Teaching One degree-of-freedom vibration on the WWW

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Abstract

This paper describes an extensive set of WWW pages that include the transient and forced vibration of a single degree of freedom system. There are Java applets that allow animation, parameter variation and self-test questions with diagnostic feedback. In addition there are more conventional notes with access to theoretical derivations. As far as possible there are links to other pages where definitions of terms are given and illustrated. As the volume of material increased to even include non-linear vibration the question of ease of navigation arose. It is proposed that recommended route options are a possible aid to students. It would of course be possible to explore the material in a free browsing manner but recommended routes for particular requirements are available. Thus a user who wishes to just look up definitions can follow one route, a student wishing to learn at a deeper level may follow another route and so on. The authors have effectively used sign posts similar to those available in National Parks so the preferred route may be chosen but users may choose to explore in their own way. It is concluded that such predefined routes may be very helpful to those who would otherwise get lost in the large amount of material available.

1. Introduction

The use of the WWW in teaching is becoming widespread. However much of the material produced simply presents titles codes and course content. Some lecturers have produced WWW versions of their lecture notes but have not made use of any of the other possibilities afforded by the WWW. This applies to the topic considered in this paper, that of teaching vibration. A search of the WWW found only a few sites that had any material on vibration. Appendix A lists some sites of interest and what they offer. From previous experience using computers in teaching it has been found [1,2] that a subject which involves motion can be better comprehended by students if they can observe the motion. It is even more beneficial if the student is able to interact with the computer and see the effects of making changes. Finally it has been found [3-5] that improved exam performance can be achieved by having on-line quizzes, especially if the problems have immediate diagnostic feedback. When the currently available WWW material on vibration is examined most of these desirable features are missing. This paper describes such material for teaching one degree-of-freedom (DOF) vibration. In addition because of the amount of material a recommended route is provided. Students may make excursions out of interest but the route may easily be re-joined.

2. Overview of Content

The one DOF package is the first of a series that is being written to cover the topic of vibration. It is expected that a common format and notation will be used throughout. The contents of the first package as seen on the WWW is shown in Figure 1. There is much that would be common to any text book treatment of the topic. However because of the advantages of the WWW and Java applets it is possible to cover some material that is rarely treated but is of significant practical importance. With treatments of any topic there is always the concern about what may reasonably be assumed to be prior knowledge. To avoid boring those with prior knowledge it has been assumed that all necessary dynamics is known. However where it is considered that the prior knowledge may not be known there are links to explanations. This is explained in the general information as shown in Figure 2. The information shown on selecting degree-of-freedom is not included because of space limitations. However all the material is freely available on the WWW at http://www.mech.uwa.edu.au/bjs/Vibration/OneDOF/ Figure 2 also shows the information on a "recommended path" but more of this later.

