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The Art of Effectively Teaching Math to Engineering Students

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Session XXXX

The Art of Effectively Teaching Math to Engineering Students

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ABSTRACT

Throughout more than forty years of teaching an abstract subject like math to engineering students, I have been guided by a strong conviction that understanding concepts is the key to problem solving. Thus, I have striven to present the lectures with an emphasis on analytical, logical and intuitive thinking; geometric reasoning; and physical description in a way that is more congenial and receptive to engineering students. All the mathematical formalism and much of the interesting minutiae of the discipline, though are important and necessary to math students, are nonetheless secondary to our primary goal of getting the concepts across to the students so that they can solve problems by their own means.

In this paper I discuss seven effective techniques to boost students' motivation and confidence in learning mathematics. Each technique is specifically designed to achieve its own unique objective. The first indispensably important technique is to foster a positive relationship with the students in order to gain their trust in us and their willingness to follow us on their journey in learning math. The successive techniques are all geared to stimulate students' interest and curiosity to build their motivation and confidence through introducing them to mathematical mysteries and counterintuitive problems with unbelievably surprising results. The last but yet very important technique is to assist students to discover patterns as "to understand is to discover". Patterns are key factors in understanding mathematical concepts. Mathematics is virtually based on pattern and structure.

INTRODUCTION

To begin with, it should be noted that engineers and mathematicians don't seem to be in complete agreement as to how mathematics should be taught. Mathematicians tend to stress and favor rigor and formalism, wherefore theorems are proved rigorously at a level that engineering students are not adequately prepared and ready for the abstract presentation, and consequently they will not learn as well as expected. Truthfully, to an engineer, a formal proof is often unnecessary or even counterproductive. On the contrary, engineers tend to stress and favor using informal language and present the essentials of the subject as crisp and thorough, yet comprehensible and useful to the students. The emphasis is on understanding concepts which are pivotal to problem solving.

With the students' math background being generally poor coupled with math being generally an abstract and hard-to-learn subject, it has become a huge challenge to teach math to engineering students. In this paper, I will discuss the following seven effective techniques to meet the challenge:

1. Fostering a positive relationship with students
2. Creating an effective class starter
3. Meeting students where they are
4. Fostering mathematical curiosity to building motivation
5. Introducing students to mathematical mysteries
6. Ingenious use of counterintuitive questions to stimulate learning in math
7. Discovering patterns

EFFECTIVE TECHNIQUES

1. Fostering a Positive Relationship with Students.

First and foremost, fostering a positive relationship with students is an important first step to build their trust in us and belief in us, making it easier for us to work with them and to motivate and help them. I have encouraged students to ask questions for there is no such thing as a dumb question. It is through answering the questions that we can effectively pique students' curiosity about something they are naturally motivated to learn, thus they will be better prepared to learn subjects that they would normally consider difficult and boring.

2. Creating an Effective Class Starter

Through years of teaching experience, I have found that the first few minutes of the class period sets the tone for the entire class. Thus, we should carefully plan ahead and effectively utilize the first few minutes to get a good start for the class. Normally, I emailed students my own lecture notes prior to the class so that they know what to expect in the forthcoming class. In the lecture notes I clearly stated the learning objectives so that they can self-assess whether the objectives have been met at the end of the class. In some classes, we may start the class with a warm-up problem (mostly homework problem) as a way to review or assess students' prior knowledge in preparation for the material to be covered.

3. Meeting Students Where They Are

Our students came with different levels of ability in math. Meeting the needs of the students is an ideal goal of each instructor, but it can be a challenging task to accomplish. We do our best to respond to students' individual needs in a way that fosters optimal intellectual growth for all.

Whether math problems require problem-solving skills, inferential thinking, or deductive reasoning, we meet students right where they are. By this, it is by no means that we are lowering the bar for everyone, leading to a never-ending spiral of remediation. We would make an earnest effort to nurture and assist struggling students and bring them up to speed with the class by working closely with them outside the class. We challenge the mathematically talented students, who have the natural propensity to know more about the "hows" and "whys" of mathematical ideas than the computational "how-to" processes, by assigning them open-ended questions. These open-ended questions are broad and may have multiple right answers. The answers to open-ended problems will open up stimulating discussion among all students. In this way, every student can benefit from the open discussion by sharing ideas and learning from one another.

