

# Teaching Freshman Engineering Students to Solve Hard Problems

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## 1. Introduction

The typical freshman engineering student encounters subject matter that is more complex and delivered at a faster rate than what they experienced in high school. Combined with weak problem solving skills and inadequate study habits, this can be a recipe for disaster. “Judgment day” for many students occurs when the first round of midterms are handed back. Some students give up and make plans to switch majors. Others continue to struggle in spite of increased study effort. A common complaint is, “I understand the material, but I can’t seem to work the problems on the exam correctly.”

We are developing a course to make freshman engineering students better problem solvers. We focus on how to set up problems that are unfamiliar to the students, but can be solved with the math concepts the students are already familiar with.

We emphasize the use of hard problems and explanation. In hard problems the solution method and concepts needed for solution are not immediate. Exercises are designed to train students to expend effort to obtain a mathematical understanding of a problem sufficient to allow its solution, recognize when such understanding has been achieved, and rely on a wide spectrum of prior knowledge.

The emphasis on explanation appears to be a unique feature of our course. We emphasize explanation as a learning skill by fostering self-explanation, in which students’ process worked examples by explaining the concepts and purposes behind each step. Students also explain their solutions to others in the context of cooperative learning exercises and short class presentations. Explaining helps students clarify the concepts in their solutions, which facilitates transfer of those concepts to new situations, and builds self-efficacy, or domain specific self-confidence, in both the explainer and listener.

The authors are teaching the course to one of 8 sections of GES 131 Foundations of Engineering I, an existing introduction to engineering course. The impact of our course was assessed through a pre and post test and a pre and post math/problem solving self-efficacy questionnaire given to both our section and a control section, matched for ACT scores. Students were selected for the experimental and control sections to avoid the volunteer effect. A key goal of our course is the transfer of problem solving skills to future coursework. To assess transfer, we will compare performance of the experimental and control groups on certain exam problems in follow-on math and physics courses.

## 2. Rationale for the Course

The goal of our course is to teach mathematical problem solving in a way that leads to transfer of knowledge and skills to future coursework. We felt several factors were essential:

1. Hard problems, in which the solution method and mathematics concepts needed are not immediate.
2. Explanation to facilitate learning with understanding. This includes self-explanation of worked examples, and students explaining their work in both cooperative learning groups and class presentations.

We focused on the solution of difficult, but well defined problems because this is a major deficiency in the students we see.

To develop problem solving skills, it is important for students to solve problems in which they must select the mathematical tools to solve the problem<sup>11</sup>. The use of cumulative review problems, drawing on all previous material, resulted in increased problem solving ability<sup>9</sup>. In contrast, typical textbook exercises rely mainly on recent content, and give students sufficient clues to select solution techniques without understanding the concepts well enough to apply them in new contexts.

Problem solving courses for engineers exist at many universities. Specific instruction about the problem solving process has had positive effects, but often did not transfer to future coursework without significant integration into the entire four-year curriculum, as in the McMaster Problem Solving Program<sup>15</sup>. This is difficult to accomplish at most institutions. Many introductory problem solving courses provide the students with a 4-6 step method, but do not emphasize explanation and hard problems to develop the cognitive skills required, and facilitate transfer. We believe an emphasis on hard problems and explanation is critical for transfer of skills.

For successful transfer to occur, the students must understand well how to solve problems, and not back off when confronted with a difficult problem<sup>2</sup>. To achieve transfer, several factors are important: i) Self-Efficacy, or students' confidence in their ability to solve mathematical problems<sup>10</sup>; ii) Learning with understanding. Students must understand the fundamental principles behind solutions to problems<sup>2</sup>; iii) Beliefs. Students must believe the concepts they have learned are real, and will still be real in a different context, such as the next course.

Self-efficacy is promoted in four ways<sup>1</sup>: 1. Mastery experiences, where students demonstrate mastery by overcoming obstacles through effort. 2. Reinforcement by seeing peers achieve mastery. 3. Social encouragement. 4. Learning to manage emotional and physical reactions, such as math anxiety, or the knot in the stomach when facing a challenging problem on a test. We promote mastery by using hard problems, and emphasizing explanations. We use cooperative learning, teaming, and class presentations to create a learning environment in which students see the mastery experiences of peers. Students were repeatedly given encouragement that the problems are hard, but can be solved with hard work.

