Innovative Way of Teaching Magnetic Circuits with Minimum Vector Algebra

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Abstract- **In schools of electrical engineering around the nation and abroad, some curriculums offer a limited electromagnetic course or no course at all. This is an issue of curriculum constraints, thererefore; magnetic circuits need to be taught in other courses to be able to assure that the students are exposed to the material. An appropriate course that we use is the energy conversion class, sometimes called "Electrical Machines." It is no wonder that almost all textbooks [1] used in the area have extensive chapters dealing with magnetic circuits. This paper is a study of an effective way of transitioning a topic from one area of electric engineering to another (from Electromagnetic Fields to Electric Power Systems)**

Symbols:

"E" electric field [V/m]
"H" magnetic field intensity [A/m]
"B" magnetic field intensity [Webbers/m^2]
Or [T]
"ϕ" magnetic flux [Webbers] or [Wb]
"D" displacement flux density [C/m^2]
"J" current density [A/m^2]

I. INTRODUCTION

Historically magnetic circuits are included in "Electromagnetics" courses and consequently these courses became prerequisites for electric energy conversion. In many programs nationwide there is no required electromagnetic course prerequisite anymore and students end up not knowing the basics of magnetic circuits.

However, the background necessary to assure understanding of the material comes from the first and second college physics courses. Furthermore, if the energy conversion course is offered, it is done as an elective in most institutions.

The effect of graduating students with an electrical engineering degree without an energy conversion course is very detrimental to their basic knowledge. We will discuss this issue further down the road in this paper. Staying focused on the transition, we will show how we teach the concepts and how the student's background from their regular physics classes is sufficient to understand, comprehend and learn the material.

II. MAGNETIC CIRCUITS

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Magnetic circuits deal with magneto-static equations discovered by Maxwell[1]. These equations can be of integral and/or differential form. We are presenting them here for future reference.

The first one is Faraday's law

$$\oint_c E \bullet dl = -\frac{d}{dt}\int_s B \bullet dS \qquad (1)$$

A simple explanation is; in a wire the rate of change of a magnetic flux density induces a voltage.

The differential form of this equation is shown in equation 2:

$$\nabla \times E = -\frac{\partial}{\partial t}(B) \qquad (2)$$

Where;

∇x <u>curl of a vector</u> (appendix 1)

Where, "∇" pronounced "del" is a three dimensional vector with coordinates in the Cartesian, Cylindrical and Spherical coordinate system. Remember that we are just trying to show the way we develop a concept and how it is presented to our students.

If we follow the same way of thinking we can obtain Maxwell's equation for Ampere's law.

$$\oint_c H \bullet dl = \int_s J \bullet dS + \frac{d}{dt}\int_s D \bullet dS \qquad (3)$$

Which can be explained as follows; *the flow of current in a wire will induced a magnetic flux*

In differential form we get:

$$\nabla \times H = J + \frac{\partial}{\partial t}(D) \qquad (4)$$

The remaining Maxwell's laws are the ones for Gauss's law for the electric field and Gauss's law for Magnetic fields.

$$\oint_s D \bullet dS = \int_v \rho dv \qquad (5)$$

Again, this equation reads as follows; *the flow of charges in a wire creates a flow of current in a wire.*

In differential form;

$$\nabla \bullet D = \rho \qquad (6)$$

Where

$\nabla \bullet$ <u>Divergency of a vector</u> (appendix 1)

Then Maxwell's equation for Gauss's law of magnetic fields is:

$$\oint_s B \bullet dS = 0 \qquad (7)$$

This equation reads as follows: *The net magnetic flux density in a closed surface area is zero, i.e. the flux entering an area is the same as the one that leaves that same area.*

In differential form;

$$\nabla \bullet B = 0 \qquad (8)$$

III. TRANSFORMATIONS

For practical engineering use, we carry on some simplifications in order to reduce the complexity of the equations and be able to apply them to our engineering problems. We must keep in mind that if our graduating students get involved in a sophisticated electromagnetic problem, they should at least be capable of understanding the problem and communicate with the expert in the field.

Setting up a table with the equations used most frequently, we obtain the following.