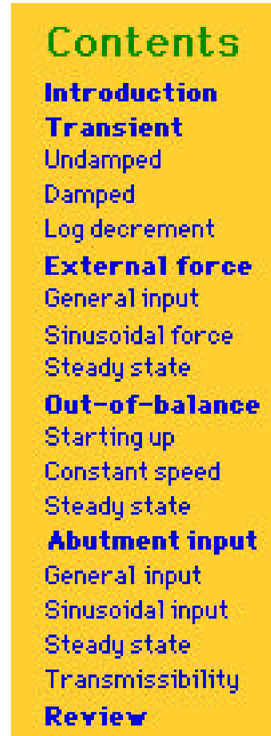


Figure 1 - Contents

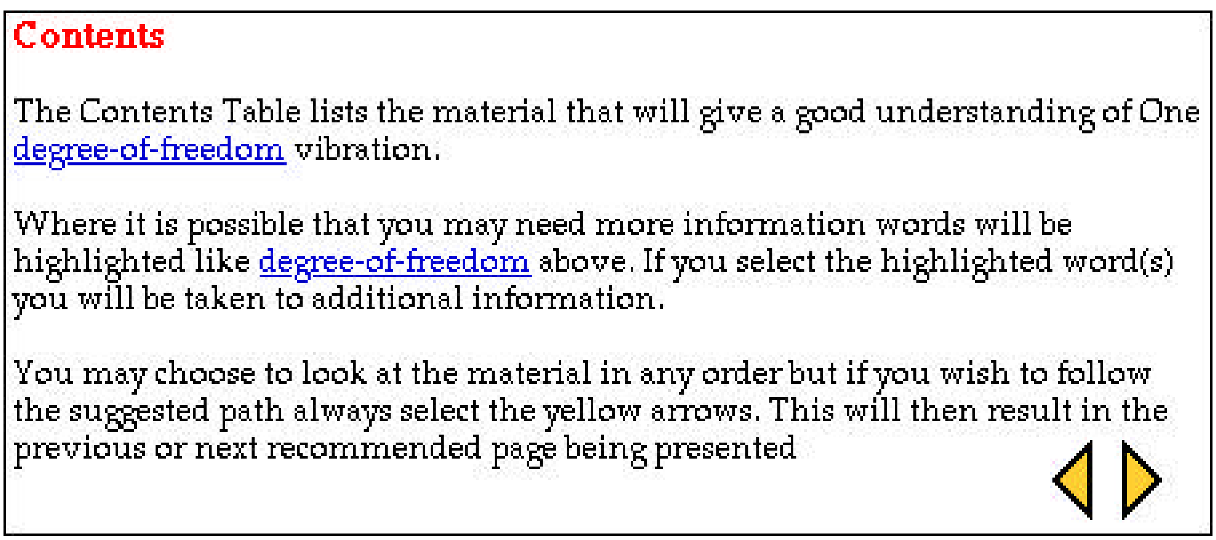


Figure 2 General information given at the start.

3. Levels

The material is presented at different levels. As far as possible mathematical derivations have been kept out of the top level. Figure 3 shows part of the "Damped" page which is in the "Transient" section. The various links may be followed as "excursions" as necessary. The next recommended section deals with "Log decrement" as shown in Figure 4.

Damped

Real vibration systems have a source of energy dissipation and it is convenient to represent this by a massless viscous damper as shown. This produces a drag force opposing the motion and which depends on the velocity of the mass.

Thus the damping coefficient c, of the damper, results in an additional force -cx'(t) on the mass. Thus from Newton's second law of motion using a free-body diagram, the equation of motion is,

$$mx''(t) = -kx(t) - cx'(t) \quad \ldots\ldots\ldots\ldots \quad (2)$$

It is useful to divide equation (2) by m so that rearranging we obtain,

$$x''(t) + 2\xi\omega_n x'(t) + \omega_n^2 x(t) = 0 \quad \ldots\ldots\ldots \quad (3)$$

Where ω_n is the undamped natural frequency as before and the viscous damping ratio is defined as

$$\xi = \frac{c}{2\sqrt{km}}$$

The solution of equation (3) has a different form depending on the value of ξ. If the initial conditions are x(0) and x'(0) then for

$\xi < 1$

$$x(t) = e^{-\xi\omega_n t}\left[x(0)\cos\omega_n t\sqrt{1-\xi^2} + \frac{[x'(0) + \xi\omega_n x(0)]\sin\omega_n t\sqrt{1-\xi^2}}{\omega_n\sqrt{1-\xi^2}}\right]$$

$\xi = 1$

$$x(t) = e^{-\omega_n t}\left[x(0) + [x'(0) + \omega_n x(0)]t\right]$$

$\xi > 1$

$$x(t) = e^{-\xi\omega_n t}\left[x(0)\cosh\omega_n t\sqrt{\xi^2-1} + \frac{[x'(0)+\xi\omega_n x(0)]\sinh\omega_n t\sqrt{\xi^2-1}}{\omega_n\sqrt{\xi^2-1}}\right]$$

The different types of motion may be seen by running the animation program.

