AC 2008-76: TECHNIQUES MOTIVATING PROJECT-DIRECTED MATHEMATICS

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IMPLEMENTING TECHNIQUES FOR PROJECT-DIRECTED MATHEMATICS

Abstract

This study is the third in a series examining ways to motivate learning of contemporary mathematics among design students at Virginia Commonwealth University Qatar (VCUQ).

In the first study the authors examined the learning preferences work of Ricki Linksman, founder of the National Reading Diagnostic Institute in the United States and author of *How to Learn Anything Quickly*. They theorized that female MATH 131 students at VCUQ were probably visual and tactile right-brained learners based on their artistic interests in three design majors and the characteristics of these types of learning preferences. Based on these learning preferences, the authors then suggested effective teaching strategies for motivating the students to master the concepts in this contemporary math course by relating the concepts to their culture and to interior, graphic, or fashion design fields.

A follow-up study took the previous work one step farther by *testing* the learning preferences of female students in two VCUQ MATH 131 classes in Fall 2006 through Spring 2007 semesters and examining the effectiveness of projects upon their concept mastery (*project-directed mathematics)*. Documenting the students' preferences revealed that over 65 percent were visual or tactile learners and, surprisingly, they were fairly evenly distributed between right-brained and left-brained preferences. The students found sample projects helpful, but their evaluations of creating their own projects were mixed.

The current study builds on the authors' earlier hypothesis that project-directed mathematics will facilitate better learning than will traditional lectures and problem-solving assignments in three primary ways: First, it continues tracking the students' "super-links" (or fastest, most effective modes of learning) by adding data from the Fall 2007 and Spring 2008 Semesters. Next, it continues observing the effectiveness of using projects as both samples and assignments in MATH 131. Finally, it examines techniques for effectively motivating project-directed mathematics.

The techniques implemented to motivate projects are focused on the design of both individual and group activities that engage students in high-level thinking and mathematical problem-solving. As a team, the professor and students discuss deeper mathematical properties illustrated within the students' projects. Because they are design majors, the students are very motivated to create unique projects that draw upon their artistic talent and creativity. Combined with a "mathematical spin" required by the professor, these projects offer a pilot for mathematical understanding. Preliminary results indicate that the project-directed approach is much more successful than the traditional lecture and problem-solving techniques because students are eager both to collaborate with their peers and professor and to compete against each other in developing the most creative projects.

Introduction

During the academic years 2003-2008, Dr. Schmeelk instructed several sections of a contemporary mathematics course in Doha, Qatar, a small peninsula of 4,400 square miles extending into the Persian Gulf and attached to Saudi Arabia. The size is comparable to the state of Connecticut in the United States. The population is approximately 770,000, and its leader is His Highness Sheikh Hamad Bin Khalifa Al Thani, Emir of Qatar. In 1997 Virginia Commonwealth University, located in Richmond, Virginia, USA, (VCUR) and Qatar's Emir collaborated to form the Shaqab College of Design Art located in Doha, Qatar, and primarily supervised by Qatar Foundation, an organization chaired by Her Highness Sheikha Mozah Bint Nasser Al Missned. In 2002 the school was officially renamed Virginia Commonwealth University Qatar (VCUQ) and began to operate more fully under the direction of VCUR.

Within the curriculum for design students in both VCUR and VCUQ, one general education requirement for all students is a contemporary mathematics course (MATH 131). Tailoring this course to fit the unique needs and interests of VCUQ majors became a unique and exciting challenge that gave rise to the authors' 2006 study entitled, "Making Connections Among Culture, Personality, and Content in Analytical Courses," which was presented at the March 2006 Conference of Middle Eastern Teachers of Science, Mathematics, and Computing in Abu Dhabi.

In their previous studies, the authors relied upon the work of Ricki Linksman, an expert in accelerated learning who founded the National Reading Diagnostic Institute in the United States and who popularized her research on accelerated learning in *How to Learn Anything Quickly.* Basically, Linksman claims that students learn best when new material is presented in ways that appeal to their favorite learning style (visual, auditory, tactile, or kinesthetic) and brain hemispheric preference (i.e., right-brain, left-brain, mixed, or integrated preference). The combination of learning style and hemispheric preference favored by an individual is known as that person's "super link," according to Linksman, and appealing to someone's super link is the fastest way for that person to learn.

In the first study of MATH 131, the authors deduced that the majority of the students would be right-brained visual or tactile learners based on observations of design students' characteristics and Linksman's definitions of these super links. The second study set out to test that deduction by recording students' super links after administering Linksman's tests for determining learning style and brain hemispheric preferences (see Appendices A and B). The tests were given early in the term shortly after introducing the course and its project-directed concept, and the results were discussed with the students, who also received handouts of Linksman's characterizations for each of the learning styles and brain hemispheric preferences.

Because Arabic art and architecture are strongly influenced by geometrical designs, MATH 131 includes several chapters of the mathematical text[6] that expand upon rotations, reflections, and translations. In addition, the course begins with mathematical formulas that speak to the issue of geometric shapes, followed by an intense development of the Fibonacci sequence and several of its properties illustrating the utility of the sequence in the "real world." In the current study, students were shown some past student projects submitted as partial fulfillment in the previous

MATH 131 courses to introduce each new topic visually and were required to complete a much more comprehensive project component (hence the term *Implementing Techniques for Project-Directed Mathematics*). The students were very much impressed by the past projects and wanted to compete with each other to find new projects that illustrate mathematical principles. The authors demonstrate and illustrate the procedures for several of these course topics, beginning with sequences and series.

Sequences, Series, and Fibonacci Numbers

To motivate students' interest, the instructor then begins with discussing mathematics in the Muslim Community, since most of the students are Muslims and VCUQ exists in a Muslim country. The first (Fig. 1 below) indicates several Muslim achievements such as inventing Algebra and the notion of zero, etc. Muhammad ibn Musa al-Khwarzimi wrote the famous *Kitab al-Jabr wa al-Mugabala*, the first book on algebra [6].

The Fibonacci sequence is presented as the first sequence since it enjoys a rich history. The professor and students consider Fibonacci as an Italian mathematician, and the students research him on the web. The topic is introduced by showing the Leaning Tower of Pisa to place the mathematician in an Italian setting (see Fig. 2 below). This is followed by a discussion of the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, …) and illustrated with past students' projects on the topic. One past project is a poster used to motivate computing and working problems on the white board (see Fig. 3). The students then draw during class time the Fibonacci spiral. A rough draft for such a drawing is shown in Fig. 4. The spiral is drawn using a rectangular coordinate system whereby the box lengths are 1, 1, 2, 3, 8, 13, 21, … centimeters. Two squares, each 1 cm long, are drawn side by side, and then another square, 2 cm long, is drawn above the two original

LIFE | SCIENCE

▲ ABOVE In 875, Ibn Firnas built a glider and launched himself from a mountain. He was 70 years old

DID YOU KNOW?

Muslims were the first to...

Mathematics
■ Introduce the number zero (0) and the decimal system.
■ Invent logarithms and the science of algebra.
■ Discover fractions.
■ Discover the laws of sine and cosine.

