

2006-1625: THE “BACK TO THE FUTURE” EXPERIENCE OF GRAPHICAL ANALYSIS

Thomas Malmgren, University of Pittsburgh-Johnstown

Thomas Malmgren, P.E. Associate Professor of Mechanical Engineering Technology at the University of Pittsburgh at Johnstown. B.S. Mechanical Engineering from Pennsylvania State University, M.S. Industrial Engineering from University of Pittsburgh. Registered professional engineer in Pennsylvania. Interests include CAD, design, thermodynamics, enjoying the outdoors (swimming, hiking, hunting, fishing, kayaking) and my family. Worked as a journeyman machinist for General Electric, U.S. Navy Machinery Repairman, and methods engineer for Elliott Company (manufacturer of turbo-machinery) prior to teaching Mechanical Engineering Technology.

THE “BACK TO THE FUTURE” EXPERIENCE OF GRAPHICAL ANALYSIS

Abstract

Graphical analysis has always been an integral part of the medley of techniques to solve engineering problems. In the past, the accuracy of the graphical analysis solution was limited by the precision of the instruments associated with producing it. Rudimentary tools such as pencils, scales, protractors, T-squares, planimeters, slide rules, etc., provided a solution within acceptable limits of error. However, the precision, accuracy, and flexibility available now with computers antiquated the hand-drawn graphical analyses methods for solving engineering problems.

The advent of computer-aided drafting (CAD), re-engineered the old concept of graphical analysis. CAD brought it out of the past and into the future with a renewed respect, appreciation, precision, and applicability to a wide variety of engineering problems. Thus, graphical analysis has become a champion of faculty seeking to provide solutions to problems and offer an entirely new perspective to engineering problem solutions. “Graphical thinking” provides a tremendous tool to engineers seeking to develop an extensive conceptual knowledge base via greater visualization capabilities.

Vector solutions provide an excellent example of the versatility of graphical analysis. The types of vectors commonly encountered in engineering problem-solving include displacement, velocity, acceleration, force and moments. Using “graphical thinking”, one can visualize vector components and vector manipulations such as vector addition, subtraction, vector dot and cross products. A graphical picture of the vector solutions makes problems easier to conceptualize, providing an intuition check on the correctness of the vector solutions that one does not readily get from utilizing matrix solution techniques or even vector component manipulation analyses.

The application of common CAD software to a multitude of engineering problems significantly enhances the students’ ability to solve an endless variety of engineering problems. Requiring students to use a CAD-based graphical solution in place of or in addition to a traditional analysis provides students with a greater understanding of the solution to the problems on which they are working. Stressing a CAD-based graphical solution in the courses of Engineering Statics, Dynamics, and Mechanics of Material provides a strong foundation for using “graphical thinking” to solve upper level course problems as well as have a positive impact on engineering design coursework. This paper will emphasize and focus on the application of CAD-based graphical solutions to Statics, Dynamics, and Mechanics of Material courses taken by the majority of engineering and engineering technology students.

Introduction

Times have certainly changed for undergraduates in engineering and engineering technology programs. We have gone from using slide rules to calculators to computers for computations. Engineering graphics were a major part of the curriculum and involved T-squares, triangles, protractors and a variety of scales. Unfortunately, some of the basic concepts from the “old days” have been lost despite an increase in the level of sophistication in engineering analysis methods. Graphical analysis techniques were routinely used in many courses such as Statics,

Dynamics, and Mechanics of Materials. Many instructors demonstrated these techniques in class and required their use in homework assignments as well as in quizzes and exams. Many of these basic techniques have fallen by the wayside and have been replaced with more precise numeric methods. During that same time, mechanical drawing was replaced by computer-aided drafting (CAD) software on computers. Traditional graphical analysis tools, powered by the accuracy of CAD, are now capable of being utilized with solution results having the same precision as the numeric methods.