[1] Maxwell James Clerk developed his set of equations in 1873, in Cambridge University England

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Maxwell's Differential Equations
Faraday's Equation
$\oint_c E \bullet dl = -\frac{d}{dt}\int_s B \bullet dS$ Transformation Equations $\oint_c X \bullet dl \Rightarrow \nabla \times X$ $\int_s X \bullet dS \Rightarrow X$ $\frac{d}{dt}\oint_s X \bullet dS \Rightarrow \frac{\partial}{\partial t}(X)$ Final Equation Differential form $\nabla \times E = -\frac{\partial}{\partial t}(B)$
Ampere's Equation
$\oint_c H \bullet dl = \int_s J \bullet dS + \frac{d}{dt}\int_s D \bullet dS$ Transformation Equations $\oint_c X \bullet dl \Rightarrow \nabla \times X$ $\int_s X \bullet dS \Rightarrow X$ $\frac{d}{dt}\oint_s X \bullet dS \Rightarrow \frac{\partial}{\partial t}(X)$ Final Equation Differential Form $\nabla \times H = J + \frac{\partial}{\partial t}(D)$
Gauss's Electric Field Equation
$\oint_s D \bullet dS = \int_v \rho dv$ Transformation Equations $\oint_c X \bullet dl \Rightarrow \nabla \times X$ $\int_s X \bullet dS \Rightarrow X$ $\int_v X \bullet dl \Rightarrow X$ Final Equation in Differential Form $\nabla \bullet D = \rho$
Gauss's Magnetic Field Equation
$\oint_s B \bullet dS = 0$
Transformation Equation $\oint_s X \bullet dS \Rightarrow \nabla \bullet X$ Final Equation Differential form $\nabla \bullet B = 0$

IV. ENGINEERING SIMPLIFICATIONS

After obtaining Maxwell's equations in differential and integral form, we proceed to make engineering assumptions to further simplify them in their differential form.

At this particular point we emphasize the engineering approach; Let **E** and **H** be fundamental fields and **D** and **B** derived fields. The relationship between those fields is as follows:

We are assuming that B is uniform across the magnetic material (which is assumed in most engineering applications).

D = ε **E**

And

B = μ **H**

Where:

$\varepsilon = \varepsilon_o \varepsilon_r$

$\mu = \mu_o \mu_r$

ε Permittivity of the material

ε_o Permittivity of free space (8.85×10^{-12} F/m)

ε_r Relative permittivity of the material

μ Permeability of the material

μ_o Permeability of free space

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$((4\pi x 10^{-7} H/m)$

μ_r Relative permeability of the material

In a conductor of radius "r" Faraday's law LHS is:

$$\oint_c H \bullet dl = \int_0^{2\pi} H_\varphi r d\varphi = 2\pi r H_\varphi$$

And the RHS of Faraday's law is:

$$\int_s J \bullet dS = I_r$$

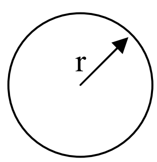

Figure1. Conductor with radius "r"

Therefore from 3:

$$2\pi r H_\varphi = I_r$$

Notice that the "displacement current" (D), was thrown out of the equation. The reason is that this current is so small compared to "J" that virtually it has no effect in our applications.

If there are "n" conductors, then;

$$2\pi r H_\varphi = nI_r$$

Therefore;

$$H_\varphi = \frac{NI}{2\pi r}$$

Since $B_\varphi = \mu H_\varphi$ we obtain $B_\varphi = \dfrac{\mu NI}{2\pi r}$

Then:

$$\varphi = \int_s B \bullet dS$$

Suppose we have a toroidal winding as shown below.

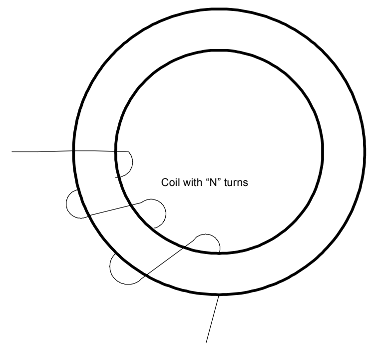

Figure 3 Toroid

Then integrating from x to y we get:

$$\varphi = \int_s B_\varphi \bullet dS = \frac{\mu nI}{2\pi}\int_x^y \frac{1}{r}dr \int_0^h dz = \frac{\mu nI}{2\pi} h \ln(\frac{y}{x})$$