4. Fostering Mathematical Curiosity to Building Motivation

Building motivation and arousing mathematical curiosity in students takes effort and experience. Unmotivated students will not learn well, no matter how well we have prepared the lectures. As instructors, we play a pivotal role in recognizing these students' trouble spots and motivating them to learn and bring back their enthusiasm.

It is no secret that curiosity makes learning more effective and enjoyable. Curious students not only ask questions, but also actively seek out the answers. Curiosity puts the brain in a state that is more receptive to learning and retaining any kind of information like a vortex that sucks in what we are motivated to learn, and also everything around it.

One of the effective ways to motivate students is to pique students' curiosity in a mathematical problem of genuine interest to them and is related to what is currently being discussed, and then encourage them to actively justify that mathematical curiosity. As I have worked with engineering students for many years, most of them have a natural propensity for working with numbers and find it easier to learn mathematics this way. Especially to those students who are simply intrigued by numbers, we will entertain them with some famous sets of strange numbers that have curious properties and ask them to justify those curious properties.

Examples: Justifying the Mathematical Curiosities

1. Given the following set of strange numbers, justify that this sequence of numbers is in fact a Fibonacci sequence of numbers. Furthermore, justify the relationship of Fibonacci sequence to the golden ratio.

   0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946

   The above sequence of numbers fits the profile of Fibonacci sequence of numbers where each term "F_n" is equal to the sum of the two numbers before it: That is, $\mathbf{F_n = F_{n-1} + F_{n-2}}$.

   Furthermore, if you divide a Fibonacci number by the number preceding it, the ratio $\frac{\mathbf{F_n}}{\mathbf{F_{n-1}}}$ hovers around the golden ratio. And the further you go in the sequence, the closer the ratio gets to the **golden ratio** which is defined to be the number **(1+√5)/2 = 1.6180339**…. For instance, 987/610 = 1.6180328… and 10946/6765 = 1.6180339….

2. **Where does Fibonacci sequence exist in the Pascal's triangle?**

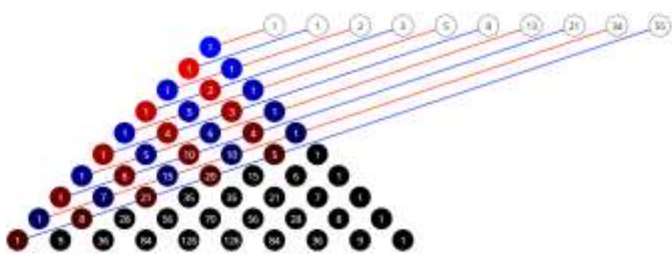

Figure 1. Showing how the Fibonacci sequence is computed[1]

Justification:

The Fibonacci sequence exists in the Pascal's triangle when you sum the numbers on the nth diagonal (from the top) of Pascal's triangle, a corresponding Fibonacci sequence term F_n is obtained as shown vividly in the above figure. To be mathematically precise, we can use the following mathematical equation to verify the accuracy of the computation of F_n.

$$F_{n+1} = \sum_{k=0}^{[\frac{n}{2}]} \binom{n-k}{k}$$

As an illustrative example, we will compute F_{10} as shown below:

$$F_{10} = F_{9+1} = \sum_{k=0}^{[\frac{9}{2}]} \binom{n-k}{k} = \binom{9}{0} + \binom{8}{1} + \binom{7}{2} + \binom{6}{3} + \binom{5}{4}$$

$$= 1 + 8 + 21 + 20 + 5$$

$$= 55$$

Note: The answer is precisely correct and it unambiguously matches every term on the 10th diagonal.