Medicine
■ Discover the existence of blood capillaries.
■ Provide the first detailed study of the human eye, and the formation of image in the retina.

Chemistry
■ Discover the use of chemistry for medical purposes.
■ Establish apothecary shops.
■ Discover and identify three chief acids: nitric, sulfuric, and hydrochloric.
■ Discover the arts of distillation, crystallisation and oxidation.
■ Classify substances into animal, mineral and vegetable origin.

Physics
■ Discover the Principle of Pendulum, which was used to measure time.
■ Discover the principle of inertia.
■ Discover the reflection of light in the anatomy of the eye, and the formation of image in the retina.

Geography and astronomy
■ Establish correctly the circumference of the earth.
■ Make detailed studies of solar and lunar eclipses.

Ibn Sina (who is more frequently referred to as Avicenna) wrote 246 books, including *Kitab Al-Shifa* (*The Book of Healing*) and *Al-Qanun fit Tibb* (*The Canons of Medicine*). The *Qanun* was a primary guide for medical science in the West from the 12th until the 17th centuries. Dr William Osler, writer of *The Evolution of Modern Science*, says, "The *Qanun* has remained a medical Bible for a longer period than any other work."

The House of Wisdom also counted Al Kindi among its many geniuses – he is considered to be the first Arab philosopher. He also made important contributions to a number of practical disciplines, such as physics, optics, reflection of light, tides, geography and metallurgy, as well as being an expert in music.

Outside the society known as the House of Wisdom, other minds were also at work, inventing devices and machines to make the world an easier and more enlightened place.

As well as being the instigator of many scientific, literary and artistic achievements, Ibn Firnas devoted much of his career to assembling an operational flying machine. In 875, at the age of 70, Ibn Firnas built his own glider and launched himself from a mountain. He actually managed to fly using a bird structure without sustaining any major injuries.

The exhibition also hosts eight fully-operational interactive exhibits, representing the ingenious devices of Al Jazari, who is known as the father of modern-day engineering. Six of Jazari's water-raising devices can be explored in the mall's Tunisia Co... An ingenious 13th century Musl... mechanic and engineer, Al Jazari worked on these innovative proj... between 1174 and 1200.

Visitors can also view an ancie... astronomical computer called the Islamic Astrolabe. An astrolabe (f... the Arabic word, *isterlab*) is both ... observational tool and a calculati... tool. It can be used to calculate th... of day, using the height of the su... a star, and also to discover their r... and setting times. Astrolabes wer... used by Muslims to determine pr... times and the direction of Mecca ... the beginning of the 19th century.

A whole host of other exhibits ... be viewed throughout the mall, including Al Jazari's giant Castle Clock: the medieval Muslim eng... invented this device using sophisticated mechanics to harne... water, and to be able to tell the ti...

An exhibition on medical discoveries is also on show in the mall's India Court. The invention... Zheng He, a Chinese Muslim wh... the admiral of a vast fleet of ship... the 15th century, can also be view...

This unique and innovative retrospective has already been commended and commissioned by historians and scientists from important museums and research centres around the world. The exhibition is open to visitors until the end of this year.

102 | oryx August 2006

Fig. 1: Muslims' Achievements

Fig. 2: The Leaning Tower of Pisa

squares. A fourth square 3 cm long is drawn to the right side of all three squares. Continuing in this fashion and connecting each square with a spiral line completes the Fibonacci Spiral.

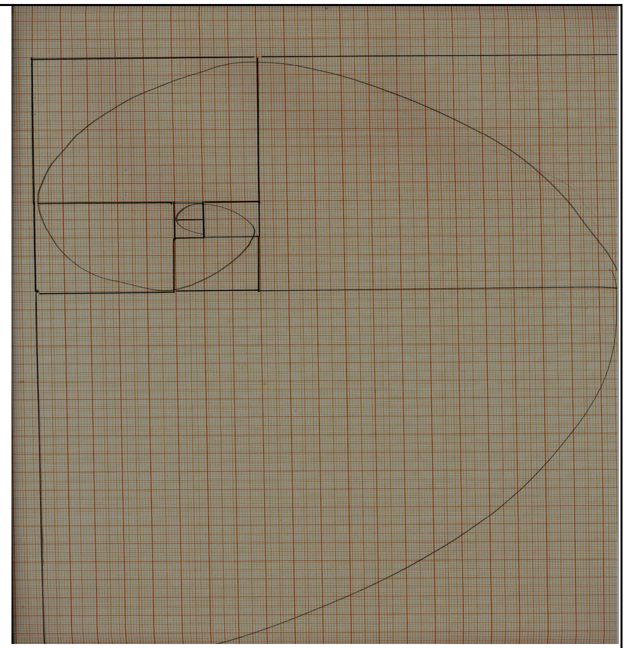


Fig. 3: Prior Students' Fibonacci Sequence Posters

Fig. 4: Student's Draft of Fibonacci Spiral

The students use some artistic talent and draw several Fibonacci Spirals. Some were displayed in Fig. 5 below on a bulletin board during the Fall term 2007. Another project was produced by a student in Spring 2006 who named it "The Fibonacci Tower" (Fig. 6). In this tower, the idea is that each level is approximated to be 1, 1, 2, 3, 5, 8, and 13 square inches. When they saw this project, several students commented that "I didn't know what these numbers mean, and now I know I understand them."

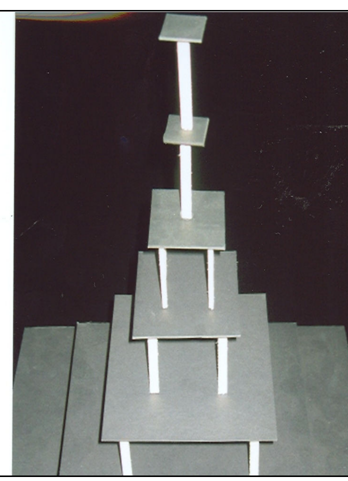


Fig. 5: Artistic Fibonacci Spirals

Fig. 6: The Fibonacci Tower

The Fibonacci numbers also occur in nature. Schilling and Harris's chart shown in Table 1 illustrates the presence of the Fibonacci sequence in flowers. Until Fall 2007, the Mat 131 students were all females (and the classes continue to be predominantly female), and they enjoy the flowers. Several flower illustrations taken from a web site are shown to students on a screen. Two are shown below in Figures 8-9. The Nautilus is a seashell (see Fig. 10) that also has the shape of a Fibonacci Spiral.