There are several primary reasons why CAD-enhanced graphical methods are not being used in solving Statics, Dynamics, and Mechanics of Materials problems. First, the latest CAD software has evolved to the point that it has become fairly complex, has frequent version changes and requires extensive training to use it effectively. Second, faculty teaching Statics, Dynamics, and Mechanics of Materials typically may not teach courses in CAD, leaving them unfamiliar with CAD's capabilities. Finally, students tend to compartmentalize learning and do not realize how skills learned in one course (CAD) can be applied to other coursework. Faculty should realize that students are proficient with CAD software from previous coursework and could easily apply graphical techniques in other core engineering courses but are not encouraged to do so nor are they shown how to do it.

The lack of using graphical techniques to solve problems hinders many students' engineering analysis abilities. Using graphical thinking to solve engineering problems helps students develop a conceptual understanding of the problem while relying less on formula memorization, and can save time. Graphical analysis offers the advantage of creating a strong link between visual (spatial) learning with logical (mathematical) learning to give students a deeper understanding of engineering fundamentals¹. For example, a vector's magnitude, sense and direction can be visualized and drawn. Types of vectors commonly encountered in engineering problem solving include displacement, velocity, acceleration, force, and moments. Using graphical thinking, students can visualize vector components and vector manipulations, such as vector addition. A graphical picture makes the problem easier to conceptualize (e.g. free body diagram or vector addition) and provides an intuition check on the correctness of solutions that one does not get from utilizing matrix solution techniques or other vector component manipulation analysis.

Other graphic concepts that can be evaluated in CAD that are used in engineering problem solving include the concepts of "areas", centroids, and area moments. Mathematically, areas are calculated as integrals identified by their boundaries (perimeter). CAD can be used to find areas of complex beam or part geometries that would be difficult to do by other means. Of interest would be areas of beam cross-sections, areas and centroid locations of distributed loads, and area moments of inertia. Utilizing CAD software for solid modeling, students can also determine the mass, center of gravity, mass moments of inertia, surface area, and other attributes of parts. In other coursework, such as Thermodynamics, areas on pressure-volume plots are used to determine work, and heat transfer can be found based on temperature-entropy areas on these plots.

Historically, graphical solutions done by "hand" had poor precision based on 2-D mechanical drawing and measurement skills. While the precision of linear and angular measurements using scales and protractors can be fairly accurate (two or three significant figures), measuring areas required special equipment (such as a planimeter) and practice to develop acceptable skill and precision. CAD requires less manual skill and can be applied to 2-D and 3-D graphical

problems. Accuracy, along with precision drawing, measuring and calculating capabilities are built into the program. Almost any CAD software (e.g. Pro/ENGINEER[®], AutoCAD[®], CADKEY[®], etc.) can be used for solving problems. They all can be used for accurate positioning, drawing, dimensioning, and conducting queries or analysis as well as accessing other information or properties of geometric features. In turn, these capabilities provide the ability to solve engineering problems using graphical techniques producing results similar to other calculator or computer techniques with the same inherent accuracy. With the visualization dimension of CAD or “graphical thinking”, reliance on typical “plug-and-chug” formulas and other abstract methods are greatly reduced. This gives students more confidence in their engineering analysis abilities and insight into their problem analysis techniques. The significant edge for CAD to perform graphical analysis techniques to accurately solve problems is shown in examples in that follow.

Concept Development and Implementation

One of the first hurdles that needs to be overcome involves convincing CAD-challenged faculty that it is to their and the student’s advantage to learn and use some of the basic CAD techniques. Developing several elementary examples should convince these faculty members that the cost/benefit for learning CAD is well worth the effort. The problem solution examples that follow will illustrate the use of CAD-based graphical problem solution to faculty unfamiliar with CAD software to encourage CAD-based graphical analysis in student assignments as well as spark interest in its use for themselves.

The first rudimentary example of using graphical analysis with CAD as the solution method represents a typical problem found in Engineering Statics. This force analysis problem requires finding the tension in two cables supporting a 200 pound weight. One cable (A) is 45° from the horizontal upward toward the left while the other (cable B) is 15° from the horizontal upward toward the right. From a Free Body Diagram (FBD) shown on the left side of Figure 1, we see that we know the magnitude and direction of the weight, but we only know the direction of the tension forces in the cable. We must determine the magnitude of the tension forces in the cables. A standard approach would be to sum the horizontal components of tension forces (X direction) and set them equal to zero. Then, sum the components of the tension forces in the vertical direction (Y direction) and set them equal to zero. With two equations and two unknowns, the magnitudes of the tension in each cable can be found by solving the two equations simultaneously.