3. How does Golden ratio relate to Fibonacci spiral?

Shown below is the Fibonacci sequence being portrayed as a spiraling shell. Each of the squares illustrates the area of the next number in the sequence and the areas fit together nicely as the ratio of two consecutive numbers approaching very close to golden ratio. We then connect the opposite corners of each of the squares to graph the Fibonacci spiral as shown in the figure below:

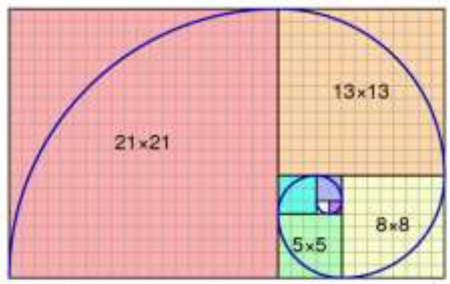

Figure 2. Showing how the Fibonacci spiral is drawn[2]

4. How does Fibonacci sequence idealize growth of rabbits' population?

Fibonacci sequence **idealizes** the rate rabbits breed family. It assumes that it takes a single newly born pair (one female and one male) of rabbits a month before they are able to mate and produce another pair at the end of second month. It further ideally assumes that the rabbits never die and a mating pair always produces a new pair from second month onwards. The growth pattern during the first six months is shown below:

The Puzzle: How many pairs of rabbits will there be in six months?

Month																			No of Pairs		
Jan																				1	
Feb																				1	
Mar																				2	
Apr																				3	
May																				5	
Jun																				8	

Figure 3. Showing the growth pattern of rabbits during the first six months

5. Introducing Students to Mathematical Mysteries

Especially when we are about to start a new topic, such as Fractal Geometry, it could be very useful to introduce students to the closely related mathematical mysteries such as there are geometric shapes whose **dimensions are non-integer and whose perimeters are infinite but areas are finite**. These mysteries will excite and instill in students a strong desire to know or learn. The inquisitive students will initiate a diligent search for clues so as to seek out the answer actively, and will be mentally prepared and eager to engage in the forthcoming discussion in class. Fractal Geometry is a fascinating subject for students to explore. It is a branch of mathematics, but the renderings are so beautiful that they have become an integral part of the art world.

A question that the curious students will naturally ask is "what exactly is a fractal?" It is a complex, never-ending pattern. It possesses two intriguing features: **self-similarity** and **non-integer dimension**. It exhibits mind-boggling, perplexing properties: **infinite perimeter but finite area**. This mathematical mystery will capture students' imagination, and will naturally set the stage for their curious brains to be more receptive for learning, and as they learn they enjoy the sensation of learning.

As an illustrative example, the Koch snowflake is built iteratively, in a sequence of stages. It starts with an equilateral triangle, removing the inner third of each side, building another triangle at the location where the side was removed, and then repeating the process indefinitely. The perimeters of the successive stages increase without bound, while the areas enclosed by the successive stages converge to 8/5 times the area of the original triangle. Consequently, **the snowflake encloses a finite area, but has an infinite perimeter.**

The **fractal dimension** of the Koch curve is ln 4/ln 3 ≈ **1.26186**[3].

Figure 4. Showing the first, second, and third iteration of snowflake[3]

6. Ingenious Use of Counterintuitive Questions to Stimulate Learning in Math

One way to engage, arouse curiosity, and stimulate intellectual capacity in learning mathematical concepts is to use counterintuitive questions. There are many interesting questions in the mathematics realm that are often counterintuitive. For example, the birthday problem is one of the most famous problems in combinatorial probability. The problem is famous because the answer is unexpectedly surprising, and even unbelievable. It is indeed a good question to be openly discussed in class so as to effectively motivate students in basic belief in probability.

The classical statement of the problem is to find how many people do we need to choose in order to have a 50% or better chance of having a common birthday among the selected individuals? First, we ignore leap day so that there are 365 possible birthdays. Next, we assume that birthdays are uniformly distributed throughout the year. With these assumptions, the birthday problem is really a sampling problem.