Fibonacci Sequence

Reference: Digital Signal Pocessing
Robert J. Schilling & Sandra Harris

Iris	3 petals
Wild Rose	5 petals
Delphinium	8 petals
Corn Marigold	13 petals
Aster	21 petals
Pyrethrum	34 petals
Michaelmas Daisy	55 petals

Table 1: Flower petals show the Fibonacci Sequence

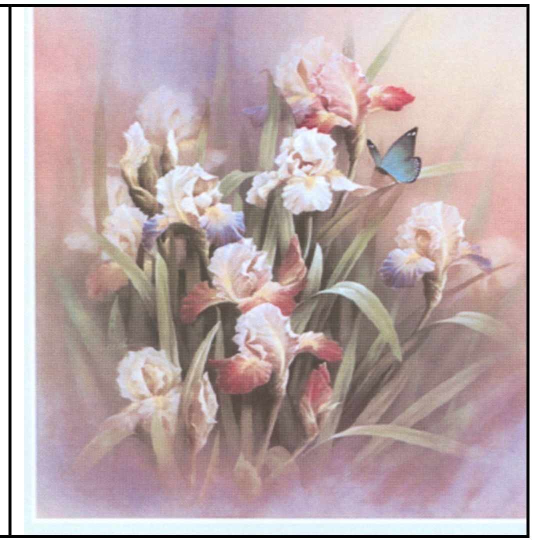


Fig. 8: A painting of irises (3-petal flowers)

Fig. 9: A painting of Asters (21 petals per flower)

Fig. 10: Nautilus Seashell

The next topic is the "Golden Number," often termed "The Divine Proportion," leading to the golden rectangle. The book, *The Golden Ratio*[5], refers to The Divine Proportion, since the Golden Number is used as a reference during this topic. The Golden Number, $(1+\sqrt{5})/2$, approximated to be 1.61803…, can be found in lectures on art history, architecture, and numerous other places. This topic generates much student discussion. Two project posters (Figs. 11 and 12) illustrate some features using the Fibonacci sequence.

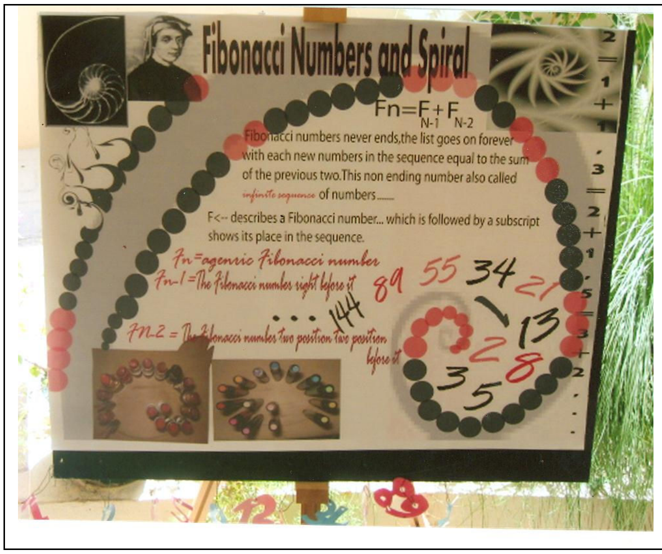


Fig.11: Student Poster A on Fibonacci

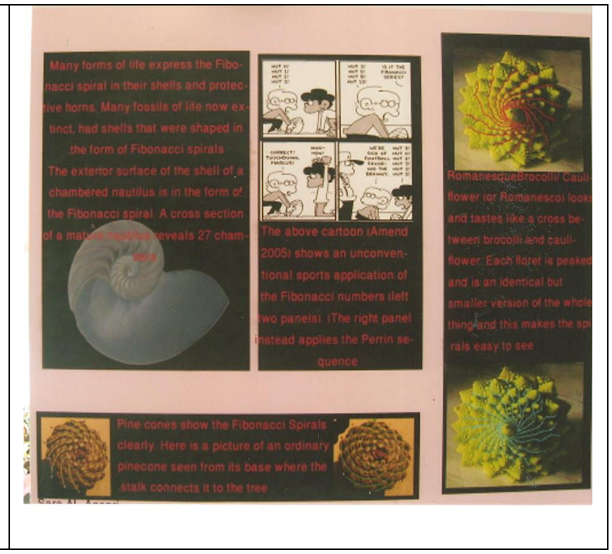


Fig. 12: Student Poser B on Fibonacci

We conclude this section with the standard arithmetic (linear) and geometric (exponential) sequences and series. We distribute the formula sheets to the class (see Appendix C). The linear and geometric sequences and series are displayed in pie and bar graphs using Matlab-6.7. This also introduces the students to the software package Matlab-6.7. We exhibit two figures illustrating the standard geometric sequence and series, $\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},\frac{1}{32},\ldots$ (see Figs. 13 and 14).

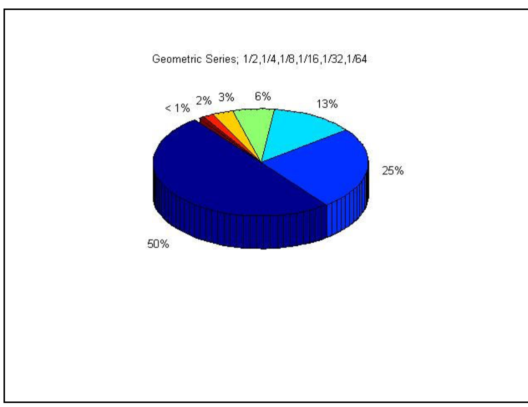


Fig 13: Pie Graph
Geometric Sequence and Series

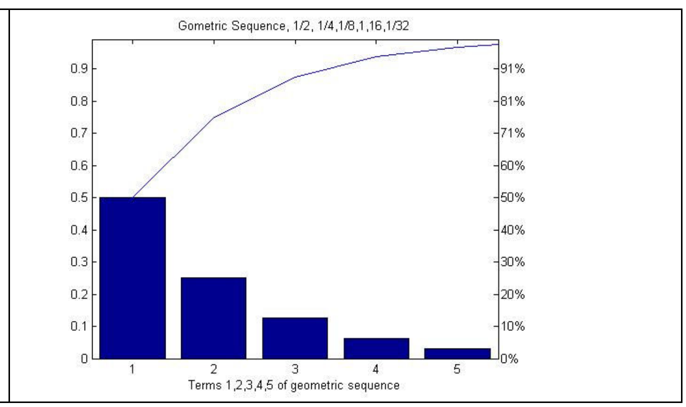


Fig 14: Bar Graph
Geometric Sequence and Series

Symmetry

Symmetry is motivated by several photos illustrating Doha's mosques. Figures 15, 16, and 17 are pictures of Doha mosques about 75 years, 45 years, and 10 years of age, respectively. Being mostly Arab Muslims, the students can immediately identify symmetry found in the mosques. Since all the men in their families are obliged to go to the mosque five times daily, the young women are constantly exposed to mosque life and also are encouraged to attend mosque with their families and friends whenever possible. (Attendance obligations differ for men and women.)

The reference book, *Architecture of the Islamic World*[2], was circulated among the students so they could observe architectural structures designed by Islamic architects. The illustrations in the book precipitate the students' understanding of Islamic architects' extensive use of structural symmetry. The hope is that the students will be motivated to understand the underlying principles displayed in the mosques and other buildings containing so much symmetry in Qatar.

