Teaching "Virtual Work" in Engineering Statics by a project

Engineering Statics curricula in most institutes do not comprise virtual work (VW), though the subject appears in most standard textbooks. In essence, potential energy of a mechanical system due to force (or load), moment, and gravity can be formulated as a function of the incremental displacement or rotation, for instance, $U(\theta)$ where θ is the titling angle of a solid. At equilibrium, $U(\theta)$ is at a stationary position such that $dU = 0$, or $(\partial U/\partial\theta) = 0$. A local maximum $(\partial^2 U/\partial\,\theta^2) < 0$ leads to an unstable equilibrium, a local minimum $(\partial^2 U/\partial\,\theta^2) > 0$ corresponds to a stable equilibrium, while an inflexion $(\partial^2 U/\partial\,\theta^2) = 0$ yields a neutral equilibrium. When a small disturbance or virtual linear or angular displacement is introduced, the resulting virtual work causes the system to either, topple or return to equilibrium. In classical physics, VW is equivalent to the principle of least action used to investigate the stability of a mechanical system. One possible reason that VW is not covered in Statics is the shift from the classical approach to solve equilibrium problems using force and moment balance only to a more abstract concept of work and energy. A more intense mathematical analysis is necessary to establish the system stability. Because VW introduces the important concept of energy, which is essential in understanding many mechanical concepts and even materials science, we decided it is worthwhile to include VW in our Statics course. A more novel pedagogy is necessary, however, due to the shift from the equilibrium approach to the abstract concept. This paper details a project that combines experiment and theory to give students an understanding of VW.

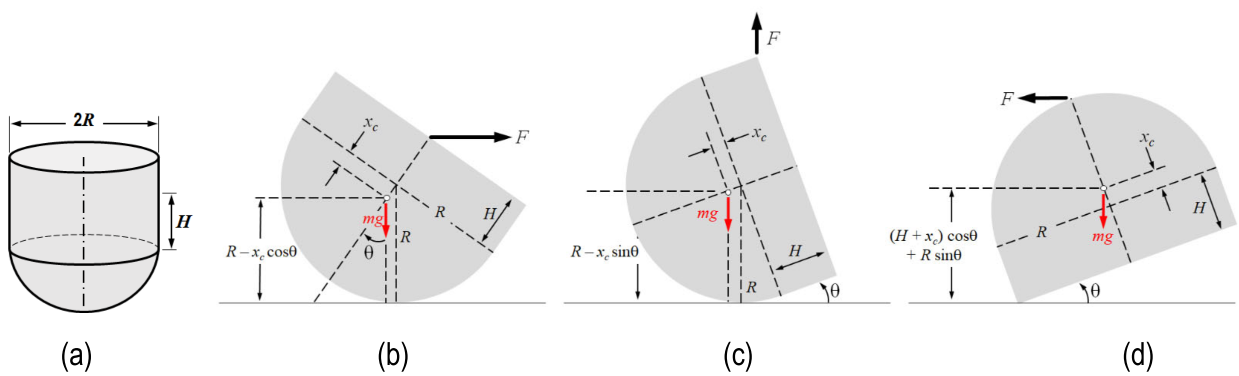

Figure 1. (a) Sketch of a roly-poly comprises a top cylinder with fixed height H and a hemispherical base with radius R. (b) Horizontal force F applied at the center of base area tilts the roly-poly at an angle θ. Center of gravity is x_c from the cylinder-hemisphere interface. (c) Vertical force is applied at one corner while the roly-poly lies on its side. (d) Horizontal force is applied at the apex of the curved surface while the roly-poly sits on its flat surface.