More simplifications can be done if we use the geometry of the magnetic circuit. For example in figure 3, the average length is "l_{avg}"then $\oint_c H \bullet dl = Hl_{avg}$, this relationship is very powerful in magnetic circuits. Furthermore:

$$\oint_c H \bullet dl = \int_s J \bullet dS$$

$$Hl_{avg} = I$$

For several turns "n";

$$Hl_{avg} = NI$$

If $B = \mu H$ then; $B = \dfrac{\mu NI}{l_{avg}}$

Since $\varphi = BA$

Then $\varphi = \dfrac{\mu NI}{l_{avg}} A$

Let F be the magneto-motive force, "MMF" and it is defined as: $F = NI$ [A-turns]

Then $\varphi = \frac{F\mu A}{l_{avg}}$.Finally, if we define

reluctance as $R = \frac{l_{avg}}{\mu A}$ then; $F = \varphi R$

We can make an analogy from magnetic circuits to electric circuits; this is shown in the table below:

Example
The magnetic circuit shown if figure 4 has

Magnetic Variable	Electric Equivalent
R (Reluctance)	R (Resistance)
F (MMF)	V (Voltage)
φ (Magnetic Flux)	I (Current)

the following lengths:

$$l_{m1} = 17.78cm$$
$$l_{m2} = 15.24cm$$
$$l_{m3} = 13.7cm$$
$$l_{m4} = 10.16cm$$
$$l_{m5} = 9.96cm$$
$$width = 7.6cm$$
$$l_{g1} = 0.25cm$$
$$l_{g2} = 0.2cm$$
$$l_{w\_right} = l_{w\_center} = l_{w\_left} = 3cm$$

Where l_{w_xxx} is the width of the leg

The material is M-27, 24 gage. The number of turns is 150. Let the magnetic flux in the right leg be 2 mWb find:

- ϕ_{center}
- ϕ_{left}
- Current necessary in the coil

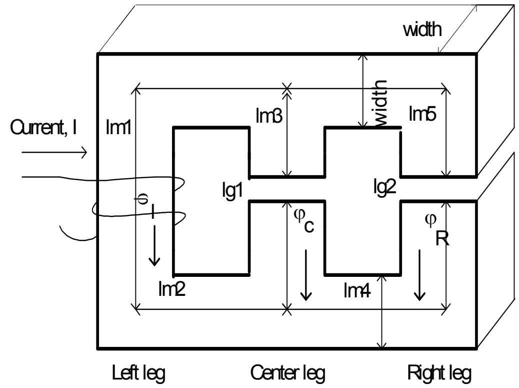

Figure 4 Magnetic-circuit for example

The magnetization curves for magnetic materials are shown in figure 5.

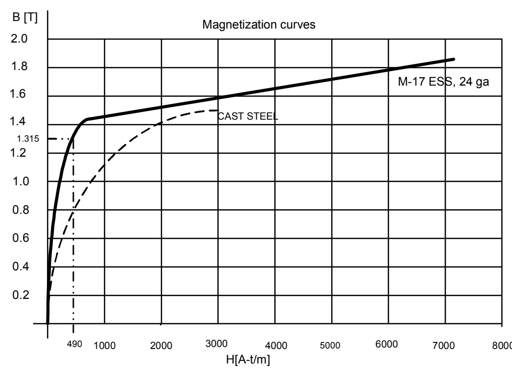

Figure 5 Magnetization curve for magnetic materials

Solution

$$\mu_o = 4\pi 10^{-7}$$
$$= 12.566e^{-7}$$

The magnetic flux density in the right leg is calculated using; $\phi = BA$, then:

$$B_{right} = \frac{\phi_{right}}{A_{right}} = 1.135[T]$$

Enter curve in figure 5 and read
$$H_{right} = 490[A - t / m]$$

The equivalent magnetic circuit is shown in figure 6. All the electrical theorems hold true for equivalent magnetic circuits. For example, current division, voltage division Kirchoff's laws, etc. are all applicable.