Fig. 15: One of the oldest Doha mosques, about 75 years old

Fig. 16: A Doha mosque, about 45 years old

Fig. 17: A modern Doha mosque showing Spanish influence (about 10 years old)

Fig. 18: A symmetrical lighting fixture inside a mosque

A mosque lighting fixture (Fig. 18) is shown to the students, since a lighting course is required for interior design students, as the starting point to present the symmetric notions of rotations, reflections, and glide reflections. The concepts of rotations and reflections are presented in the text, implementing the notation D_n to indicate symmetry having exactly *n* rotations combined with exactly *n* reflections. The notation Z_n indicates exactly *n* rotations and no reflections. Notational requirements present problems to most novice math students, but MATH 131 students relate these ideas clearly and concisely when working with projects.

One of the classes took a field trip to the neighboring Ritz Carlton Hotel, a glamorous five-star hotel in Doha. The young women were enchanted by the symmetry found within the building. Since the young women are Muslim, photographing them studying the contents is a forbidden practice. We select a few photographs of the hotel interior to illustrate the notion of symmetry. One rug in the Ritz Carlton (see Fig. 19) illustrates several properties of symmetry in the medallions, as does the side panel within the corridors (see Fig. 20).

Fig. 19: Rug in Ritz Carlton

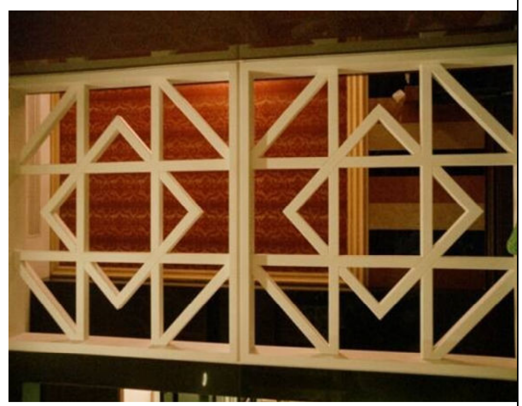


Fig. 20: Panel Divider in Corridor of Ritz

The medallions on tribal carpets are so prevalent that the instructor introduced a set of cards, each showing a different medallion. Figure 21 shows one of the medallions. The students carefully investigate the medallions and identify the symmetry properties using the D_n and Z_n notations if applicable.

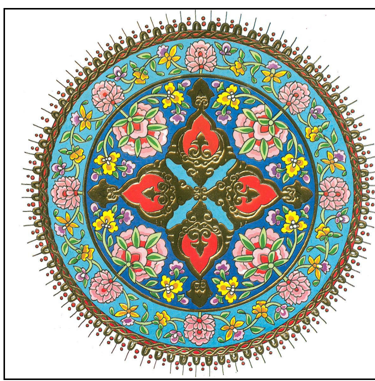


Fig. 21: Medallion on a card

Student projects on symmetry using these notational conventions are shown in Figures 22-25.

Fig. 22: A student's project on symmetry

Fig. 23: A student's multi-poster project on symmetry

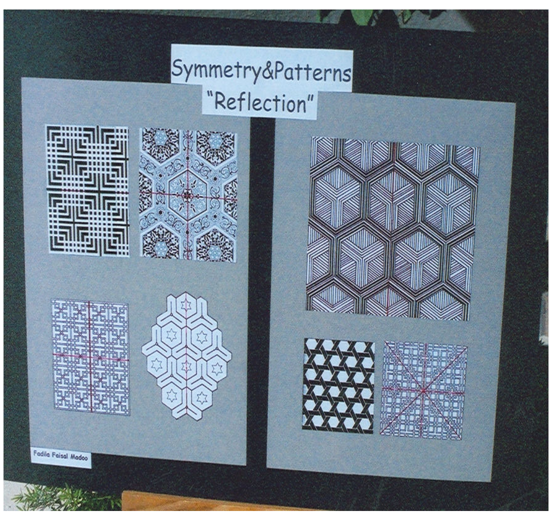


Fig 24: Student project on symmetrical patterns and reflections

Fig. 25: A student's project illustrating symmetry in the design of kick plates on a stairway

Fractals

The journey through fractal designs begins with showing a fractal contained within a fractal gallery located on the web (Fig. 26) and then fractal calendars created by previous MATH 131 students (Fig. 27).

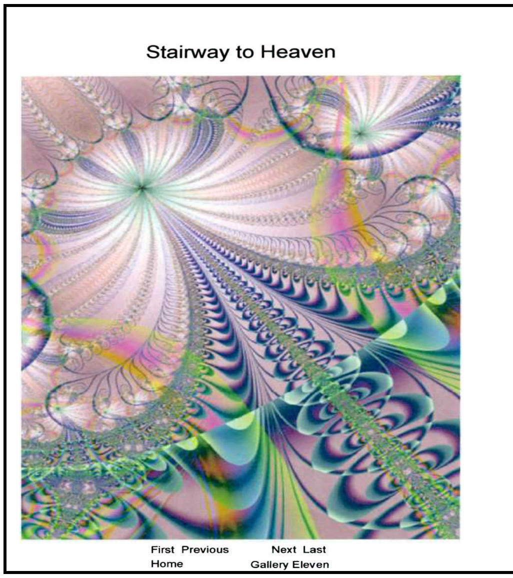
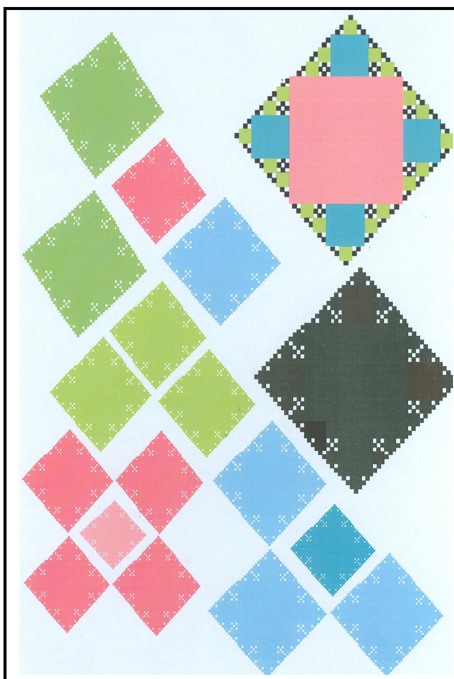


Fig. 26: A famous fractal from a web site

Fig.27: Sample calendars by previous MATH 131 students

The students collaborate to investigate various fractal galleries found on the web, and then professor and students develop the Koch Snowflake (Fig. 28). The students are again given graph paper to create the square Koch Snowflake together with a worksheet (Appendix D) to guide the students to carefully examine the recursive formulas used in the Koch snowflake fractal. A student project illustrates some other fractals (Fig. 29).