Graphically, this is a simple vector addition problem where the sum of all the force vectors must equal zero (i.e. vectors drawn tip to tail must form a closed loop; in this case, a triangle). Referring to the center portion of Figure 1, draw a vertical line at a length to represent the magnitude of the weight (e.g. 1” = 1 lb.). Next, draw a construction line of arbitrary length through the lower end of the line at a 15° angle to represent the tension in cable B. Now, draw an arbitrary length line (to represent the magnitude of the tension in cable A) through the upper end of the vertical line at 45°. Finally, trim or extend the lines representing the cable tension forces to create a triangle and then dimension their lengths to determine their magnitudes (as shown to the right in Figure 1). Results shown to the nearest pound indicate the tension in cable A (T_A) is 223 lbs. while the tension in cable B (T_B) is 163 lbs. There are also other drawing commands and positioning techniques that could be used depending on the specific CAD system being used, but this is a simple, straight-forward method for both faculty and students. Note that

this example is a 2-D (planar) problem, but for a 3-D problem with 3-D force vectors, the same 3-D closed-loop vector addition concept applies & results can be viewed in 3-D.

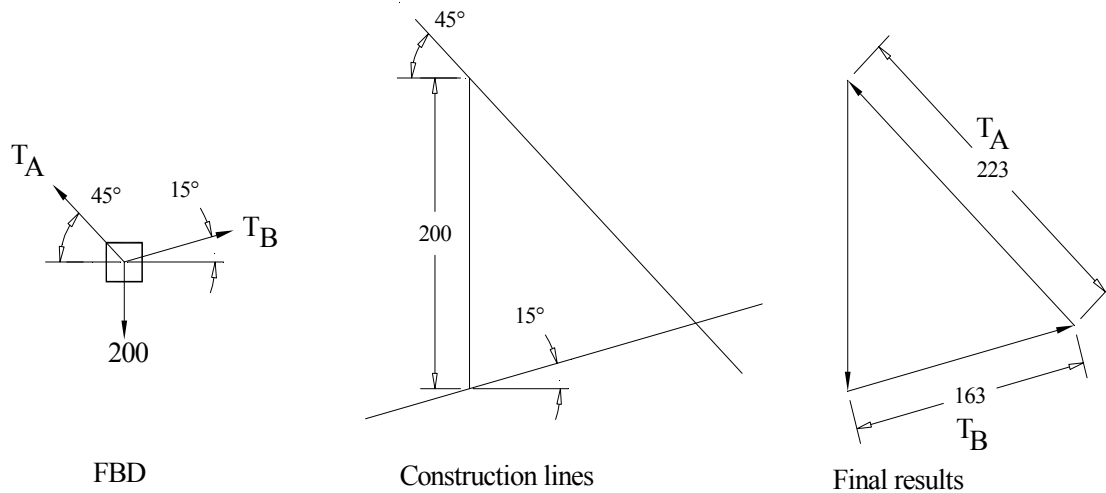


Figure 1 – Force analysis

At this point, one might think that this is a trivial problem with more work involved in solving it graphically than by using force components via trigonometric relations combined with algebra. However, the visual impact of seeing the vector components in a “picture” to gain insight and understanding into the relative magnitudes of these forces and the impact that the cable angles (geometry) have upon the magnitude of the tensions in the cables. Another distinct advantage with CAD visualization involves being able to modify the problem. As an example, what happens to the tension in the cables if the angles of cables A and B are increased or decreased? Can you visualize that from the picture?