To achieve learning with understanding, we have the students explain problems, solutions and math concepts in small groups and class presentations. We also used instruction and practice in self-explanation of worked examples. Worked-out examples play a prominent role in engineering education. Learning from worked out examples requires active processing that often does not occur in our students, creating an illusion of understanding that results in poor test performance<sup>3</sup>. Successful students practice self-explanation, and process worked-out examples

by relating solution steps to basic principles, and anticipating the next step<sup>5,12</sup>. Examples of explanation exercises include developing explanations for the purpose and concepts involved in each step of a worked example, or developing a frequently asked questions list for a worked example. One of the authors has seen a positive impact of self-explanation exercises on problem solving<sup>7</sup>.

Student beliefs are a major factor in problem solving<sup>14</sup>. Students must believe the concepts they have learned are real, and will still be real in a new context. Connecting concepts with reality gives students confidence those ideas will still be true in the next class. The use of physical, hands-on exercises integrally connected to mathematics problems was used to convince students that the ideas they are learning apply in the real world. To become successful problem solvers, students must also believe it is necessary to have a good understanding of the problem before plunging in to the solution.

For simplicity we used Polya's problem solving framework of Understand, Plan, Do, Reflect<sup>11</sup>. These were taught as phases of problem solving, rather than steps that are completely followed in order. Students were also given instruction and practice in identifying which phase of problem solving they were in, so they could develop meta-cognitive skills to better control their problem solving activity.

## 2. Description of the Course

In Fall 2004 the course was taught as one of eight sections of GES 131 Foundations of Engineering I at the University of Alabama. GES 131 is a two credit course that meets three times per week, twice for 50 minutes, and once for 110 minutes. The 110 minute section is useful for extended exercises, student presentations, or special projects. Two sections of GES 131 were set apart for first semester freshmen who had the appropriate math placement scores (Calculus ready or one semester before Calculus). One section received the experimental problem solving instruction, while the other served as a control. About 35 students were enrolled in each section. Students were assigned to these sections during summer advising, without knowledge of the experimental nature of our section, so there was no "volunteer effect." All students in the experimental and control sections signed informed consent forms as required by the Institutional Review Board (IRB). The experimental and control sections were formed to balance ACT scores as much as possible.

The textbooks for the course were *Schaum's Outline on Precalculus*<sup>13</sup> and *The New Way Things Work*<sup>8</sup>. The Schaum's Outline was used to create a common baseline of pre-calculus mathematics knowledge. Homework was assigned out of this book for review as well. The worked examples were also used in self-explanation exercises. The New Way Things Work was used to help students look at complex devices in terms of simple machines and identify common basic physical principles. The identification of common basic principles in different situations supports creative problem solving, design and transfer. This book was often used for reading assignments on devices, such as a mouse or floppy disk drive, to prepare students for upcoming problems related to those devices.

Planning the course was a challenge because there were many topics which ought to be covered right at the beginning of the course, and this is, of course, not possible.

We began the course with a presentation of the engineering and mathematical analysis that went into the Wright Brothers' first flight as in introduction to engineering.

Students were given an overview of problem solving in terms of Polya's framework of Understand, Plan, Do, Reflect<sup>11</sup>. Many sets of problem solving 'steps' have been published. We felt Polya's framework was simple and included the most important activities. These four phases of problem were presented as such, and not as steps that mechanically lead to an answer in a linear fashion without iteration. To help students become more aware of their own problem solving behavior, we used cooperative learning exercises in which students identified which parts of a worked solution corresponded to each phase of problem solving. It became apparent that understanding the problem was the key step.

Polya<sup>11</sup> and others have written that most students' difficulties lie in understanding the problem. This also agrees with the authors' experience. By understanding the problem, we are not referring to mere reading comprehension, but a mathematical formulation of the problem that captures the important features and facilitates the solution. This involves introducing notation, drawing and labeling figures, putting the problem data on a figure, identifying conditions and equations, and identifying relevant mathematics concepts. We constantly communicated to the students that understanding the problem requires time and effort. Since this is a more descriptive activity that does not immediately move toward the solution, it is often skipped by students, who are then unable to solve the problem. Understanding the problem often requires the student to connect a physical intuition about a problem with a precise mathematical description. This is also an important skill for practicing engineers that is not covered in high school or, for that matter, most engineering classes.

As an example, students were given the conditions for a see-saw to balance ( $W_1D_1 = W_2D_2$ ), and asked to determine conditions for a board with weights on each end to not tip over when placed across two saw-horses. This problem is easily solved once it is understood by drawing a diagram of the board in both the tipped and untipped positions. One group of students used their textbooks to make this into a hands-on exercise. Only a few students were able to solve this exercise because they did not draw the diagrams required to understand it. Those who solved it did so with significant help from the instructors. Many students produced equations, rather than the correct inequalities that make physical sense.