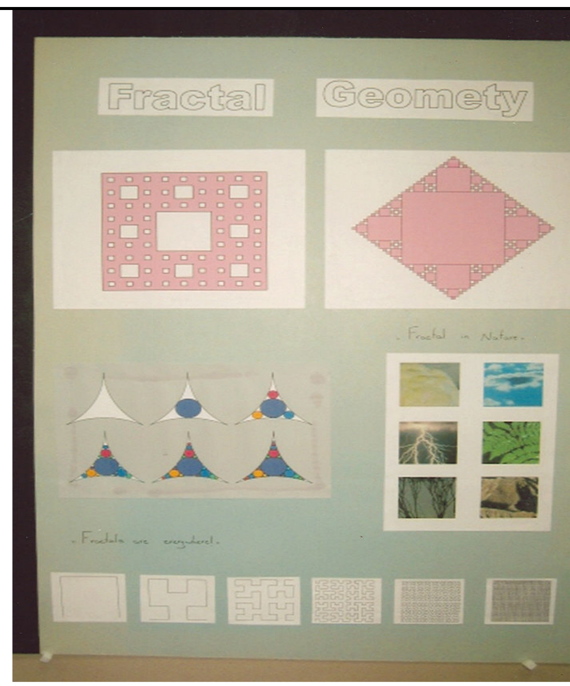


Fig. 28: Square Koch Snowflakes

Fig. 29: Calendars designed by MATH 131 students using fractal illustrations

The already mentioned worksheet on the Koch Snowflake is given to the students, and they also are given a Serpinski gasket worksheet (Appendix E). This enables the students to better understand the recursive formulas for the number of edges, perimeters, areas, etc. This method is somewhat successful and requires concentration and careful development by the students. Several students selected snowflake and fractal calendars for their projects, which are shown in Figures 30-31.

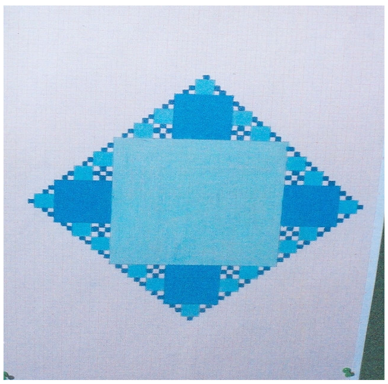


Fig. 30: Development of a Koch Snowflake by a student.

Fig. 31: Calendars designed by MATH 131 students

The Mandelbrot set (Fig. 32-33) is shown to the students and carefully examined, and the mathematical technique is studied implementing both real and complex numbers. This allows students to introduce artistic development by employing colors for the escaping, periodic, and attracting sequences within fractals. A short MatLab program (see Fig. 32) generates 301 by 301 seeds and iterates each seed 50 times before it is returned into the picture displaying the color. This helps the student understand the iteration of the point and the need for computer power to obtain good fractal results. Work in progress involves using software to create some interesting fractals. Again, the students can introduce artistic creativity by employing hot and cool colors for the escaping, periodic, and attracting sequences.

```
%Mandelbrot Set-saved as Mandel.m
h=waitbar(0,'Computing…');
x=linspace(-2.1,0.6,301);
y=linspace(-1.1,1.1,301);
[X,Y]=meshgrid(x,y);
C=complex(X,Y);
Z=max-1e6; it_max=50;
Z=C;
for k=1:it_max
        Z=Z^2+C;
        waitbar(k/it_max)
end
close(h)
contourf(x,y,abs(Z)<Z_max,1)
title('Mandelbrot Set', 'FrontSize',16)
```

Figure 32: Program

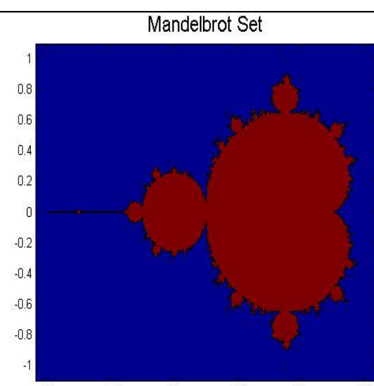


Figure 33: The Mandelbrot Set

The course concludes with a very brief introduction of graph theory. Several topics are developed from graph theory, such as Euler and Hamilton graphs. The maps shown in Figures 34-35 help students visualize the famous Konigsberga seven-bridge problem. We also cover the famous Kruskal's algorithm, which can select the best route on a graph to maximize profit. The algorithm is demonstrated in the student's project shown in Figure 36. The project implements the algorithm by finding the least expensive route to maximize profit to ship cargo from Doha to several neighboring airports.

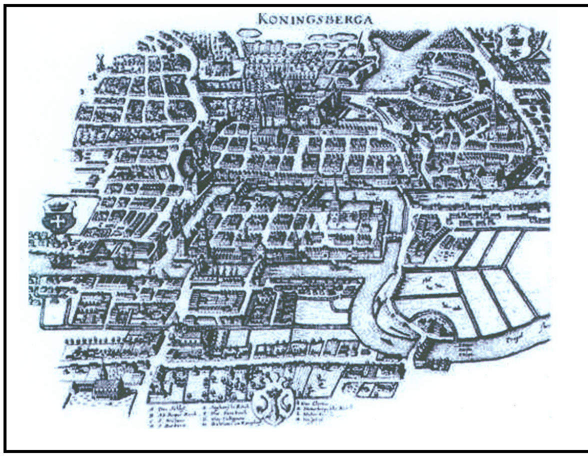


Fig. 34: Map of Konigsberga illustrating its bridges

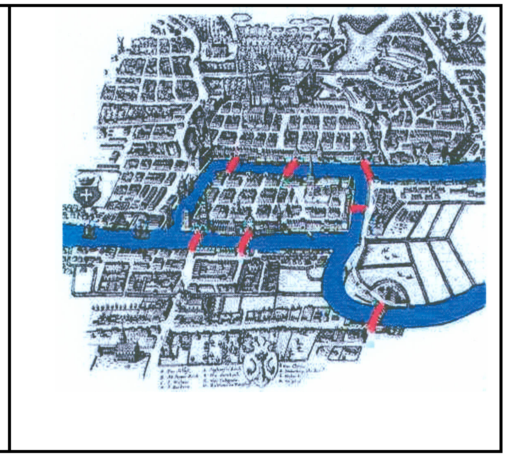


Fig. 35: Bridges highlighted

Fig 36: Student project on airports using Kruskal's algorithm.

Results

(NOTE: The following information is current as of Spring 2007. It will be updated when 2007-2008 results are available.)

The results of MATH 131 students' learning style and hemispheric preference tests documented that students' actual super links were not as anticipated. The authors expected design students to prefer primarily visual and tactile learning styles with a right hemispheric preference. Instead, students scored highest in visual preference (50 percent), next highest in kinesthetic preference (25 percent), third in tactile preference (16.67 percent), and lowest in auditory preference (8.33 percent).

Surprisingly, the majority of students exhibited *left* brain preferences (50 percent) with only 42 percent having the expected right brain preference. (Eight percent exhibited a mixed preference, using the two hemispheres interchangeably for different tasks.) Left-brain learners process information linearly and prefer working with symbolic language, such as numbers, which is advantageous for studying math. On the other hand, right-brain learners think globally, connect seemingly unrelated ideas, and are extremely creative, all characteristics expected of design students.

In follow-up surveys and interviews with students about the course content and methods, the authors learned that 100 percent of students found the sample projects "helpful" or "very helpful." However, students expressed mixed preferences when given a choice between taking a test and producing a project, and their preferences, surprisingly, usually favored taking the test.