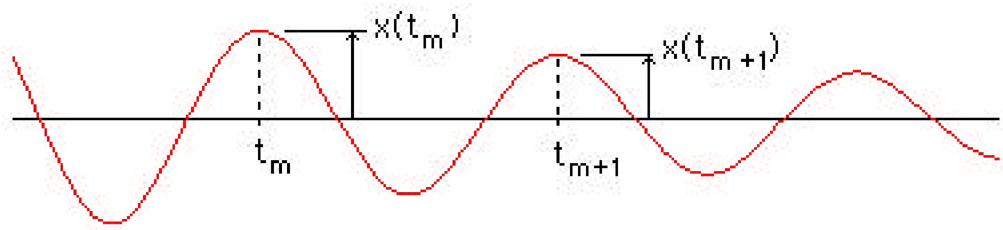
Figure 3 Part of the top level page for damped transients.

Log decrement

When the damping ratio $\xi < 1.0$ then vibration will occur and the motion is defined by

$$x(t) = e^{-\xi\omega_n t}\left[x(0)\cos\omega_n t\sqrt{1-\xi^2} + \frac{[x'(0) + \xi\omega_n x(0)]\sin\omega_n t\sqrt{1-\xi^2}}{\omega_n\sqrt{1-\xi^2}}\right]$$

and looks like



It can be shown that, if the amplitudes on any two succesive peaks are measured, the ratio of these amplitudes is constant. For any value of m, the log decrement will be

$$\delta = \ln[x(t_m)/x(t_{m+1})] = 2\xi\pi/\sqrt{1-\xi^2}$$

This equation can be rearranged to give,

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

Figure 5 Part of the "Log decrement" top level page.

To understand log decrement it is crucial that the student understand the top-level "damped" page. As a student is not told what needs to be carried forward then it may be necessary to return to the previous page to check the first equation. The yellow trail arrows easily allow this even when the page was *not the one previously being viewed*. As an illustration of level two material, Appendix B shows what happens when the shown link in Figure 5 is selected. This paper in fact uses the same strategy as the WWW pages. Detailed maths is "relegated" from the main body of the text. As a colleague who peer reviewed [6] the pages commented,

I looked at the web address and like it. I would much prefer that treatment to that of a text book(except that an equation editor would be helpful). The animations help but also the concise economical manner of presentation. A general comment on the subject at large (not your treatment) is that although it starts off innocently, it quickly degenerates into very complex mathematics. If you do manage to master the mathematics, there are limited applications waiting to reward your efforts. Is this being too harsh?

4. Quizzes

Figure 6 shows the NOTE section which would be seen along side the material shown in Figure 5. These sections are used to stress crucial and frequently misunderstood ideas.

At this stage it is appropriate to make an excursion. Students need quizzes to test their understanding and to have misconceptions corrected. The use of diagnostic WWW based testing has been extensively developed by Scott [3,4] and stand-alone quiz questions have been included in the material. Thus students may choose to check their understanding. If the red button is selected the problem shown in Figure 7 is presented. This problem has been designed to recognise standard errors that students tend to make in this topic. Part of the code for the problem applet is given in Figure 8. It is expected that students may just use the ratio of the two numbers as the decrement even though not consecutive peaks. If they do this then their wrong answer is anticipated and appropriate diagnostic feedback given. Also students sometimes use $\log_{10}$ rather than natural log. Again this is spotted even if the wrong ratio has also been used.