For most of the problems assigned in the course, understanding the problem meant coming up with a good diagram, or a set of simultaneous equations. Since we wanted to review several mathematics concepts, such as plane geometry and trigonometry, we started with simultaneous equation problems and introduced geometric problems as soon as possible.

Explanation was a key factor in the course. Students were given specific instruction in how to self-explain (explain to yourself) worked examples by explaining the principles behind each step, and the purpose of each step. In class and home exercises required students to explain worked examples by explaining each step, or developing a frequently asked questions list. As part of

instruction on oral and written communication, students were also given specific instruction on how to explain their solutions. Here we noted that the audience will typically not have done the work to understand the problem, and will need this part explained (Where did you get that equation?).

Since the course focused on problem solving skills rather than mathematical content, no attempt to introduce new mathematics concepts was made. We reviewed concepts of solving simultaneous equations, polynomial curve fitting, plane geometry involving triangles and circles, trigonometry, and exponential and logarithmic functions. Numerical exercises in Excel accompanied some of these reviews. Each review involved several math concepts integrated into a single problem. Students were also assigned problems in Schaum's Outline on Precalculus<sup>13</sup> for math review.

The ABET description of the course includes ethics. We focused on cheating as an issue of immediate relevance to the students. The students worked in groups and came up with how they would respond in different situations in which they experienced peer pressure to cheat. Some students were very honest and said they would not try to stop another student from cheating. Given the research indicating that engineers who cheat as students are more likely to engage in unethical behavior in the workplace, we felt this was a good treatment of ethics for freshmen<sup>4</sup>.

Students in the course received both instruction and practice in teamwork. Most of the students formed three person teams on their own. As these were working well together, we stayed with the student selected teams, rather than assign them. Developing good interpersonal communication skills was a main point of the teamwork instruction.

Most of the class consisted of active and cooperative learning exercises in problem solving. To break the mindset that a problem should be solved in 10 minutes or abandoned, we gave problems that took several class periods to solve completely. Once again, most of this activity consisted of understanding the problem.

The last two to three weeks of the course consisted of an extended design project, in which teams of three students were asked to design a device that would fit inside a 15" cubic box. A toy car would be dropped into a hole in the top of the box, stay inside for at least 3 seconds, and then be propelled out of the box for a distance between 5 and 50 feet. The device also had to have a switch operated light, make a unique sound, and cost less than \$25. Students submitted a written report and made 8 minute presentations on their designs.

To help students become more aware of themselves as learners, we had students complete and score the Learning Combinations Inventory<sup>6</sup>. The LCI assess the degree to which students tend to use four learning patterns:

1. Sequential pattern. Likes clear instructions and step by step procedures.
2. Precise pattern. Likes to be sure things are right. Asks a lot of questions.
3. Technical pattern. Likes hands on learning and learning how things work.
4. Confluent pattern. Likes unstructured situations that require creativity. Likes to develop their own ideas rather than learn the ideas of others.

Needless to say, most engineering students score high in the Technical learning pattern, as found in our class and other studies. This underlines the importance of hands-on experience in engineering classes. We liked the LCI and associated Let Me Learn process, since it involved making students aware of their strengths and weaknesses, and encourages them to be responsible for compensating for weaknesses. Much of learning style testing tells students they will do well if they are in the right environment for them, which can be unhelpful.

We also noted that many students scored lower in the Precise learning pattern. Since successful problem solving involves getting things right, particularly in the understanding the problem phase, this was a serious issue. This was reflected in observations by the instructors of students proceeding to solve problems with assumptions that clearly violated the original problem statement. Specific exercises to develop this precise learning pattern may be extremely helpful in improving student performance in both learning and problem solving tasks. Students must correctly determine if they understand a new class concept, or if they correctly understand a problem. We also noted students discussing the results of the LCI later in the semester in connection with the problems they were solving.

In considering the learning experiences of engineering students, most courses require the Sequential and Precise patterns. We also noted our design project required the Technical and Confluent patterns. This may account for the frequently observed dichotomy between student GPA's and performance in design courses.