Another basic problem which would not intimidate a CAD novice involves Engineering Dynamics to include the mysterious “acceleration” term which tends to bewilder students. The problem focuses on determining the total acceleration of a sliding collar on a straight rod rotating about a vertical axis when the rod’s current position is at 45° as shown in Figure 2 on the left. At this instant, the point P on the collar has a outward velocity along the rod of 4 m/s (relative to the rod as shown) when the distance is 2 meters from point O, the center of rotation. The velocity of the collar along the rod is increasing at a rate of 3 m/s². The angular velocity of the rod is 5 radians/s counterclockwise (ω) and its angular acceleration is 10 radians/s² clockwise (α). Since the absolute acceleration of point O is zero, the problem involves considering five components of acceleration, including Coriolis acceleration. The total acceleration of point P on the collar is equal to the acceleration of a point B on the rod, which is coincident with point P on the collar (consider normal and tangential components, $(a_B)_n = r\omega^2 = 2*5^2 = 50 \text{ m/s}^2$ directed toward point O, and $(a_B)_t = r\alpha = 2*10 = 20 \text{ m/s}^2$ directed perpendicular to the rod and downward to the right), plus the acceleration of point P relative to point B (consider normal and tangential components, $(a_{P/B})_n = 0$ since the path is straight and $(a_{P/B})_t = 3 \text{ m/s}^2$ outward from B and directed along the rod) and Coriolis acceleration ($a_{\text{Coriolis}} = 2\omega V_{P/B} = 2*5*4 = 40 \text{ m/s}^2$).

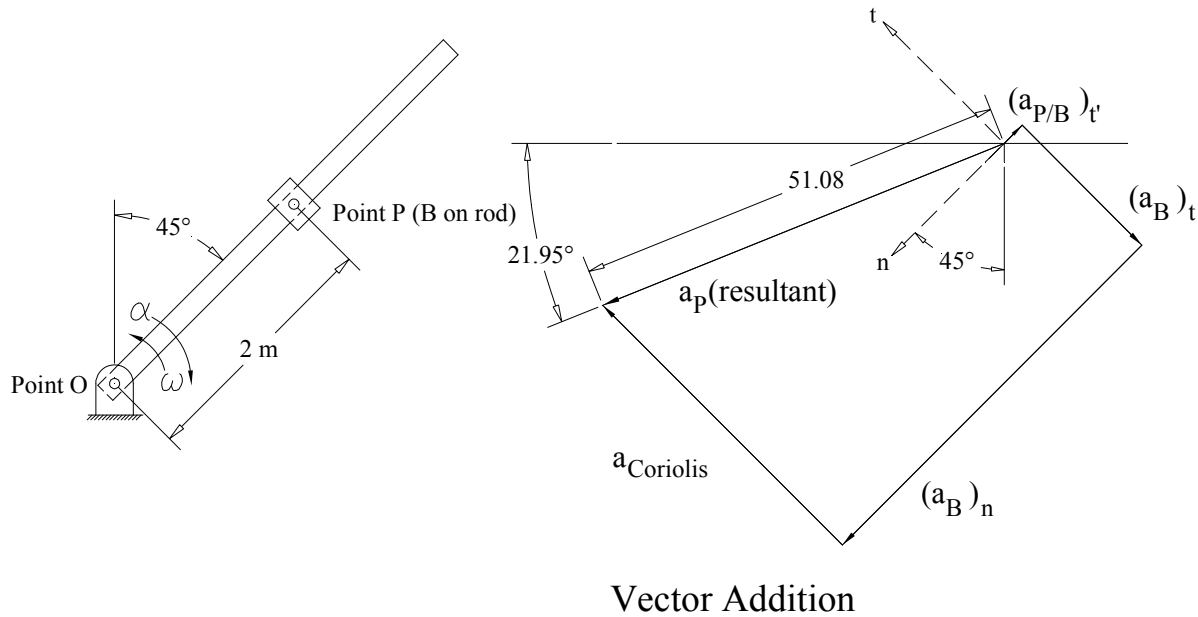


Figure 2 – Acceleration of point P

Thus, the total acceleration equation becomes:

$$a_p = (a_B)_n + (a_B)_t + (a_{P/B})_n' + (a_{P/B})_t' + a_{\text{Coriolis}} \quad \text{Eqn. 1}$$