Since derivation of potential energy and application of virtual work require strenuous efforts to formulate the time-consuming problems, it is excessive to be included in a mid-term test or final exam. A term project is introduced with the experiential elements of experimental design, prototype fabrication, mechanical measurement, and mathematical analysis. Owing to the pandemic and campus lockdown, the project is designed to be performed at home using primitive homemade equipment. The project was introduced in Fall 2020 and will be repeated in a variant manner in Fall 2021. Students are asked to fabricate a few roly-polies using rudimentary materials such as ice, paper, chocolate, play-doh, wood, metal etc. which are readily available at home, or lathe and machining, 3-D printing, and molding if the equipment is available. The prototype comprises simple geometric primitives of a cylinder, hemisphere, and cone. Figure 1(a) shows a cylinder attached to the base of a hemisphere. The most important parameter characterizing the behavior of the roly-poly is the ratio of the cylinder height, H, to the hemisphere radius, R, or, $\lambda = H/R$. Students are asked to build a few versions of the roly-poly with different λ values. With small values of λ, the prototype should be at a stable equilibrium when sitting on its hemispherical base with its axis vertical in the absence of external load $F = 0$. For sufficiently large values of λ the prototype is unstable and topples on its own weight. For the intermediate values of λ, experiments are to be performed by applying vertical and horizontal forces to roly-poly in different orientation and configurations as shown in Figure 1. Force is measured a function of titling angle, $F(\theta)$, until toppling occurs, if at all. Figure 2 shows typical prototypes made by students and the force measurements.

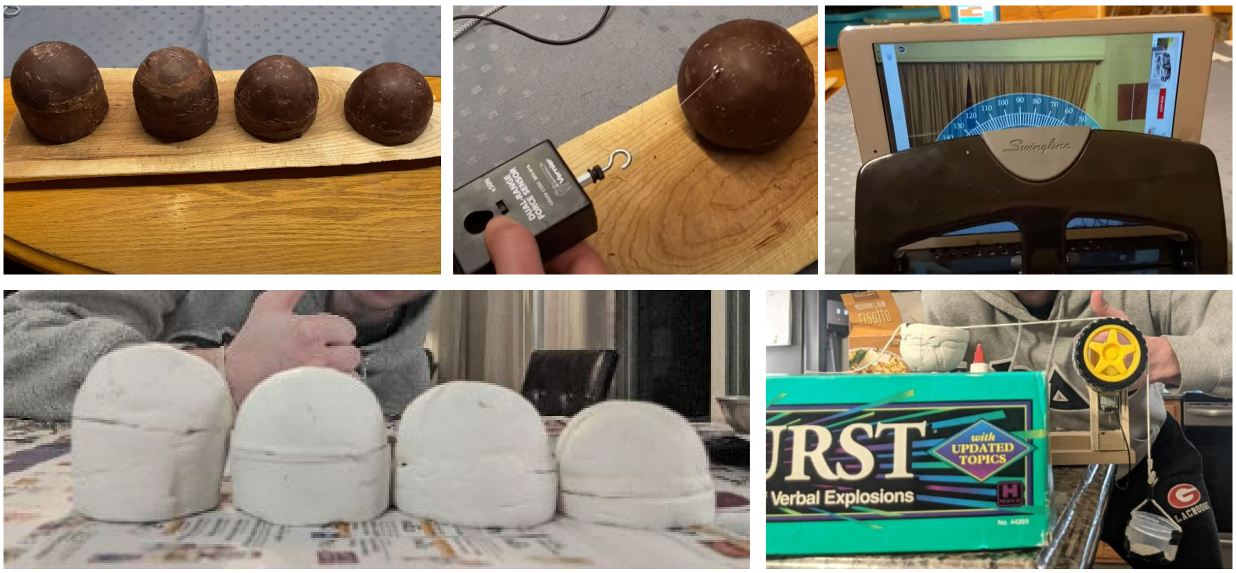

Figure 2. Typical student projects. Roly-polies made of chocolate and clay have a range of geometrical dimensions. Force measuring devices are acquired from the internet shops (above) or homemade using strings, pulley and grains.