In order to promote self-efficacy, students were called on frequently to make presentations of their solutions, or the work they had done to understand a problem, which typically was a diagram. This way the class saw their peers succeeding in solving hard problems. Students used an ELMO projector, so projection from a handwritten page was very easy. Students who presented received a round of applause for their work, and courage getting up in front of the class. PowerPoint presentations of the design projects were also required, with each team of three students making an 8 minute presentation.

Quizzes were used to assure student attendance and test problem understanding skills. Often a quiz would ask students to draw a diagram for a problem without solving it. This communicates the value of understanding the problem as a rewarded task. On exams, 40-60% partial credit was given to students who drew good diagrams that expressed a mathematical understanding of the problem. This communicates in a real way that understanding the problem is a valued task.

Homework assignments were used for several purposes. Assignments in Schaum's Outline were used to review math concepts. Assignments in The New Way Things Work were used to help students see similar basic principles behind seemingly different devices, and to familiarize students with devices they would work problems on in later classes. Home assignments were also used for extensions of in-class exercises.

### **3. Assessment Methods**

Assessment was both formative and summative. Formative assessment focused on what the students found helpful or not. Summative assessment focused on changes in student problem

solving ability and self-efficacy. Transfer will be assessed by student performance on specific problems in future Calculus and Physics problems.

For summative assessment we developed a six question math problem solving test designed for a 60 minute time period. A copy of this test is included at the end of this paper. Students were informed the test would be counted toward their grade on a good faith attempt basis. For assessment purposes, each problem was graded on a scale of 0 – 4, with 4 being the highest. Algebra errors were not counted, however students arriving at a correct answer by a trial and error process received only a score of 2/4. This test was administered to students in all eight sections of GES 131 at the beginning of the semester as a pre-test, and again at the end of the semester as a post test. This test was also administered to students in the two honors sections of GES 145 as a pre-test. The pre-test results of the honors sections were very high, and a post-test was not given to the honors sections.

We also developed a 25 question self-efficacy scale that focused on mathematical problem solving for students at the level of Engineering freshmen. A copy of this questionnaire is included at the end of this paper. Existing self-efficacy scales were designed for students at a much lower level, and would not be useful for this study. Two types of questions were used on the scale. The first section of 10 questions asked students to agree or disagree on a scale of 1 – 10 with statements such as “I am good at solving mathematics problems,” with 1 being strongly disagree, 3: Disagree, 5: Neutral, 8: Agree, 10: Strongly agree. The second section of 15 questions asked students to express their confidence in their ability to solve specific math problems on a scale of 1 – 10, with 1 being Cannot solve it at all, 5: Moderately certain I can solve it, and 10: Certain I can solve it. A copy of the self-efficacy scale we developed is given at the end of this paper. This self-efficacy assessment was given to all 8 sections of GES 131 at the beginning of the semester, and repeated for the experimental and control sections at the end of the semester. Further analysis is needed to refine and validate this instrument.

For formative assessment, we asked students to write one-minute papers several times during the semester. Student comments on formal course evaluations are not available. Students identified what aspects of the course they found helpful, and what could be improved. Instructor observations were also part of formative assessment.

#### 4. Assessment results

The results of the pre and post math problem are shown below in Table 1. Results are only included for students taking both the pre and post tests. An effect size of .148 was calculated using the standard deviation of the control group.

	Pre-Test Mean	Pre-Test St. Dev.	Post-Test Mean	Post-Test St. Dev.	Delta Mean	Delta St. Dev.	Effect Size
Experimental Problem Solving Section N=26	12.27	4.61	14.92	4.41	2.65	3.85	.148
Control Section N=30	9.5	4.47	11.53	4.07	2.03	4.18	

Table 1. Mathematical Problem Solving Test Results for Questions 1-6 (all questions).

This was a disappointing result. Close examination of the data revealed that most students in the experimental section performed better on the post-test on Questions 2 through 5. Many students in the experimental section performed significantly poorer on the Post-test Question 1, and about the same on post-test Question 6. Question 1 involved estimating the answer to an arithmetic calculation (which was not covered in our problem-solving course) and Question 6 was a very challenging logic problem. To better understand the effect of our problem-solving course midway through our project, we reanalyzed the data using only Questions 2 through 6. The results are shown in Table 2 below. When Questions 2 – 6 are considered, the mean score of the experimental section increased significantly more than did that of the control section. The effect size was .658.

	Pre-Test Mean	Pre-Test St. Dev.	Post-Test Mean	Post-Test St. Dev.	Delta Mean	Delta St. Dev.	Effect Size
Experimental Problem Solving Section N=26	9.19	3.91	12.58	3.88	3.38	3.16	.658
Control Section N=30	7.3	3.88	8.87	3.39	1.57	2.75	

Table 2. Mathematical Problem Solving Test Results for Questions 2-6.