Since the magnitudes of each component have been determined and the directions are all known, it becomes a matter of adding the vector components tip to tail to find the resultant vector. By starting at the origin (0,0), the magnitudes and angles of each component can be input using relative polar coordinates in CAD to display the vector addition. The resultant vector is then drawn from the origin to the end point of the last vector. As shown in Figure 2, the magnitude of the absolute acceleration of point B (found by dimensioning the resultant vector in the figure) is 51.08 m/s^2 at an angle of 21.95° in the direction shown (found by dimensioning the angle of the resultant vector). Again notice how the directions of the vectors and their magnitudes impact the resultant acceleration vector. (i.e. Observe which acceleration vectors have the biggest affect on the total acceleration's magnitude and direction.) These relationships tend to get lost in the mechanics of a totally mathematical vector component solution approach.

Another straight forward Engineering Dynamics example problem using CAD software requires determining the velocity of the center of a roller given the velocities at the top and bottom of the roller. This problem can be solved using the instant center of zero velocity method with CAD software. In Figure 3 on the left, the 0.25 m diameter roller is shown with the known velocities at the top ($V_t = 0.5 \text{ m/s}$ to the right) and bottom ($V_b = 0.8 \text{ m/s}$ to the left) of the roller. Laying out the problem in CAD, a 0.25 m diameter circle is drawn along with a line representing the velocity vector at the top drawn to the right at a length of 0.5 m. Next, another line is drawn at the bottom to represent its velocity vector at a length of 0.8 m to the left. Now a vertical line is created to connect the top and bottom of the circle (this line is always drawn perpendicular to the velocity vector) and another to connect the other ends of the vectors. Where these two lines cross the vertical line is the instant center (IC) of zero velocity. The velocity vector of the center (V_c) is drawn from the center at right angles to the vertical line until it reaches the diagonal line. Using the CAD dimensioning tools, the location of the center of zero velocity is revealed to be

0.154 m from the bottom of the roller. Similarly, the dimension for the length of the line representing the center's velocity vector (0.15 m/s) is the magnitude of the velocity of the center of the roller (V_c) and its direction is to the left as shown in the figure. From the graphic, it is easy to visualize what happens to the velocity of the center if the magnitudes of the velocity vectors at the top or bottom are changed.

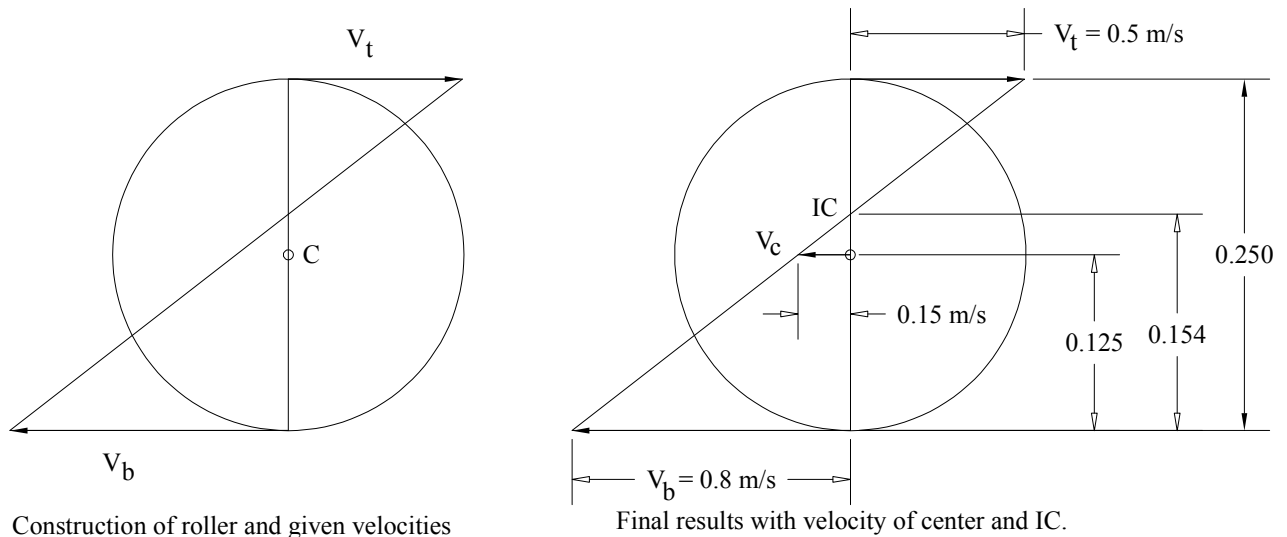


Figure 3 – Instant center of zero velocity and the velocity of the center of the roller.