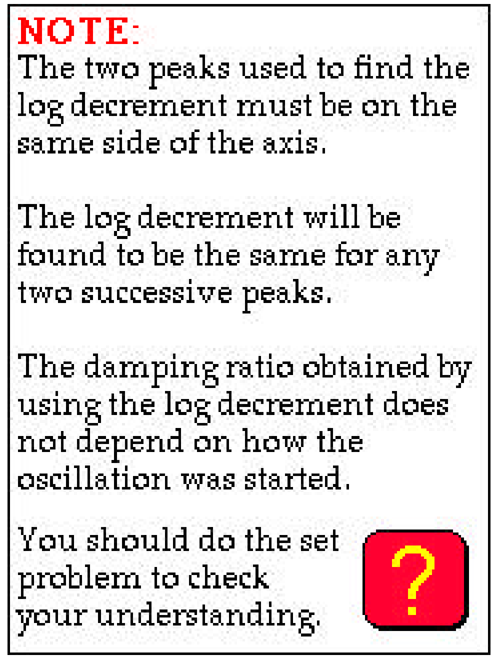


Figure 6 Stress of main points and quiz

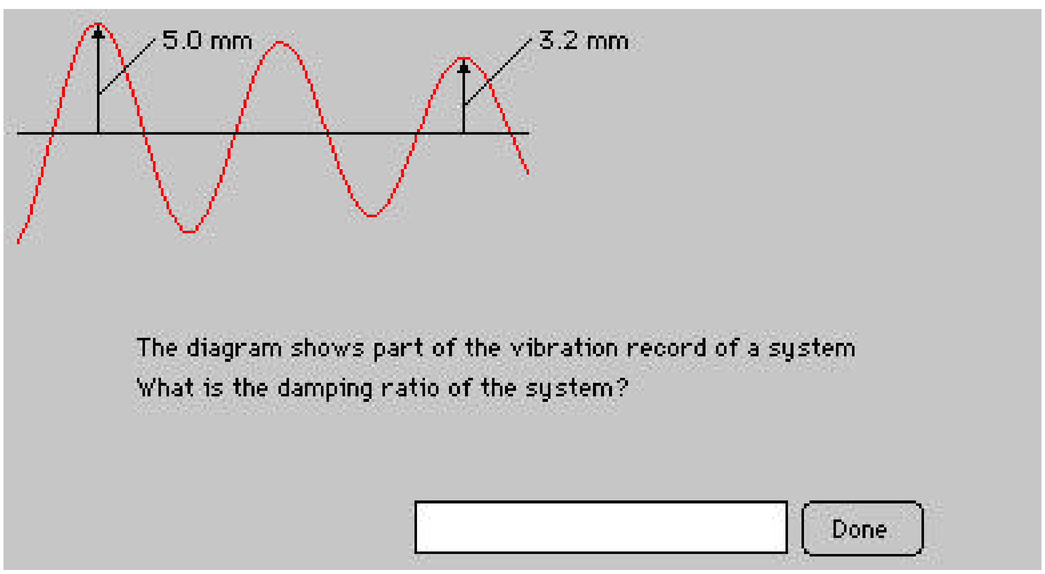


Figure 7 Diagnostic question for "Log decrement"

Scott [3,4] developed a serving system for such problems where students were given different problems and their performance could be monitored. The WWW pages described in this paper present all students with the same problem. Thus it would be possible to just have the right and wrong answers tested. However the code shown in Figure 8 was written so that the problem could be used in future in the serving system developed by Scott. More mature students will test themselves honestly, whereas less mature students need formal quizzes.

```
double Pisquared = Math.PI*Math.PI;
x1 = 5.0;// mm
x3 = 3.2;// mm
x2 = Math.sqrt(x1*x3);
delta = Math.log(x1/x2);
ClearErrorList();
// remember that the first "error" is the correct answer.
LoadError((delta/Math.sqrt(4.0*Pisquared+delta*delta)), "", "", "", 0, 1, 0, 0);
// ans, units, title, message, severity, c0, c1, c2 [competence classes]
delta = Math.log(x1/x3);
LoadError((delta/Math.sqrt(4.0*Pisquared+delta*delta)), "", "Ratio.html", "You must use the amplitude ratio on adjacent peaks.", 5, 1, 0, 0);
delta = Math.log(x1/x2)/Math.log(10.0);
LoadError((delta/Math.sqrt(4.0*Pisquared+delta*delta)), "", "Natural.html", "Use natural logs.", 5, 1, 0, 0);
delta = Math.log(x1/x3)/Math.log(10.0);
LoadError((delta/Math.sqrt(4.0*Pisquared+delta*delta)), "", "Natural.html", "Use natural logs.", 5, 1, 0, 0);
```

Figure 8 Part of Java code for quiz question.