We can also consider the percent of correct answers (4 out of 4 points) on problems 1-6 and 2-6. These are shown in Table 3. Once again, the results on questions 1-6 are comparable for both sections, however the results for questions 2-6 show significant differences.

	Questions 1-6			Questions 2-6		
	Pre-Test	Post-Test	Delta	Pre-Test	Post-Test	Delta
Experimental Problem Solving Section N=26	32.1	45.5	13.4	23.8	46.9	23.1
Control Section N=30	24.4	35.0	10.6	20.7	31.3	10.6

Table 3. Percent of correct answers (4 out of 4 points) on the Math Problem Solving Test.

Considering the scores on questions 2-6 only, 50% of the students in the experimental section increased their scores by 4 points or more out of 20 from the pre to post test, compared to 20% of the students in the control group. 26.9% of students in the experimental group had post test scores of 16 out of 20 or above, compared to 3.3% in the control group.

Scatter plots of the pre and post test scores (Questions 2 – 6) of the two sections sorted by pre test scores show another important result. In the control section, students with lower pre test scores made significant gains, while those with higher test scores did not. In the special problem solving section, significant gains were made by students across the entire range of test scores, as shown in Figure 1 and Figure 2. One interpretation of this effect might be that just being at the university for one semester, taking both mathematics and an introduction to engineering class will have a remedial effect for the lower scoring students. Additional studies would be required to support this hypothesis. Since the honors students had been placed in separate sections, it seems that the problem solving session especially benefited the mid-range students, i.e. those scoring higher on the pretest, but not in honors classes.



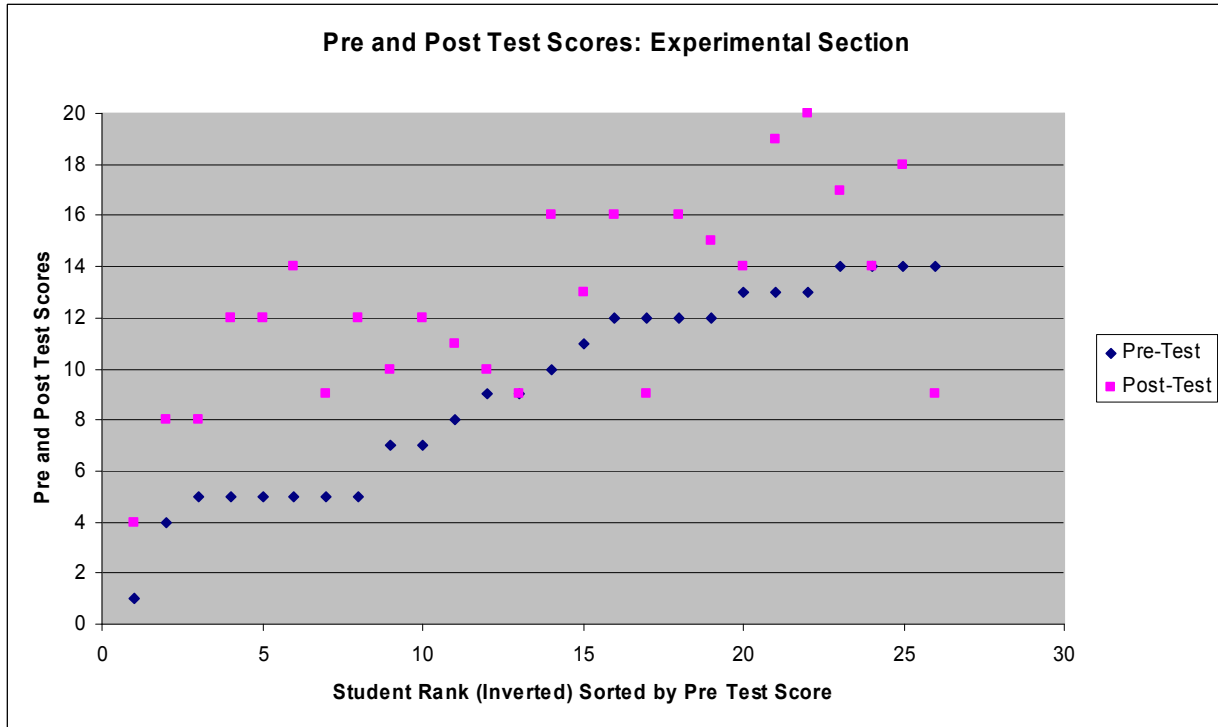


Figure 1. Pre and post test scores in the problem solving section sorted by pre test score.