Conclusions

(NOTE: The following information is current as of Spring 2007. It will be updated when 2007-2008 results are available)

Although the majority of MATH 131 design students were visual learners, as anticipated, the numbers who preferred a kinesthetic style involving muscles and movement were unexpected. Instead, the authors did expect tactile, or hands-on, preferences but underestimated the movement preference that would overshadow students' tactile tendencies. From this standpoint, allowing students to handle and examine sample projects and to create their own projects is an effective learning tool for design students taking MATH 131.

While students' responses strongly favored the sample projects, their preference for taking a test rather than creating their own projects was surprising given their visual and kinesthetic learning style preferences. One explanation may lie in the high number of left brain hemispheric preferences, which would find the linear style of a test more appealing. Another explanation may be that design students find themselves creating numerous projects in their design courses, so the novelty of creating projects is diminished, and producing numerous projects may make completion time a serious consideration when students are faced with creating yet another required project. In addition, although the tradition is changing, pre-college Qatari students traditionally have learned by rote memorization and recitation, so test-taking may represent a familiar habit that is preferable to unfamiliar creative requirements. Finally, because classes are small (usually 8 to 12 students) and naturally collaborative (a cultural norm), one ringleader student may influence other students' choices by expressing her preference. More research is needed to determine the actual factors resulting in this choice.

Appendix A

Dr. John Schmeelk NAME: ___________________________________
MATH 131
Spring 2008

Project-Directed Mathematics

Learning Style Preference Assessment

Please circle the letter of the answer that BEST fits you most of the time. ***Please give only ONE best-for-you answer for each question.*** *If you do not understand a word or a question, please ask your instructor for help. This test is designed to help you learn your own preferences, so there are no right or wrong answers. Answering as truthfully as you can will give you a more accurate picture of your own learning style.*

1. When you meet a new person, what do you FIRST notice about him or her?
   A. What he or she looks like and how he or she dresses
   B. How the person talks; what he or she says; or his or her voice
   C. How you feel about the person
   D. How the person acts or what he or she does

2. Days after you meet a new person, what do you remember the most about that person?
   A. The person's face
   B. The person's name
   C. How you felt being with the person even though you may have forgotten the name or face
   D. What you and the person did together even though you may have forgotten the name or face

3. When you enter a new room, what do you notice the most?
   A. How the room looks
   B. The sounds or discussion in the room
   C. How comfortable you feel emotionally or physically in the room
   D. What activities are going on and what you can do in the room

4. When you learn something new, which way do you need to learn it?
   A. A teacher gives you something to read on paper or on the board and shows you books, pictures, charts, maps, graphs, or objects, but there is no talking, discussion, or writing
   B. The teacher explains everything by talking or lecturing and allows you to discuss the topic and ask questions, but does not give you anything to look at, read, write, or do

C. The teacher lets you write or draw the information, touch hands-on materials, type on a keyboard, or make something with your hands

D. The teacher allows you to get up to do projects, simulations, experiments, play games, role-play, act out real-life situations, explore, make discoveries, or do activities that allow you to move around to learn

5. When you teach something to others, which of the following do you do?
   A. You give them something to look at, like an object, picture, or chart, with little or no verbal explanation or discussion
   B. You explain it by talking but do not give them any visual materials
   C. You draw or write it out for them or use your hands to explain
   D. You demonstrate by doing it and have them do it with you

6. What type of books do you prefer to read?
   A. Books that contain descriptions to help you see what is happening
   B. Books containing factual information, history, or a lot of dialogue
   C. Books about characters' feelings and emotions, self-help books, books about emotions and relationships, or books on improving your mind or body
   D. Short books with a lot of action, or books that help you excel at a sport, hobby, or talent

7. Which of the following activities would you prefer to do in your free time?
   A. Read a book or look at a magazine
   B. Listen to an audiotaped book, a radio talk show, or listen to or perform music
   C. Write, draw, type, or make something with your hands
   D. Do sports, build something, or play a game using body movement

8. Which of the following describes how you can read or study best?
   A. You can study with music, noise, or talking going on because you tune it out
   B. You cannot study with music, noise, or talking going on because you cannot tune it out
   C. You need to be comfortable, stretched out, and can work with or without music, but negative feelings of others distract you
   D. You need to be comfortable, stretched out, and can work with or without music, but activity or movement in the room distracts you

9. When you talk with someone, which way do your eyes move? (You can ask someone to observe you to help you answer this question.)
   A. You need to look directly at the face of the person who is talking to you, and you need that person to look at your face when you talk
   B. You look at the person only for a short time, and then your eyes move from side to side, left and right
   C. You only look at the person for a short time to see his or her expression; then you look down or away
   D. You seldom look at the person and mostly look down or away, but if there is movement or activity, you look in the direction of the activity

10. Which of the following describes you best?
    A. You notice colors, shapes, designs, and patterns wherever you go and have a good eye for color and design
    B. You cannot stand silence, and when it is too quiet in a place, you hum, sing, talk aloud, or turn on the radio, television, audiotapes, or CD’s
    C. You are sensitive to people’s feelings; your own feelings get hurt easily; you cannot concentrate when others do not like you, and you need to feel loved and accepted in order to work
    D. You have a hard time sitting still in your seat and need to move a lot, and if you do have to sit you will slouch, shift around, tap your feet, or kick or wiggle your legs a lot

11. Which of the following describes you the best?
    A. You notice when people’s clothes do not match or their hair is out of place and often want them to fix it
    B. You are bothered when someone does not speak well and are sensitive to the sounds of dripping faucets or equipment noise
    C. You cry at the sad parts of movies or books
    D. You are restless and uncomfortable when forced to sit still and cannot stay in one place too long

12. What bothers you the most?
    A. A messy, disorganized place
    B. A place that is too quiet
    C. A place that is not comfortable physically or emotionally
    D. A place where there is no activity allowed or no room to move

13. What bothers you the most when someone is teaching you?
    A. Listening to a lecture without any visuals to look at
    B. Having to read silently with no verbal explanation or discussion
    C. Not being allowed to draw, doodle, touch anything with your hands, or take written notes, even if you never look at your notes again
    D. Having to look and listen without being allowed to move

14. Think back to a happy memory from your life. Take a moment to remember as much as you can about the incident. After reliving it, what memories stand out in your mind?
    A. What you saw, such as visual descriptions of people, places, and things
    B. What you heard, such as dialogue and conversation; what you said; and the sounds around you
    C. Sensation on your skin and body and how you felt physically and emotionally
    D. What actions and activities you did, the movements of your body, and your performance

15. Recall a vacation or trip you took. For a few moments remember as much as you can about the experience. After reliving the incident, what memories stand out in your mind?
    A. What you saw, such as visual descriptions of people, places, and things
    B. What you heard, such as dialogue and conversation; what you said, and the sounds around you
    C. Sensation on your skin and body and how you felt physically and emotionally
    D. What actions and activities you did, the movements of your body, and your performance