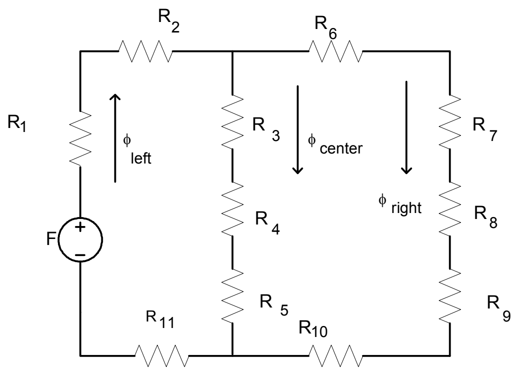

Figure 6 Equivalent magnetic-circuit

From now on we will state the expressions and curves that will help us solve the problem. The arithmetic is left for the student to do and practice. Grouping reluctances as if they were resistors, we obtain:

$$R_3 + R_4 + R_5 = R_{center}$$

$$R_6 + R_7 + R_8 + R_9 = R_{right}$$

$$R_{11} + R_2 + R_1 = R_{left}$$

The flux flowing in the right leg produces an MMF using the following relationship:

$$F_{right} = \phi_{right} R_{right}$$

Taking KVL on the right side of the circuit we obtain:

$$F_{center} = \phi_{right}(R_6 + R_{10}) + F_{right}$$

Since the MMF in the center leg is also:

$$F_{center} = \phi_{center} R_{center}$$

And solving for the magnetic flux in the center leg, we get:

$$\phi_{center} = \frac{F_{center}}{R_{center}}$$

Using KCL in the magnetic circuit, we find the magnetic flux produced by the coil:

$$\phi_{left} = \phi_{center} + \phi_{right}$$

The magnetic flux density is found as follows:

$$B_{left} = \frac{\phi_{left}}{A_{left}}$$

Using figure 5 we find the magnetic field intensity. Enter B_{left} and read off H_{left} .

Finally we can determine the current in the coil that is necessary to produce the magnetic flux in the right leg as stated in the beginning of this problem.

$$F_{left} = NI = H_{left}(l_{m1} + 2l_{m2})$$

And the current is calculated as follows:

$$I = \frac{F_{left}}{N}$$

V. CONCLUSIONS

Due to the low frequency operation used in electric power systems, and energy conversion, some terms in the different Maxwell's equations can be neglected. This is called "lumped –circuit theory." Then the time-varying term in equation 4 can be eliminated. This term is:

$$\frac{\partial}{\partial t}(D)$$

Consequently the "time-varying" term in equation 2 can be neglected as well.

Wave propagation and transmission lines theories are based on the complete set of Maxwell's equations with all the terms included.

As it can be observed the basic application of Maxwell's equations is not that difficult. However, the proofs and mathematical developments can be overwhelming to the average student. It is for this reason that we focus in the understanding of the equations, and their application. As a reference we use simple examples of vector algebra and definitions as is shown in appendix I just to

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clarify the math involved with the curl and divergency operator.

Furthermore, we do not use more than two lectures to explain all the material presented in this paper. We explain an equation, derive its solution, simplify it and then apply it. Then homework is given to reinforce the concept. While this approach is not perfect the response of the students has been positive and induced a better understanding of the material than the classic mathematical approach used in the past.

We will start documenting, assessing, and the results of this approach will be better as soon as we get significant data. So far we have used feedback from students in informal conversations.

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VII APPENDIX I

(A) Curl of a vector.

Cartesian: $$\nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical: $$\nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ A_r & rA_\varphi & A_z \end{vmatrix}$$

Spherical:

$$\nabla \times A = \begin{vmatrix} \frac{i_r}{r^2 \sin\theta} & \frac{i_y}{r\sin\theta} & \frac{i_z}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & r\sin\theta A_\varphi \end{vmatrix}$$

Example:
Determine the curl of the following vectors:

$$A = 4yi_x - 2xi_x$$

$$A = 2i_\varphi$$

Solution:

$$\nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4i_x & -2i_x & 0 \end{vmatrix}$$

$$= -6i_z$$

Also:

$$\nabla \times A = \begin{vmatrix} i_x & i_y & i_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & 2r & 0 \end{vmatrix}$$

$$= \frac{2i_z}{r}$$

(B) Divergence of a vector

Cartesian:

$$\nabla \bullet A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial_z}$$

Cylindrical:

$$\nabla \bullet A = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Spherical:

$$\nabla \bullet A = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\varphi}\frac{\partial A_\varphi}{\partial \varphi}$$

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