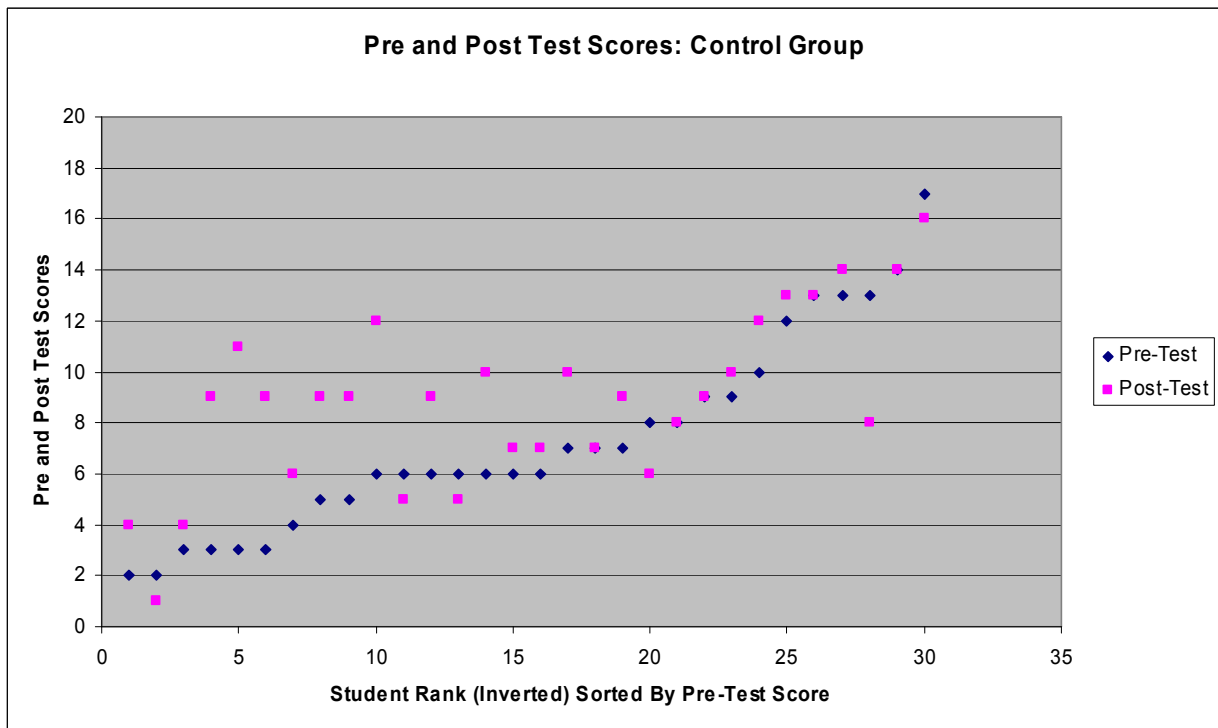


Figure 2. Pre and post test scores in the control section sorted by pre test score.

The self-efficacy scales showed only a small increase (about 8 out of 250) in average score from the pre and post course questionnaires. Six students in the experimental group increased their score by at least 20 points out of 250. Three students in the control group increased their scores by 20 points or more. The authors note that there were some severe discrepancies between self-efficacy questionnaire results and demonstrated problem solving ability for some students.

One minute papers by the students consistently showed that the students felt that working in teams was a very positive experience. They liked hands on exercises, and the two textbooks. The students indicated they would like a higher level of organization in the course, with clearer expectations. These comments lead to changes in the way assignments were made. In general the student teams were self selected, and this worked well.

Instructor observations showed substantial improvement in producing suitable diagrams. This is however still an area where there was room for improvement. A second issue that arose was student difficulties dealing with problems involving the passage of time, in which the solution required was a function of a time variable  $t$ , rather than a number or set of numbers. In Spring 2005, the course will include more material on passage of time problems. These difficulties correlate well with difficulties students are having in diverse courses like Dynamics and the transient response portion of Electric Circuit courses.

## **5. Discussion.**

The pre and post assessments showed that although problem solving performance improved slightly in both the experimental and the control sections, the experimental section showed a statistically significant higher gain than the control section based on results for Questions 2 through 6 only. This result agrees with our classroom observations over the semester in which we saw a few students make dramatic improvements in problem-solving ability, many students make slight improvements, and other students make no improvements.