16. Pretend you have to spend all your time in one of the following places where different activities are going on. In which one would you feel the most comfortable?
    A. A place where you can read; look at pictures, art work, maps, charts, and photographs; do visual puzzles, such as mazes, or find the missing portion of a picture; play word games, such as Scrabble or Boggle; do interior decoration, or get dressed up
    B. A place where you can listen to audiotaped stories, music, radio or television talk shows or news; play an instrument or sing; play word games out loud, debate, or pretend to be a disc jockey; read aloud or recite speeches or parts from a play or movie, or read poetry or stories aloud
    C. A place where you can draw, paint, sculpt, or make crafts; do creative writing or type on a computer; do activities that involve your hands, such as playing an instrument; play games such as chess, checkers, or board games; or build models
    D. A place where you can do sports, play ball or action games that involve moving your body, or act out parts in a play or show; do projects in which you can get up and move around; do experiments or explore and discover new things; build things or put together mechanical things; or participate in competitive team activities

17. If you had to remember a new word, would you remember it best by:
    A. Seeing it?
    B. Hearing it?
    C. Writing it?
    D. Mentally or physically acting out the word?

TOTAL ANSWERS MARKED A: _______

TOTAL ANSWERS MARKED B: _______

TOTAL ANSWERS MARKED C: _______

TOTAL ANSWERS MARKED D: _______

Your preferred learning style is: ______________________________

****

The Brain Hemispheric Preference Assessment

Please circle the letter of the answer that BEST fits you most of the time. ***If both answers suit you equally well in this section, CIRCLE BOTH A AND B.*** *If you do not understand a word or a question, please ask your instructor for help. Remember, this test is designed to help you learn your own preferences, so there are no right or wrong answers. Answering as truthfully as you can will give you a more accurate picture of your own hemispheric preference.*

1. Close your eyes. See red. What do you see?
   A. The letters *r-e-d* or nothing because you could not visualize it
   B. The color red or a red object

2. Close your eyes. See three. What do you see?
   A. The letters *t-h-r-e-e,* or the number *3,* or nothing because you could not visualize it
   B. Three animals, people, or objects

3. If you play music or sing:
   A. You cannot play by ear and must read notes
   B. You can play by ear if you need to

4. When you put something together:
   A. You need to read and follow written directions
   B. You can use pictures and diagrams or just jump in and do it without using directions

5. When someone is talking to you:
   A. You pay more attention to words and tune out their nonverbal communication
   B. You pay more attention to nonverbal communication, such as facial expressions, body language, and tones of voice

6. You are better at:
   A. Working with letters, numbers, and words
   B. Working with color, shapes, pictures, and objects

7. When you read fiction, do you:
   A. Hear the words being read aloud in your head?
   B. See the book played as a movie in your head?

8. Which hand do you write with?
   A. Right hand
   B. Left hand

9. When doing a math problem, which way is easiest for you?
   A. To work it out in the form of numbers and words
   B. To draw it out, work it out using hands-on materials, or use your fingers

10. Do you prefer to:
    A. Talk about your ideas?
    B. Do something with real objects?

11. How do you keep your room or your desk?
    A. Neat and organized
    B. Messy or disorganized to others, but you know where everything is

12. If no one is telling you what to do, which is more like you?
    A. You do things on a schedule and stick to it
    B. You do things at the last minute or in your own time, and/or want to keep working even when time is up

13. If no one were telling you what to do:
    A. You would usually be on time
    B. You would often be late

14. You like to read a book or magazine:
    A. From front to back
    B. From back to front or by skipping around

15. Which describes you best?
    A. You like to tell and hear about events with all of the details told in order
    B. You like to tell the main point of an event, and when others are telling you about an event you get restless if they do not get to the main idea quickly

16. When you do a puzzle or project, do you:
    A. Do it well without seeing the finished product first?
    B. Need to see the finished product before you can do it?

17. Which method of organizing notes do you like best:
    A. Outlining or listing things in order?
    B. Making a mind map, or web, with connected circles?

18. When you are given instructions to make something, if given the choice, would you:
    A. Prefer to follow the instructions?
    B. Prefer to think of new ways to do it and try it a different way?

19. When you sit at a desk, do you:
    A. Sit up straight?
    B. Slouch or lean over your desk, lean back in your chair to be comfortable, or stay partly out of the seat?

20. When you are writing in your native language, which describes you best?
    A. You spell words and write numbers correctly most of the time

    B. You sometimes mix up letters or numbers or write some words, letters, or numbers in reverse order or backward

21. When you are speaking in your native language, which is more like you?
    A. You speak words correctly and in the right order
    B. You sometimes mix up words in a sentence or say a different one than what you mean, but you know what you mean

22. You usually:
    A. Stick to a topic when talking to people
    B. Change the topic to something else you thought of related to it

23. You like to:
    A. Make plans and stick to them
    B. Decide things at the last minute, go with the flow, or do what you feel like at the moment

24. You like to do
    A. Art projects in which you follow directions or step-by-step instructions
    B. Art projects that give you freedom to create what you want

25. You like:
    A. to play music or sing based on written music or what you learned from others
    B. to create your own music, tunes, or songs

26. You like to play or to watch:
    A. Sports that have step-by-step instructions or rules
    B. Sports that allow you to move freely without rules

27. You like to:
    A. Work step-by-step, in order, until you get to the end product
    B. See the whole picture or end product first and then go back and work the steps

28. Which describes you the best?
    A. You think about facts and events that really happened
    B. You think in an imaginative and inventive way about what could happen or what could be created in the future

29. You know things because:
    A. You learn from the world, other people, or reading
    B. You know them intuitively, and you can't explain how or why you know

30. You like to:
    A. Stick to facts
    B. Imagine what could be

31. You usually:
    A. Keep track of time
    B. Lose track of time

32. You are:
    A. Good at reading nonverbal communication
    B. Not good at reading nonverbal communication

33. You are:
    A. Better at directions given verbally or in writing
    B. Better at directions given with pictures or maps

34. You are better at:
    A. Being creative with existing materials and putting them together in a new way
    B. Inventing or producing what is new and never existed

35. You usually work on:
    A. One project at a time, in order
    B. Many projects at the same time

36. In which of the following environments would you prefer to work?
    A. A structured environment where everything is orderly, someone is telling you what to do, a time schedule is kept, and you do one project at a time, step-by-step and in order
    B. An unstructured environment where you have freedom of choice and movement to work on what you want, where you can be as creative and imaginative as you want, keep your belongings any way you want, and do as many projects as you wish simultaneously, without any set time schedule

TOTAL ANSWERS MARKED A ONLY: _______

TOTAL ANSWERS MARKED B ONLY: _______

TOTAL ANSWERS MARKED <u>BOTH</u> A AND B: _______

Your preferred hemispheric preference is: ____

Appendix B

SCORING INSTRUCTIONS FOR THE LEARNING STYLES PREFERENCE AND THE HEMISPHERIC PREFERENCE TESTS

Learning Styles Assessment:

Total the scores for each letter of the assessment. If you gave more than one answer for any question, include all of the choices in the total for each letter.