5. Animations

The teaching of vibration calls out for simulations and animations. A dynamic subject should be presented in a dynamic way. It should not be overwhelmed by mathematics. As yet there are very few examples on the WWW. The WWW pages for one DOF described in this paper have many animation programs included. Such animations are best appreciated by trying them.

http://www.mech.uwa.edu.au/bjs/Vibration/OneDOF/

Animation is not just used for showing the motion of the one DOF system. It is possible to also use animation to show the connection between rotating vectors and sinusoidal motion. This can also be helpful in showing the significance of phase between two sinusoidal motions. Figure 9 shows the static level 2 page defining phase angle. There is no need to access this page from the top level (the recommended path) unless the definition is required. If after examining the page the definition is not clear there is a link to an animation program that should clarify matters. A static screen-shot from this program is shown in Figure 10. Again the benefits of animation can only be appreciated by running the program.

With so much to be gained from illustrating vibration by means of animations it is surprising that apparently there are very few examples on the WWW. This may be the result of a lack of Java skills. Scott and Stone [7 - in these proceedings] have developed various Java *shells* for the work described in this project and a detailed description of them is given in their paper.

6. Conclusions

This paper has given an overview by illustration of an extensive set of WWW pages for teaching vibration. As the material grew in volume it was clear that an aid was required to help students find their way through. As explained in Figure 2 there is a recommended route that the instructor suggests. For a different course or instructor an alternative route may easily be