We anticipated that teaching a complicated skill to students with a wide variety of backgrounds and abilities was a daunting challenge. We consider the modest gains experienced by the class to be a partial success. We will use our experience from the first semester to improve the course effectiveness for the next semester. Our observations on the first semester include:

1. Some students needed so much remedial mathematics that they could not benefit from our problem solving course.
2. We need to more effectively facilitate student diagnosis and correction of their own deficiencies. Students frequently did not seem to learn from their mistakes on homework and in-class problems and repeated the same mistakes on exams.
3. Many of our students were identified by The Learning Connections Inventory (LCI) as scoring low on use of the Precise learning pattern. This concurred with our observation that a very frequent source of error was inaccurate or incomplete replication of information in the problem statement. Making students aware of their own learning patterns and the need to intentionally be more precise may be helpful for some students.

4. Students in the experimental section showed improvement in selecting systematic solutions strategies over trial-and-error, and in drawing more diagrams. Students need more work in evaluating and improving the quality of their diagrams.
5. Most students did not show improvement in their ability to articulate the problem solving process.
6. Some students continued to speak negatively about their math and problem solving abilities, as if conforming to an established social norm.

A key goal of our course is transfer of improved problem-solving ability to future coursework. We will compare performance on final exam questions in future Calculus and Physics classes of students in the experimental and control groups.

## 6. Acknowledgement

This work was supported in part from NSF Grant DUE-0341463 under the Course Curriculum and Laboratory Improvement – Educational Materials Development (CCLI-EMD) program as a proof of concept study.

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**Math Problem Solving Assessment**

**Fall 2004**

Solve the following problems. Show all your work. Calculators are not permitted.

1. An experimental atomic layer deposition (ALD) process can produce a layer of iron on a silicon wafer that is one atom thick. On a certain run, the process deposits a total of 30,668,000 atoms. If there are 614,360 atoms deposited per square micron, ABOUT how many square microns are covered? Note: There are  $10^6$  microns in a meter.
2. Bill's health food store stocks only two items, Sugar Frosted Flakes and barbeque sauce. He is open for 9 hours every day, 6 days a week. One morning, when he opens the doors, he notices he has 108 items on the shelves. He has an especially good day and sells everything in the store. When counting the money at the end of the day, he notices he took in a total of \$420. If a box of Sugar Frosted Flakes sells for \$5.00, and a jar of barbeque sauce sells for \$3.00, how many boxes of Sugar Frosted Flakes did Bill sell that day? Food items are exempt from sales tax in Bill's state.
3. You want to construct an open square box with no top by taking a square piece of cardboard, cutting squares out of the corners and folding up the sides. If the sides of the box are 4" high, how large a piece of cardboard do you need to make a box that holds a volume of 144 cubic inches? You may neglect the thickness of the cardboard.
4. The following problem does not contain all the data necessary to solve it. Clearly identify what additional data is needed. Then plug in the value 2 for all missing data and solve the problem.

A rectangular swimming pool has a uniform depth of 8 feet. It takes a good swimmer 20 seconds to swim the length of the pool, and 10 seconds to swim across the pool. How long in seconds does it take to fill the pool with water?

5. A workman places a 12" by 12" square stool with four legs on top of the roof of a typical house (i. e. with a single ridge down the middle, and sloping equally on each side) so that the surface of the stool is level. The legs are square, vertical, and flat on the bottom. The space between the legs on any side of the stool is 8", and each side of the roof is at a  $33^\circ$  angle from horizontal. If the stool is 12" high, and the house is 30' high, 50' long, and 30' wide, how high is the top of the stool above the ground? You may leave any trigonometric, exponential, square root or logarithmic expressions unevaluated. So  $2 \cos(15^\circ)$  would be ok as an answer.
6. Eight robbers stole a sack of diamonds from a jewelry store. After returning to their hide-out, they divided up the diamonds and 3 were left over. In the ensuing brawl over the 3 diamonds, one robber was killed. The diamonds were again divided among the 7 robbers and 3 were left over. The gang leader decided that he should get the 3 left over diamonds and was promptly shot, so the 6 remaining robbers divided up the diamonds and again 3 were left over. Feeling despondent and in fear for his own life, one of the robbers just gave up and left. Finally, this time the diamonds were divided evenly among the remaining 5 robbers. What is the least number of diamonds in the sack?