TOTAL A: _____ If A is highest, you are VISUAL

TOTAL B: _____ If B is highest, you are AUDITORY

TOTAL C: _____ If C is highest, you are TACTILE

TOTAL D: _____ If D is highest, you are KINESTHETIC

Also note your second, third, and least preferred learning styles. Some people have developed several or all learning styles, and two, three, or all four styles may be tied.

Brain Hemispheric Preference Assessment:

Score one point for each question you answered with only A and write the total: _____
If your highest score is A, you prefer the LEFT HEMISPHERE.

Score one point for each question you answered with only B and write the total: _____
If your highest score is B, you prefer the RIGHT HEMISPHERE.

Score one point for each question you answered with both A and B and write the total: _____
If your highest score is in both A and B (tied), you are INTEGRATED.

If you have almost the same number of checks for A and B (not including the tied A and B column), you may have a MIXED preference and are using each side of the brain for different functions.

If your scores for the single A and the single B are within 1-2 points of each other, you have a MIXED preference favoring the (A/LEFT, B/RIGHT) hemisphere.

Appendix C

MATH 131
FORMULAS FOR MAT 131

FORMULAS FOR MAT 131

Fibonacci Numbers (Recursive Definition)

$$\begin{aligned} Seeds: F_1 &= 1 \\ F_2 &= 1 \end{aligned}$$

$$\text{Re}\,cursive\,rule: F_n = F_{n-1} + F_{n-2},\ n \geq 3$$

Binet's Formula

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$$

Arithmetic Sequence or the linear growth model:

$$Explicit\ description: a_n = a_0 + n \cdot d$$

$$Sum: a_0 + a_1 + a_2 + \ldots + a_{n-1} = \frac{(a_0 + a_{n-1}) \cdot n}{2}$$

Geometric Sequence or the exponential growth model:

$$Explicit\ description: p_n = p_0 \cdot r^n$$

$$Sum: p_0 + p_0 r + p_0 r^2 + p_0 \cdot r^3 + \ldots + p_0 \cdot r^{n-1} = \frac{p_0(r^n - 1)}{r - 1},\ r \neq 1$$

Compounding Formula:

$$Total\ Amount: P_N = P_0 \times (1 + \frac{i}{k})^{N \times k}$$

Appendix D

MATH 131
KOCH SQUARE SNOWFLAKE
WORKSHEET- INCLUDES THE SOLUTIONS

START:

Start with a solid square, perhaps 9 boxes wide and 9 boxes high, in the middle of a sheet of graph paper.

STEP 1:

Divide each side of the square into 3 equal segments (3 boxes wide). Attach to the middle segment of each side of the figure a solid square with dimensions equal to one third of that side. (That would be 3 boxes wide and 3 boxes out).

STEP 2:

Divide each side of the outside edges again (this now would be 1 box wide), and place them in the middle again.

STEP 3:

Try to do it again. This time the square will be very small since it would only be 1/3 of a box wide and high.

We will continue next class with the mathematics.

KOCH SQUARE SNOWFLAKE SOLUTIONS

Number of sides START	**4**
Step1 Number of sides	**5(4)**
Step 2 Number of sides	**$5(5)(4)=5^2(4)$**
Step 3 Number of sides	**$5(5^2)(4)=5^3(4)$**
………	
Step N Number of sides	**$5^N(4)$**
Also not at start we have one square At Step 1 we have 20 sides but 4 new squares. At Step 2 we have 100 sides but 20 new squares. At each step we have the number of new squares equal to the number of sides from the previous step.	
Perimeter START one side	**S**
START All sides	**4(S)**
Step 1 Length of one new side	**1/3(S)**
Step 1 Length of all sides	**5(4)(1/3S)=4(5/3)(S)**
Step 2 Length of one new side	**$1/3(1/3)(S)=(1/3)^2(S)$**
Step 2 Length of all sides	**$5^2(4)(1/3)^2(S)=4(5/3)^2(S)$**
Step 3 Length of one new side	**$(1/3)(1/3)^2(S)=(1/3)^3(S)$**
Step 3 Length of all sides	**$5^3(4)(1/3)^3(S)=4(5/3)^3(S)$**
………	
Step N Length of all sides	**$4(5/3)^N(S)$**
Also note each side gives one new square on the next step.	

Area START	**A**
Step 1 Area of one new square	**$(1/3)(1/3)A=(1/3)^2A$**
Step 1 Area of all new squares	**$4(1/3)^2A=4/9(A)$**
Step 1 Total Area	**A+4/9A**
Step 2 Area of one new square	**$(1/9)(1/9A)=(1/9)^{2(}A)$**
Step 2 Area of all new squares	**$(5)(4)(1/9)^2A=(5/9)(4/9)(A)$**
Step 2 Total Area	**A+4/9A+(5/9)(4/9)(A)**
Step 3 Area of one new square	**$(1/9)(1/9)^2A=(1/9)^{3(}A)$**
Step 3 Area of all new squares	**$5^2x4(1/9)^3A=(5/9)(5/9)(4/9)(A)$**
Step 3 Total Area	**A+4/9A+(5/9)(4/9)A+(5/9)(5/9)(4/9)(A)**

Step N Total Area=$A+(4/9)A+4/9(5/9)A+4/9(5/9)^2A+\ldots+4/9(5/9)^{N-1}A$

$=[2-(5/9)^N]A$ and as we increase steps the total area goes to =2A

APPENDIX E

MATH 131
SIEPINSKI GASKET
FRACTAL WORKSHEET

START:

Start with an equilateral triangle, perhaps using a 24-box base and 24 box lengths for the other two sides.

STEP 1:

From the middle of each side of a triangle draw another triangle. Remove this triangle.

STEP 2:

From the middle of each side of the remaining triangles draw another triangle and remove it as in the previous step.

STEP 3:

Repeat the process and fill out the mathematical worksheet attached.

SIERPINSKI GASKET
MATHEMATICS

Number of sides START	**3 Number of triangles =1**
Step1 Number of sides	**3(3) Number of triangles=**
Step 2 Number of sides	**Number of triangles=**
Step 3 Number of sides	**Number of triangles=**
………	
Step N Number of sides	**Number of triangles=**
Perimeter START one side	**s**
START All sides	**3(s)**
Step 1 Length of one new side	$\frac{1}{2}s$
Step 1 Perimeter of one new triangle	
Step 1 Perimeter of all new triangles	
Step 2 Length of one new side	
Step 2 Perimeter of one new triangle	
Step 2 Perimeter of all new triangles	
Step 3 Length of one new side	
Step 3 Perimeter of one new triangle ……	
Step 3 Perimeter of all new triangles	
Step N Length of one new side	
Step N Perimeter of one new triangle	
Step N Perimeter of all new triangle	

Area START	**A**
Step 1 Area of one new triangle	$\frac{1}{4}A$
Step 1: Total area of all new triangles	
Step 2 Area of one new triangle	
Step 2 Total area of all new triangles	
Step 3 Area of one new triangle	
Step 3 Total area of all new triangles	
…..	
Step N Area of one new triangle	
Step N Total area of all new triangles	

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