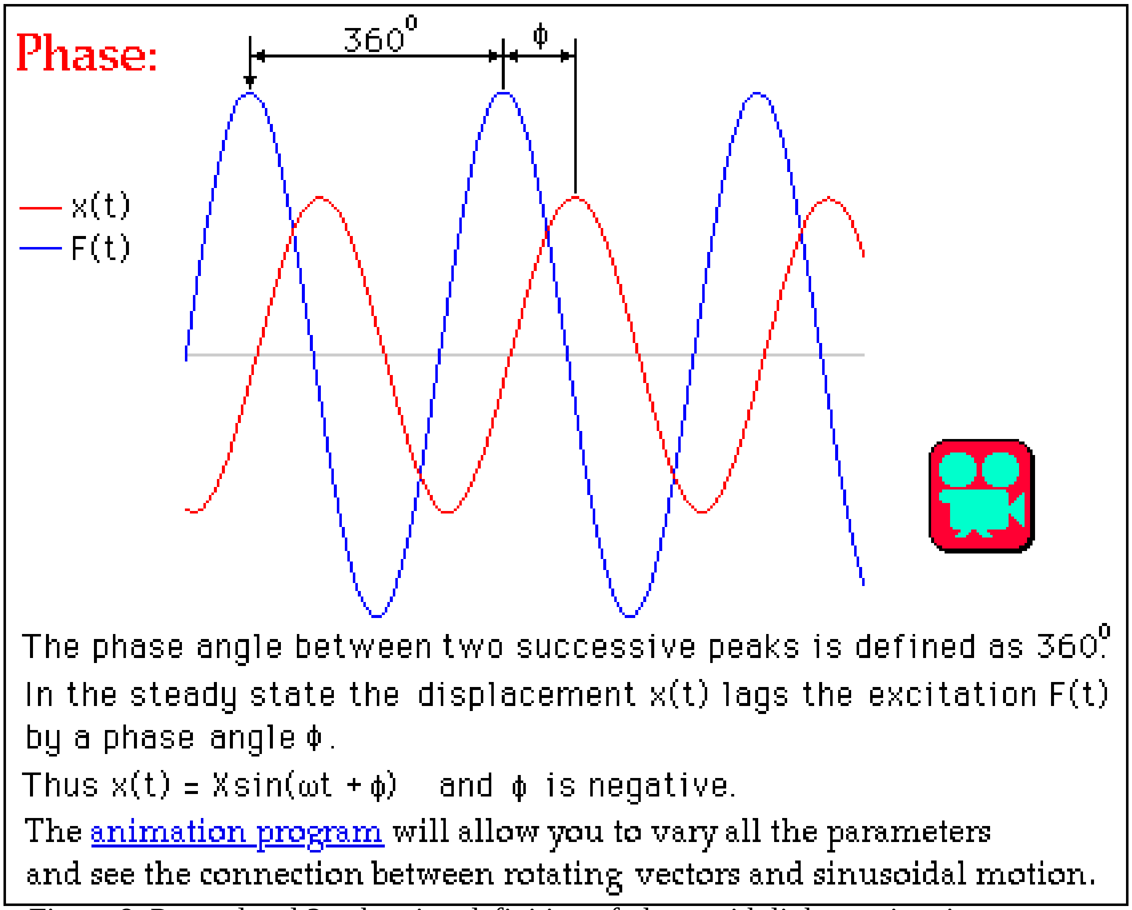


Figure 9 Page - level 2 - showing definition of phase with link to animation program.

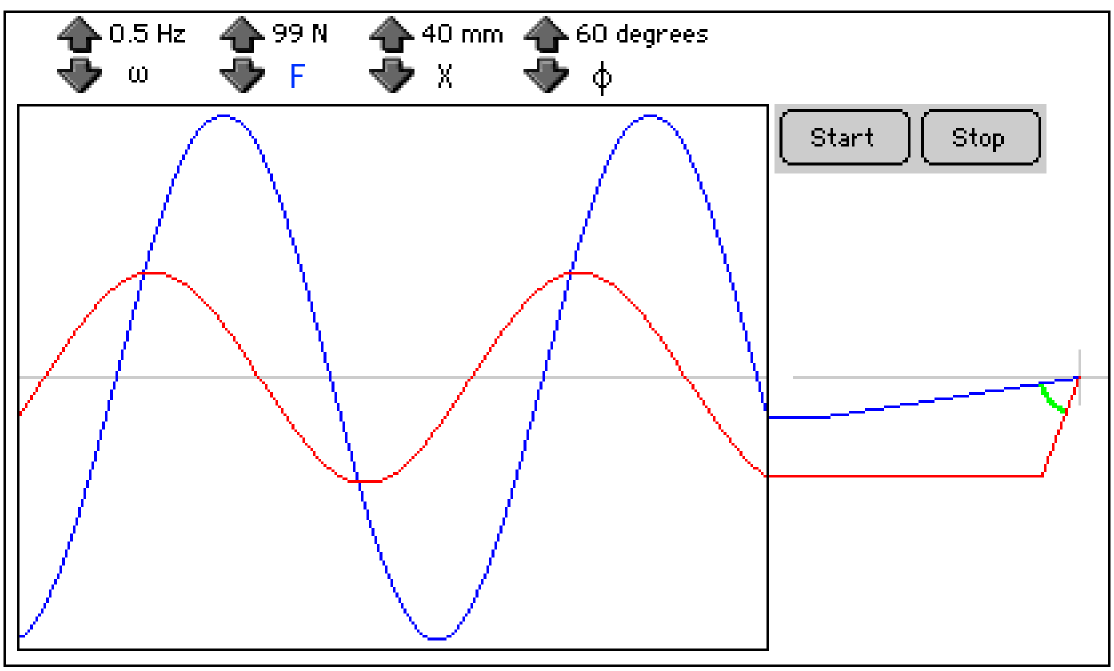


Figure 10 Animation to show connection between rotating vectors, sinusoidal motion and phase. (This diagram has been truncated to fit)

added. The basic material contained in the pages has been used for over 10 years as a set of Hypercard notes on a Macintosh for students enrolled in an advanced vibration course but who have had little or no exposure to vibration. The students (without exception) have been able to catch up by using those notes. It is expected that the WWW version will provide the same help. However there is far more information in the WWW version and it is hoped that these notes will find a use in many and diverse courses.