Angular displacement is measured using a protractor. Alternatively, a picture of the leaning roly-poly is taken by a cell phone and the images are analyzed for the tilting angles. A homemade force gage using strings, paper, pencil and sand / rice grains is designed and constructed at home. It is noted that accuracy and precision of force measurement are unnecessary, as the project aims to trace the trends only. For instance, force can be measured in terms of number of sand grains rather than Newtons or pounds. Individual final reports cover the following sections:

(i) Free body diagram of the roly-poly under the influence of external load;
(ii) Center of gravity (CG) of the composite body of a roly-poly with specific R and H, i.e., determining x_c in Figure 1 in terms of R and H;
(iii) Mechanical equilibrium in terms of force and moment balance;
(iv) Potential energy $U(\theta)$ and external force $F(\theta)$ as functions of the tilting angle θ (c.f. Figure 3);
(v) Theoretical prediction of $F(\theta)$ using VW for different λ using Matlab;
(vi) Method of fabrication;
(vii) Experimental force and angle measurements against the theoretical prediction;
(viii) Data analysis and error estimation;

(ix) Discussion and Concluding remarks;
(x) Photos of the roly-polies and force measuring devices;
(xi) Video clips of the measurements.

Project assessment is based on mathematical rigor, ingenuity and aesthetics in engineering design, and measurements and analysis. The project takes an average of 2-3 weeks to finish. Students are encouraged to additionally rock the roly-poly to perform simple harmonic motion and to measure the oscillation frequency and amplitude, which is a preamble to subsequent courses in dynamics, engineering design, controls and mechatronics.

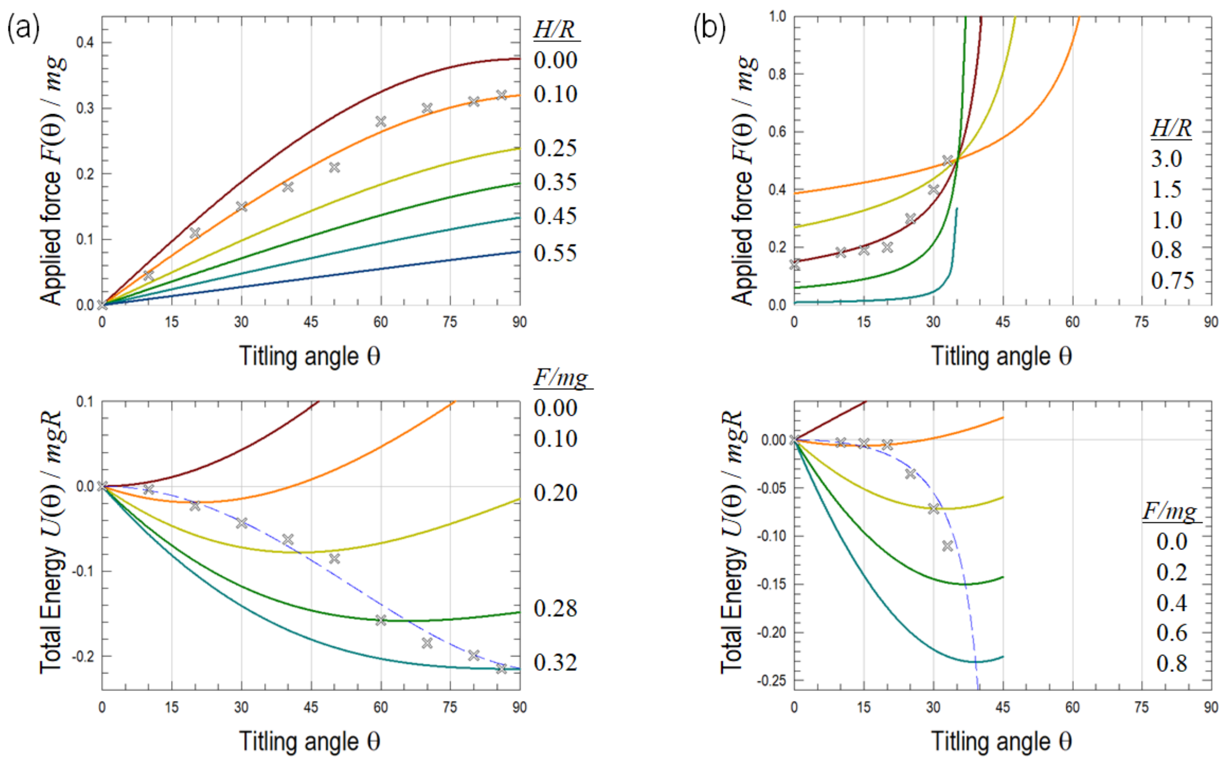

Figure 3. Typical results of theoretical curves: solid curves refer to $F(\theta)$ for specific λ and $U(\theta)$ for specific values of F, while the dashed curves are locus connecting the minimum of U(q). Experimental data are shown as symbols. Tilting angles are measured using a homemade device, and angular displacement using a protractor.

