a quiz on CLT and PERT in course A. The quiz scores have been scaled so that the highest possible score is 1. The first row corresponds to semester 1 where the lab-like exercise is not considered. The second row corresponds to semester 2 where the instructor demonstrated the laboratory in class. Notice that the mean quiz score has increased whereas the standard deviation has reduced from semester 1 to 4 (these are statistically significant at 95% level). There are two possible explanations for the observation. One is that the laboratory demonstration has indeed increased the understanding. Secondly most of the students in semester 4 had taken Course B in semester 3 where they performed the exercise as homework. This lets us conjecture that the exercise was worth the time and effort.

Semester (method)	Mean	Standard deviation	Number of students
1 (no demo)	0.864	0.285	90
4 (lab demo)	0.947	0.143	89

Table 2: Course A scores in semesters 1 and 4 on quiz covering CLT and PERT.

Now consider Table 3 where we present our findings based on Course B. The scores on a quiz based on CLT are recorded by scaling suitably so that the highest score is 1. The mean and standard deviation of the quiz scores over four semesters are tabulated. In all four semester the lab-like exercise for CLT was offered as a homework exercise. In semester 2 the quiz was given after the homework was due. In semesters 3 and 6 the homework was due well after the quiz was given. However in semester 5 the homework was due immediately after the quiz was given. Based on the mean quiz score it appears like the homework helped the quiz performance. In semester 2 almost all students that took the quiz completed the homework resulting in the best average. In semester 3 and 6 many students that took the quiz had not worked on their homework at the time of taking the quiz and hence affecting their grade. In semester 5 where some of the students completed the homework before the quiz and others did not, the grades were in between.

Semester (method)	Mean	Standard deviation	Number of students
2 (quiz after homework)	0.795	0.141	67
3 (quiz before homework)	0.515	0.287	81
5 (quiz during homework)	0.603	0.284	88
6 (quiz before homework)	0.492	0.315	54

Table 3: Course B scores in semesters 2, 3, 5 and 6 on quiz covering CLT.

In Table 4 we summarize the mean and standard deviation of the course C quiz scores on CLT. Course C has a required laboratory exercise exactly identical to the lab material presented above. The quiz scores have been scaled so that the highest possible score is 1. The first row is the mean and standard deviation when the quiz was given before the laboratory exercise during semester 7. However, in semester 8 (second row) the quiz followed the laboratory exercise. Notice that the mean quiz score has increased whereas the standard deviation has reduced from semester 7 to 8 (these are statistically significant at 95% level). Based on the observations it appears like the laboratory work by students has indeed increased their understanding.

Semester (method)	Mean	Standard deviation	Number of students
7 (quiz before lab)	0.765	0.358	39
8 (quiz after lab)	0.911	0.173	45

Table 4: Course C scores in semesters 7 and 8 on quiz covering CLT.

Now, looking at Tables 2, 3 and 4 together it appears as if the performance in course B is much worse than courses A and C. This deserves an explanation. There are three reasons for that: (1) course B uses a homework which students tend to collaborate and not follow the logic, whereas in course A students are forced to pay attention in class when it is demonstrated and in course C students individually perform experiments during a laboratory session; (2) in course B the quiz is the 10th and last quiz of the semester by which time the students are burnt out by the semester, whereas in course A it is the 3rd quiz and in course C it is the 5th quiz; (3) students are exposed to CLT for the first time in course C, however in course A students are exposed to it for the second time (first time in course B) and in course C the students see CLT in a statistics prerequisite course. Therefore it reinforces the notion that by presenting the same material multiple times, the concept finally sinks in.

Does technology boost or hinder understanding?

Although we feel the most effective way of performing the laboratory-like experiments is in class, it is unclear whether it is a good idea to have computers on students' desks during class time. These computers are either the students' own laptop or desktops provided by the university. There is a clear benefit of that technology for performing such exercises. However oftentimes students are caught browsing instead of paying attention in class. There are some software packages that can be used to either turn off the student machines or to disable web access. Unfortunately these packages do not work 100% of the time.

There are some other issues where it is debatable whether the technology boosts or hinders understanding. One question we get often in this day and age is whether the book is required or if the students can just read it on-line. There are pros and cons to this. The greatest benefits of on-line texts are cost and accessibility. However from the standpoint of time (speed to recover necessary information) and convenience of browsing through a hard copy it is unclear if an on-line textbook is the way to go.

Another question that instructors get asked often is if graphing calculators are permitted in the course. The benefit of using these calculators is not only in drawing graphs of complex functions but also being able to perform numerical integration (area under the curve) among other things. The down side is that students would never learn from the mistakes, in fact they would not even realize they made a mistake punching something on the calculator. This is an important aspect, but when calculators were first allowed (replacing slide rules and logarithmic tables), there were perhaps such similar arguments!

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