Bibliography

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3. Scott, N. W. and Stone B. J., 'A flexible Web-based tutorial system for engineering, maths and science subjects', Global Journal of Engineering Education, Vol. 2 (1998), No. 1, 7-16
4. Scott, N. and Stone, B. J., 'Continuous monitoring and class performance', 1996 ASEE Annual Conference Proceedings, 222002, 1-7.
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6. Devenish, D. G., private communication, 1999.
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Appendix A: Some WWW teaching material for vibration
Current October 1999

University of Akron, Mechanical Engineering Department
Course ME 4600 - 431: http://quinn.mech.uakron.edu/me431/
Static Class Notes (Limited Topics), Homework- Textbook problems with only the problem number, Problems and Solutions in the form of Exam Papers, Syllabus and a description of student project. Emphasis on using MATLAB and Working Model software.

US Naval Academy, Mechanical Engineering Department
Course EM 423: http://web.usna.navy.mil/~ratcliff/EM423/index.htm
Course details, Static Handouts (notes) on General data sheet comprising material properties, conversions, etc, Single Degree of Freedom (SDOF) mass- spring - damper system, Discrete Systems and Continuous systems. Notes are in detail with assignment problems at the end of each chapter. A secret link is provided where solutions for the assignments are provided which is text based.

University of Wisconsin - Madison, College of Engineering
Course - EMA 545: http://www.engr.wisc.edu/ep/ema/courses/ema545.html
WWW Resources, Goals, Syllabus, Prerequisites by topic, Credits, Reference. Uses MATLAB to solve ODE's. Few Downloads are available for solving ODE's and MATLAB files for vibration of continuous systems.

Wright State University, Mechanical Engineering Department
Course - ME 460/660: http://www.cs.wright.edu/people/faculty/jslater/vibration.html
General Materials of interest comprising course syllabus, class newsgroup and an introduction to UNIX/ MATLAB/ SMS Star Introduction. Assignments comprising problems are given in the html format along with final answer. MATLAB is used for solving problems.

The John Hopkins University, Department of Mechanical Engineering
Course - M.E.530.343: http://spray.me.jhu.edu/~llw/courses/me530343/index.html
Course description, Required text, Syllabus,
Requirements, Prerequisites, Problem sets, Laboratory sets, MATLAB examples with notes. Problem sets comprise of only the problem number of the prescribed textbook with occasional hints.

Equipment Reliability Institute
E-mail Distance Learning on Vibration and Shock

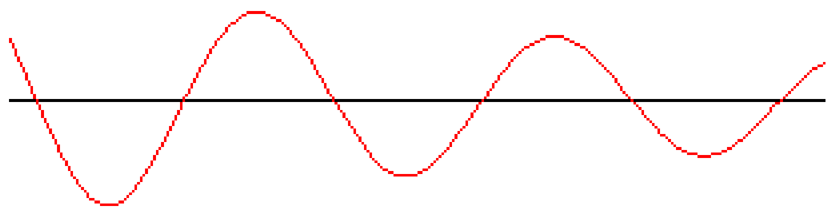
Outline of the course,
System requirements and a sample lesson.
http://www.equipment-reliability.com/ERIDist.html
This is available for a price. US$495 for individual element or US$1995 for the entire course. It comes with CD-ROM having Lessons and PowerPoint slides. Video clips are added. For example to show the fundamental mode of cantilever beam etc.

Vibrationdata.com 2445 S. Catarina, Mesa., AZ 85202, USA
Vibration Tutorials Homepage: http://www.vibrationdata.com/
Downloadable Software and Tutorials, Message board for posting questions.
Used by many universities and industry. The tutorial comprises of static notes in detail, but covers very few topics. Software that is available mostly emphasis on single degree freedom system. Interactive problems even though available, lacks animation, diagnostics and is not user friendly.

Appendix B: An example of detailed maths shown at level 2.