Another fundamental example of employing graphical analysis to solve a Mechanics of Material problem centers on using Mohr's stress circle to determine the principal stresses and maximum shear stress of a stress element in the case of plane stress. This is a wonderful picture of how a given stress state (S_x , S_y , S_{xy}) and its orientation relate to principal stresses (S_{max} , S_{min}) and the maximum shear stress (S_{smax}) in a stress element and is widely used in mechanics of materials courses. Often these problems are solved with a Mohr's circle "sketch" and a calculator with memorized or referenced equations from a textbook.

$$S_{max}, S_{min} = (S_x + S_y)/2 \pm (((S_x - S_y)/2)^2 + S_{xy}^2)^{0.5} \quad \text{Eqn. 2}$$

$$S_{smax} = \pm (((S_x - S_y)/2)^2 + S_{xy}^2)^{0.5} \quad \text{Eqn. 3}$$

These equations are not as enlightening or as meaningful as the geometry of Mohr's circle. Solving these problems at a computer using common CAD software is easier and faster and does not require a calculator or equations if you understand and utilize the basic concepts of Mohr's stress circle geometry. The circle is drawn using the stress states of two adjacent sides of the stress element (S_x , S_{xy} and S_y , $-S_{xy}$). As shown in Figure 4, the maximum and minimum principal stresses are the coordinates of the right and left sides of the circle respectively (S_{max} , S_{min}), while the maximum shear stress (S_{smax}) is the radius of the circle.