Let us derive the probability that in a group of n randomly selected people, there are at least two sharing the same birthday. It is easier to solve this problem using the complement. Let P_n represent the probability that there is not a common birthday among n people.

$$P_n = \frac{364}{365}\frac{363}{365} \cdots \frac{365-(n-1)}{365} = [1 - \frac{1}{365}]\left[1-\frac{2}{365}\right] \cdots \left[1-\frac{n-1}{365}\right]$$

where n = 2, 3, …, 365. The first factor is the probability that the first two people do not have the same birthday. The second factor is the probability that the third person doesn't have the same birthday as either one of the first two. This continues until the last factor which is the probability that the n^{th} person doesn't share the same birthday as any one of the other (n – 1) persons. Next, we keep multiplying out the above product with successive values of n until we reach the smallest n such that $P_n < 0.5$ or $1 - P_n > 0.5$. We find $P_{23} \approx 0.4927$ or $1 - P_{23} \approx 0.5073$.

The result is truly amazing and even unbelievable. It leaves the class in awe. The answer is n = 23 which is far smaller than 183 that most students would have expected.

7. Discovering Patterns

Virtually all mathematics is based on pattern and structure. Thus, an important task to us as instructors is to encourage our students to investigate the patterns to make mathematical discoveries on their own. As the psychologist Jean Piaget said, “To understand is to discover.” Students who are able to discover a variety of patterns can then use pattern-based thinking to understand and represent mathematical and other real-world phenomena. These students have a better sense of the uses of mathematics to solve problems, and are better prepared than those who are merely being given the patterns as facts, and the result is deeper understanding and greater retention.

Let us try to help our students to discover some patterns by themselves, which can be a very motivating and enlightening experience to them with a truly long lasting effect. Here we give some illustrative examples with explanation to show how “to understand is to discover”. Some questions, which could otherwise be very time consuming to solve, can be answered almost instantly and accurately by simply recognizing the patterns.

1. A simple example could be adding the numbers from 1 to 200 and then from 1 to n in general.

Rather than adding the numbers in sequence, we add the first and last (1+200 = 201), and then the second and next-to-last (2+199 = 201), and so on. Then all we have to do to get the answer is to simply compute 201(200/2) = 20100 as shown below:

$$1 + 2 + 3 + \dots 198 + 199 + 200 = (1 + 200) + (2 + 199) + (3 + 198) + \dots (99 + 102) + (100 + 101)$$
$$= 100(201) = 200(201)/2 = 20100$$

In general: $1 + 2 + 3 + \dots + (n - 2) + (n - 1) + n = (1+ n) + [2 + (n-1] + [3 + (n-2)] + \dots + [n/2 + (n/2 + 1)]$

$$= \mathbf{n(n + 1)/2} = \sum_{k=1}^{n} k$$

2. How does Pascal’s triangle relate to Sierpinski’s Triangle?

Pattern: As shown below, the Sierpnski’s pattern begins with the connection of the ***midpoints*** of the line segments of the largest triangle, thereby smaller triangles have been created. This pattern is then repeated for the smaller triangles over and over in an ongoing loop.

Figure 5. Showing the steps to generate the Sierpinski’s triangle[4]

The Sierpinski’s triangle **generates the same pattern as mod 2 of Pascal's triangle**. That is, the even numbers in Pascal's triangle correspond with the white space in Sierpinski's triangle.

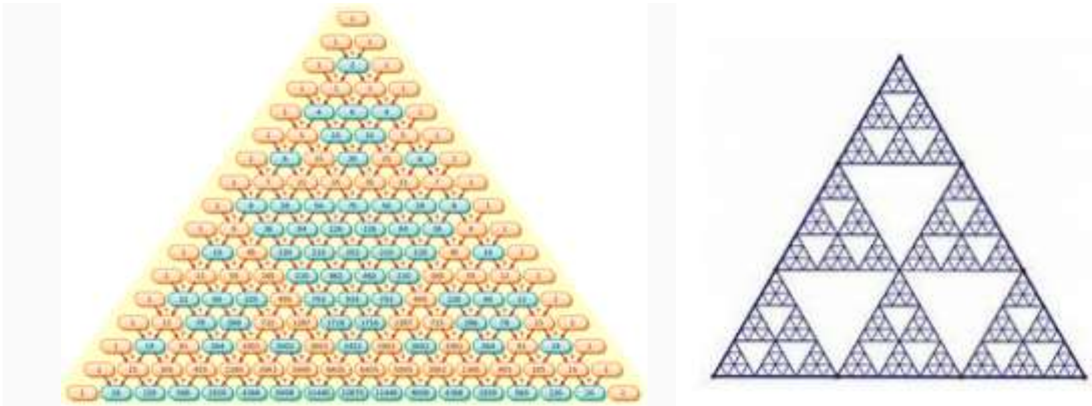

Figure 6. Showing how Pascal's triangles relate to Sierpinski's triangles[5]