Name \_\_\_\_\_ Student ID \_\_\_\_\_ GES 131 Section \_\_\_\_\_

### Self-Appraisal Inventory

Indicate the degree to which you agree or disagree with the following statements on a scale of 1 – 10.

1	2	3	4	5	6	7	8	9	10
Strongly Disagree		Disagree		Neutral			Agree		Strongly Agree

- \_\_\_ 1. I am good at solving mathematics problems.
- \_\_\_ 2. When I read a mathematics problem, I can identify the mathematics concepts I need to solve the problem.
- \_\_\_ 3. When given a mathematics problem, I usually can find more than one approach that has a good chance of successfully solving the problem.
- \_\_\_ 4. I can check my solution to a mathematics problem and feel confident my answer is correct.
- \_\_\_ 5. I am good at solving word problems.
- \_\_\_ 6. I can draw diagrams that give me insight into how to solve a mathematical problem.
- \_\_\_ 7. When I work a difficult mathematics problem, I can identify other problems I have seen before that contain ideas that help me solve the problem at hand.
- \_\_\_ 8. I can introduce mathematical notation, such as additional variable names, that helps me solve problems.
- \_\_\_ 9. When I work a complicated mathematics problem, I can find simpler sub-problems that I can use to solve the original problem.
- \_\_\_ 10. If I get stuck when working a mathematics problem, I keep working for a long time because I know that I can eventually solve it.

Suppose that you were asked the following math questions. Please indicate how confident you are that you could solve the problem correctly. Assume you can use a calculator unless stated otherwise. **Do not attempt to solve the problems.**

Please use the following scale from 1 – 10, with

1	2	3	4	5	6	7	8	9	10
Cannot solve it at all				Moderately certain I can solve it					Certain I can solve it

- \_\_\_ 11. In a certain triangle, the sum of the sides is 24 inches. The longest side is twice as long as the shortest side, and the third side is 3 inches shorter than the longest side. What is the length of the longest side?
- \_\_\_ 12. **ABOUT** how many times larger than 514,360 is 20,668,000? Do not use a calculator.
- \_\_\_ 13. Bridget buys a packet containing 9 cent and 13 cent stamps for \$2.65. If there are 25 stamps in the packet, how many are 13 cent stamps?
- \_\_\_ 14. On a certain map,  $\frac{7}{8}$  inch represents 200 miles. How far apart are two towns whose distance apart on the map is  $3\frac{1}{2}$  inches.
- \_\_\_ 15. Five math tests are given to Mary's class. Each test has a value of 25 points. Mary's average for the first four tests is 20. What is the highest possible average she can have on all 5 tests?
- \_\_\_ 16. In a small auditorium the chairs are usually arranged so there are 5 rows and 10 seats in each row. For a popular speaker, extra rows are added, and the same number of extra seats is added to each row (if you add 3 rows, you then add 3 seats to each row, including the new ones). Find the number of rows that must be added to triple the total number of seats in the auditorium.
- \_\_\_ 17. Solve  $\frac{2}{x+1} + \frac{4}{x+2} = 3$  for x.
- \_\_\_ 18. Solve  $\sqrt{2x} = \sqrt{x+1} + 1$  for x.
- \_\_\_ 19. Solve  $5^{4-x} = 7^{3+x}$  for x.
- \_\_\_ 20. Machine A can harvest all the wheat in certain field in 6 hours. Machine B can harvest the same field in 10 hours. How long would it take the two machines, working together, to harvest the field.
- \_\_\_ 21. A rectangle is inscribed inside a circle with radius r, (all four corners are on the circle). Express the area A of the rectangle in terms of the length x of one side of the rectangle and r.
- \_\_\_ 22. A function f(x) is even if  $f(x) = f(-x)$ , and odd if  $f(x) = -f(-x)$ . For any function g(x), prove that  $g(x) + g(-x)$  is even, and  $g(x) - g(-x)$  is odd.
- \_\_\_ 23. Without using a calculator, determine which number is bigger,  $\sqrt[10]{10}$  or  $\sqrt[3]{2}$  ?
- \_\_\_ 24. Find the equation of a straight line passing through the points  $(x,y) = (1, 2)$  and  $(x,y) = (-1, 1)$ .
- \_\_\_ 25. A flagpole stands in a flat open field. Fred puts his head next to the ground 50 feet from the pole, looks up at the top of the pole, and determines he is looking up at an angle of  $20^\circ$ . Find the height of the flagpole.