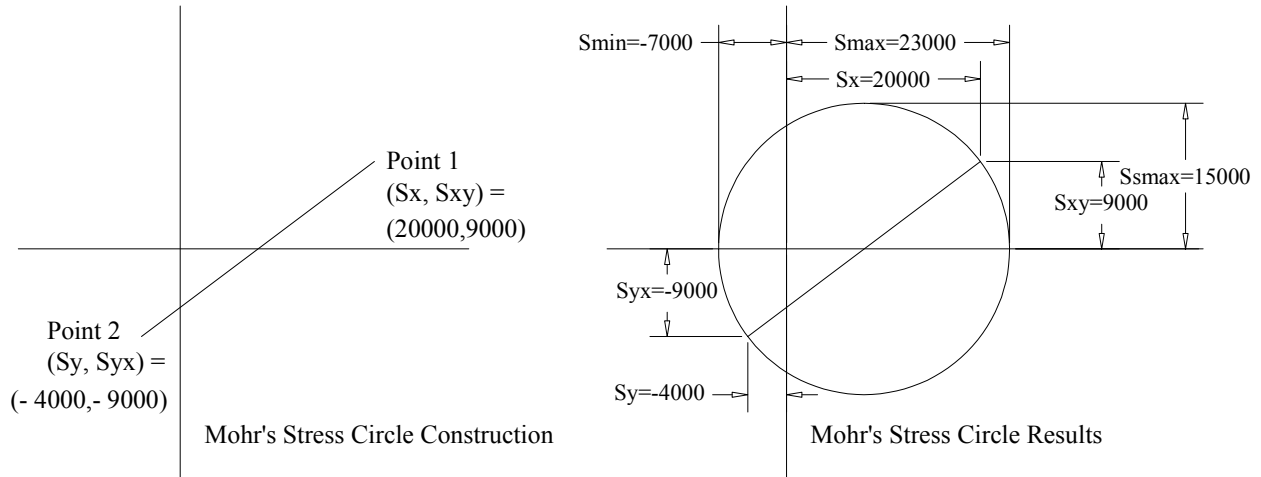


Figure 4 – Construction of Mohr's stress circle for plane stress

To create the circle, first create horizontal and vertical construction lines through the origin ($x = 0$ and $y = 0$) as shown on the left side of Figure 3. Using x, y coordinate input, create a line from point 1 ($x = S_x, y = S_{xy}$) to point 2 ($x = S_{yx}, y = S_{yx}$). Next, create a circle centered on the intersection of the horizontal construction line and line 1-2 with a radius that goes through one of the endpoints of line 1-2. Now, simply dimension the radius of the circle ($\text{Radius} = S_{smax}$) and from the origin to the right-most side of the circle for the maximum principal stress (S_{max}) and to the left-most side for the minimum principal stress (S_{min}) as shown in the figure. Note that there was NO calculator or equation required. If you want to determine the orientation angle of the principal stress element you can dimension the angle of line 1-2 and divide by two since this is twice the angle on the stress element.

Determining the area of a complex cross section or the area moment or centroid location of an area are also easily found using CAD. With solid modeling software, mass, mass moments, and center of gravity locations are available as well. This information may be necessary for doing stress analysis, beam flexure analysis, or dynamics calculations. Take the example of a beam structure with an uncommon cross-sectional shape as shown in Figure 5 below. With a single query, the area, centroid location, and area moments are determined.

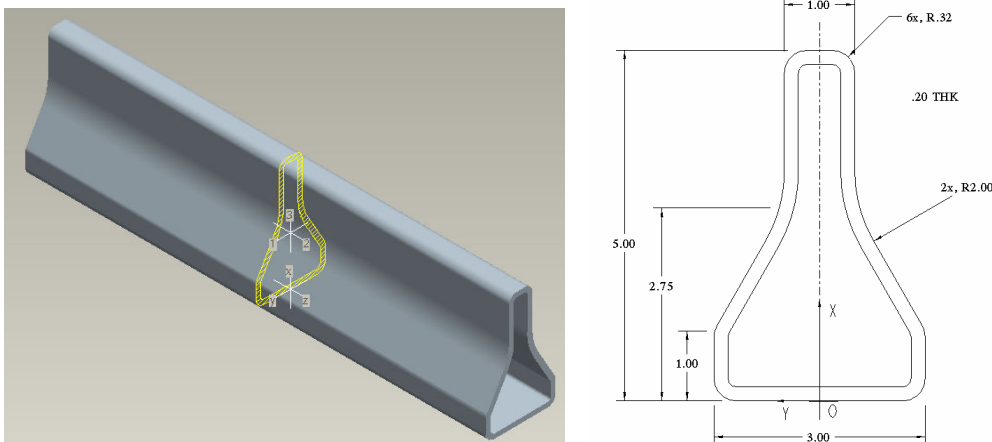


Figure 5 – Beam and beam cross section

Area, area moments, centroid location from CAD data with respect to origin at O with X as up and Y horizontal are highlighted in the results of the query shown below:

AREA = 2.6595659e+00 INCH²

CENTER OF GRAVITY with respect to O coordinate frame:

X Y 2.1137350e+00 0.0000000e+00 INCH

INERTIA with respect to _A coordinate frame: (INCH⁴)

INERTIA TENSOR:

Ixx Ixy 1.8505131e+00 0.0000000e+00

Iyx Iyy 0.0000000e+00 1.8956198e+01

POLAR MOMENT OF INERTIA: 2.0806711e+01 INCH⁴

INERTIA at CENTER OF GRAVITY with respect to _A coordinate frame: (INCH⁴)

INERTIA TENSOR:

Ixx Ixy 1.8505131e+00 0.0000000e+00

Iyx Iyy 0.0000000e+00 7.0735876e+00

AREA MOMENTS OF INERTIA with respect to PRINCIPAL AXES: (INCH⁴)

I1 I2 1.8505131e+00 7.0735876e+00

POLAR MOMENT OF INERTIA: 8.9241007e+00 INCH⁴

ROTATION MATRIX from _A orientation to PRINCIPAL AXES:

1.00000 0.00000

0.00000 1.00000

ROTATION ANGLE from _A orientation to PRINCIPAL AXES (degrees):

about z axis 0.000

RADII OF GYRATION with respect to PRINCIPAL AXES:

R1 R2 8.3414337e-01 1.6308518e+00 INCH

SECTION MODULI and corresponding points:

	MODULUS	1	2	COORD
about AXIS 1:	1.23368e+00 INCH ³	-1.1987e+00	-1.5000e+00	INCH
	1.23368e+00 INCH ³	-1.1987e+00	1.5000e+00	INCH
about AXIS 2:	3.34649e+00 INCH ³	-2.1137e+00	-1.1800e+00	INCH
	2.45078e+00 INCH ³	2.8863e+00	1.8000e-01	INCH

From this data, the cross-sectional area is determined to be 2.66 in², the centroid location is 2.114 inches from the bottom of the beam, and the area moment of inertia about the X axis is 1.85 in⁴ while the area moment about the Y axis is 18.96 in⁴.

The above five examples encompass the subjects of Engineering, Dynamics, and Mechanics of Materials. They show the application and precision of CAD-based graphical analysis. The use of graphical analysis in engineering design can significantly enhance the student's understanding of force transmission, dynamics, and how these are impacted by their mechanical design. In the sophomore engineering design course at the University of Pittsburgh at Johnstown, students are encouraged to use this approach on some of their analysis work on their designs projects. This course is taken after the students have completed Engineering Statics and while they are taking

Dynamics and Mechanics of Materials. Typically, their design projects (with students working in groups of three) involve designing a machine to perform a particular task. Through the synthesis stage of developing the machine's layout and configuration, followed by force analysis and kinematic analysis, students often just try to get "something that works." During advising sessions, they are encouraged to look at a graphical analysis approach to check their results. Many times this can be an epiphany to the students when they visually see force transmission or motion graphically, along with observing how their specific geometry relates to the forces and motion. They can then redesign to balance forces, reduce force requirements, minimize stresses, and develop better motion and functionality for their design. They take pride in developing a machine that not only "works" but in understanding "how it works." It also is appreciated as a "check" on their equation solutions and intuitive checks by playing the "what happens if we change from this to that" on the design.

Once students and faculty have a basic understanding and appreciation for CAD-based graphical solutions and its many advantages, they can further enhance its capabilities with the use of parametric modeling CAD software.² Parametric modeling allows the creation of driving parameters (variables) such as linear or angular dimensions along with a variety of constraints that can be applied to geometry. (e.g. perpendicularity, parallelism, coincidence, etc.) Further, one can get very sophisticated with relationships involving these variables. Suffice it to say that instead of using a "static graphic" and drawing inferences from that, one can simply change the value of a parameter (like a length or direction) to see the new, updated geometric relationships. The application of using parametric geometry for graphical analysis is beyond the scope of this paper.

Summary

In summary, CAD-based graphical analysis with user-friendly CAD software can be a valuable tool for solving a wide variety of problems in Engineering Statics, Dynamics and Mechanics of Materials. It has the inherent accuracy of calculator or computer solutions with the added benefits that come from visual imagery so that many basic principles are easier to conceptualize, understand, and analyze. If students have already been trained in the use of CAD software, it is a natural fit as another engineering analysis tool. Utilizing CAD to solve a variety of basic problems can greatly minimize the "Fear Factor" inherent in solving problems for the first time with CAD. As faculty become more familiar with CAD and gain confidence in themselves, they can graduate to slightly more complicated problems. In time, they will realize the value of accurate CAD-based graphical analysis and become more skilled in its use. From a student's perspective, they may develop new-found insight for CAD being a tool they can use in different courses. This realization frees them to explore different opportunities for using CAD in their analyses.

Bibliography

1. Ellis, Glenn W. and Turner, Warren A., "Improving the Conceptual Understanding of Kinematics through Graphical Analysis", 2002 American Society for Engineering Education Annual Conference, Montreal, Quebec, Canada.

2. Logue, Laurence J., "Teaching Mechanism Design Using Constraint-based Design Tools in a CAD System", 2000 American Society for Engineering Education Annual Conference, St. Louis, MO.