3. How does Pascal's triangle relate to Binomial theorem?

To see how the Pascal's triangle is related to binomial expansion, we write the coefficients of the expansions in a triangular array as follows:

$$
\begin{array}{ll}
(a+b)^0 = & 1 \\
(a+b)^1 = & 1 \quad 1 \\
(a+b)^2 = & 1 \quad 2 \quad 1 \\
(a+b)^3 = & 1 \quad 3 \quad 3 \quad 1 \\
(a+b)^4 = & 1 \quad 4 \quad 6 \quad 4 \quad 1 \\
(a+b)^5 = & 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
(a+b)^6 = & 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1
\end{array}
$$

In this array, called Pascal's triangle after Blaise Pascal (1623 – 1662), each entry other than the 1's is the sum of the closest pair of numbers in the line above it. The pattern continues forever.

$$(a + b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6x^1y^5 + y^6$$
$$= a^6 + \binom{6}{1} a^5b^1 + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3 + \binom{6}{4}a^2b^4 + \binom{6}{5}a^1b^5 + b^6$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots \binom{n}{n-1}a^1b^{n-1} + b^n$$

4. How does Pascal's triangle relate to probability?

Coin

H T
1/2 1/2
{HH} {H T, TH} {TT}
1/4 2/4 1/4
{HHH} {HHT, HTH, THH} {TTH, THT, HTT} {TTT}
1/8 3/8 3/8 1/8

Figure 7. Showing how to use Pascal's triangle to learn probability

Remark:

- Patterns are key factors in understanding mathematical concepts. Pascal’s triangle opens up many interesting patterns through which students can effectively find possibilities that bind seemingly unrelated information together as a whole, thereby making it easier for them to see the overall relationships and understand mathematics. Mathematics is virtually based on pattern and structure.
- We encourage students to use the connections between Pascal's triangle and geometry, fractals, and probability, to solve otherwise difficult problems in mathematics by ingeniously using patterns whenever possible.

Summary and Conclusions

Throughout many years of teaching as an engineering professor, I have been guided by a strong conviction that understanding concepts is the key to problem solving, especially in math. Thus, I have made every concerted effort to present the lectures with an emphasis on analytical, logical and intuitive thinking in a way that is more congenial and receptive to engineering students. I have tried to avoid the mathematical formalism and rigor that are often unnecessary or counterproductive to engineering students. Our primary goal here is to get the concepts across to the students efficiently and effectively so that they can solve the problems by their own means.

Toward achieving the ultimate goal of getting mathematical concepts across to the students, I have discussed in this paper the following seven effective techniques to boost the students’ motivation and confidence in learning math:

1. Fostering a positive relationship with students
2. Creating an effective class starter
3. Meeting students where they are
4. Fostering mathematical curiosity to building motivation
5. Introducing students to mathematical mysteries
6. Ingenious use of counterintuitive questions to stimulate learning in math
7. Discovering patterns

As we all know, unmotivated students will not learn well no matter how well we prepare and present the lectures. Thus, it is utmost important for us to foster a positive relationship with the students so that they have trust in us and belief in us. The students will then be more willing to follow us as we guide them so as to motivate them in order to bring back their enthusiasm to learn.

In this paper, I have discussed in detail seven effective ways to teach math to engineering students. One technique that stands out distinctively among all others is pattern discovery which is the most powerful one to build students’ motivation and confidence in learning mathematics. Patterns are key factors in understanding mathematical concepts, and through patterns students can effectively find possibilities that bind seemingly unrelated information together as a whole, thereby making it easier for them to see the overall relationships and understand mathematics. Mathematics is virtually based on pattern and structure.

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