In Fall 2021, the project is slightly modified by replacing the cylinder with a pyramidal cone with a circular base, to prevent students from using others' roly-polies in the previous semester. Despite the similarities between the cylindrical roly-poly and conic roly-poly, it is more strenuous to derive the center of gravity and to fabricate the slightly more intricate geometry. Similar to the cylinder-hemisphere roly-polies, the conic polies exercise stable or unstable depending on the aspect ratio between the cone height and the hemisphere radius. External forces are applied horizontally and vertically as shown in Figure 4. The potential energy $U(\theta)$ and the corresponding force $F(\theta)$ as functions of tilting angle θ are to be found based on potential energy and virtual work. Experiments will be conducted at home / dormitories and measurements checked against the theoretical prediction. Evaluation of the pedagogy and learning will be conducted in December and will be reported in ASEE 2022 conferences.

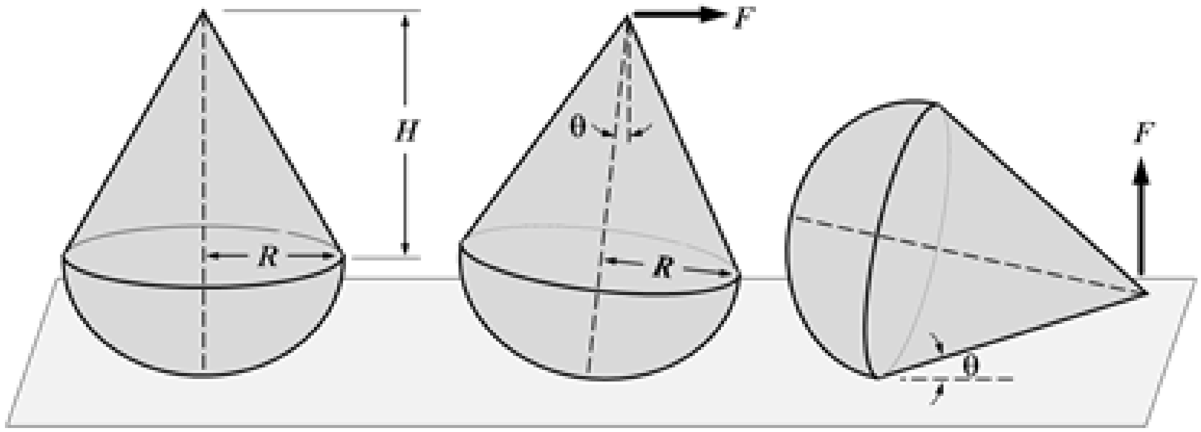

Figure 4. Roly-poly comprising a cone and a hemispherical base is under external horizontal or vertical forces applied at the apex.

Concluding remarks:

The project promotes experiential learning and arts of design and incorporates calculus-based energy method of VW into conventional Statics curriculum. A forerunner ME 2350 Statics project was implemented in Fall 2020 with 4 sections and 135 students, and an improved version is underway in Fall 2021 with 3 sections and 115 students. The course proves to be successful in a sense that students participate seriously as reflected by the many interactions with the faculty and teaching assistants. Most queries are about how to make the roly-polies, how to do the measurement, how to work out the theoretical prediction, and whether their measurement matches with the theory. Students show good understanding of the principle of virtual work. This project-based method is a viable way to teach virtual work in conventional Static courses.

It is worthwhile to mention that the prototype fabrication and force measurement are performed at home, which is particularly appealing during the campus lockdown due to pandemic. Sophisticated equipment such as universal testing machines (e.g. Instron) are available on campus, but the homemade devices turn out to be more educational in engineering design.