Solution: Log decrement

When $\xi < 1$ we have a decaying oscillation as shown. It is possible from a measured decay trace to calculate the value of ξ.



For arbitrary initial conditions the decaying motion will be given by,

$$x(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_n t\sqrt{1-\xi^2}) + B\sin(\omega_n t\sqrt{1-\xi^2})\right]$$

thus

$$x(t) = e^{-\xi\omega_n t}\, C\sin(\omega_n t\sqrt{1-\xi^2} + \varphi) \;\ldots\ldots\ldots\ldots\; (4) \quad \text{where } \tan\varphi = A/B$$

Now consider the maximum and minimum amplitudes of x(t) by finding the condition for x'(t) to be zero, differentiating (4) gives

$$x'(t) = C\left[-\xi\omega_n e^{-\xi\omega_n t}\sin(\omega_n t\sqrt{1-\xi^2} + \varphi) + \omega_n\sqrt{1-\xi^2}\, e^{-\xi\omega_n t}\cos(\omega_n t\sqrt{1-\xi^2} + \varphi)\right]$$

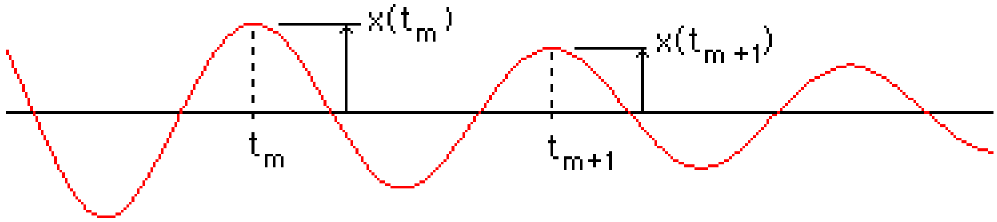
which is zero when

$$-\xi\omega_n e^{-\xi\omega_n t}\sin(\omega_n t\sqrt{1-\xi^2} + \varphi) + \omega_n\sqrt{1-\xi^2}\, e^{-\xi\omega_n t}\cos(\omega_n t\sqrt{1-\xi^2} + \varphi) = 0$$

$$\text{i.e. when } \tan(\omega_n t\sqrt{1-\xi^2} + \varphi) = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\text{or when } \omega_n t\sqrt{1-\xi^2} + \varphi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} + n\pi$$

This represents alternative max/min. Consider any two successive maxima or minima,



$$\omega_n t_m \sqrt{1-\xi^2} + \varphi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} + 2m\pi$$

$$\omega_n t_{m+1} \sqrt{1-\xi^2} + \varphi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} + 2(m+1)\pi$$

thus $\quad t_{m+1} - t_m = 2\pi/\omega_n\sqrt{1-\xi^2}$

Now $\quad x(t_{m+1}) = e^{-\xi\omega_n t_{m+1}} C \sin(\omega_n t_{m+1}\sqrt{1-\xi^2} + \varphi)$

$$x(t_m) = e^{-\xi\omega_n t_m} C \sin(\omega_n t_m\sqrt{1-\xi^2} + \varphi)$$

But $\quad \sin(\omega_n t_{m+1}\sqrt{1-\xi^2} + \varphi) = \sin(\omega_n t_m\sqrt{1-\xi^2} + \varphi) = \sqrt{1-\xi^2}$

since $\tan(\omega_n t\sqrt{1-\xi^2} + \varphi) = \dfrac{\sqrt{1-\xi^2}}{\xi}$ for $t = t_{m+1}$ and t_m

Therefore $\quad \dfrac{x(t_m)}{x(t_{m+1})} = e^{-\xi\omega_n(t_m - t_{m+1})} = e^{2\xi\pi/\sqrt{1-\xi^2}}$

The log decrement δ is defined as

$$\delta = \ln[x(t_m)/x(t_{m+1})] = 2\xi\pi/\sqrt{1-\xi^2}$$

Hence $\quad \xi = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

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