The Definition of a Function – Do Post-College Algebra Students Know It and Can They Apply It?

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Abstract

It is reasonable to assume that a student's performance in post-College Algebra math and science courses could be hindered by their failure to know and apply the definition of one of the most important concepts in mathematics – the concept of "function." Beginning with the fall 1996 term, more than 600 LSU-Shreveport (LSUS) students in post-College Algebra math courses have been surveyed on their knowledge of the definition of "function." The idea for this study of their knowledge of the definition had its beginning in a 1996 four-week Louisiana Systemic Initiative Program (LaSIP) summer workshop for Algebra I teachers in Northwest Louisiana, in which only 39.1% gave a correct response.

Initially, little background information was requested of the students, but over a period of time, the number of questions on the survey was increased. On the current questionnaire, students are asked to list the math courses completed above College Algebra, to give the term and year in which these math courses were completed, to give an example of a function, and, beginning with the 1998 summer term, to write the radius of a sphere as a function of the volume, given the formula for the volume of a sphere.

It was expected that the percent correct on the definition and the "radius of the sphere" items would vary depending on: (a) the course in which the survey was taken; (b) the number of completed math courses beyond College Algebra; (c) the number of completed math courses at the calculus level or above; and (d) the number of terms between the last completed math course and the math course in which the students were surveyed. The surprising results of this study will be presented in this paper.

Introduction

Since the 1996 summer term LaSIP participant survey, 638 post-College Algebra LSUS students enrolled in five different courses (total of 25 sections) requiring College Algebra as a prerequisite have been surveyed on their ability to define "function." Students were surveyed the first day of class in the following courses: (1) Math 131 -- Elementary Applied Calculus (three sections), populated primarily by students majoring in the College of Business; (2) Math 201 -- Discrete Mathematics (six sections), a course designed for computer science majors; (3) Math 210 – Applied Statistics (six sections), primarily filled by biological science majors; (4) Math 260 – Elementary Statistics (nine sections), a course with mostly upper-level math and non-biology science majors; and (5) Math 124 – Mathematical Concepts (one section), a course taken by Liberal Arts majors, usually as a last math course. Only Math 260 requires a calculus prerequisite.

Not all students responded to the definition of function item, but for those who did, their responses were graded as correct or incorrect. All definitions were read by two LSUS Department of Mathematics faculty members. On the few occasions in which there was disagreement on the correctness of the definition, a third math faculty member was asked to referee.

Some responses indicated the student had a nodding acquaintance with the function concept. Included in this category were: "A function is an equation in which one number gives a unique solution," "A function is a mathematical expression that for any values x, corresponding y values that show some sort of linear and correlating relationship," and "A function is an equation that for every point x, there exists a point f(x) and is continuous on a closed interval." These responses, and others like them, were graded as incorrect. Still others were not only incorrect, but revealed the student's complete detachment from the concept. Examples include: "What something does or how it works," "An operation preformed by a subset of a set," and "A skill that is learned in order to get a job done, such as cooking."

Examples of a function given by the students were sorted into six categories: (1) not a function/absurd response; (2) linear function; (3) quadratic function; (4) not linear or quadratic (included were cubic, trigonometric, exponential, and other functions); (5) graph; and (6) table, ordered pairs, or mapping.

Beginning in the 1998 summer term, surveyed students were given the formula for the volume of a sphere and asked to express the radius as a function of the volume. This item was considerably easier to grade than the definition of function item. All algebraic equivalent forms of the answer were counted as correct.

Information About the Students

(1) Of the n = 661 respondents, 197 were enrolled in Elementary Statistics, 184 were enrolled in Applied Statistics, 138 were enrolled in Discrete Mathematics, 106 were enrolled in Elementary Applied Calculus, 23 were LaSIP workshop participants, and 13 were enrolled in Mathematical Concepts.

(2) Of the n = 621 LSUS students who were surveyed regarding the number of math courses completed beyond College Algebra, 30.1% had completed no math course beyond College Algebra and 32.0% had completed just one course beyond College Algebra. The remaining 37.9% had completed more than one math course beyond College Algebra.

(3) Of the n = 621 LSUS students surveyed on the number of completed math courses at the calculus level or above, 56.7% had completed no math course at the calculus level or above, while 43.3% had completed one or more courses at the calculus level or above.

(4) Of the n = 579 LSUS students who were surveyed on the example of a function item, 34.7% gave no response, 14.9% gave either an absurd or incorrect example, 24% gave a linear function and 18.3% gave a quadratic function as an example.

(5) A total of n = 442 LSUS students were surveyed on the number of terms (fall, spring, and summer term are each counted as one term) between the last completed math course and the course in which

they were surveyed. Less than one fourth (22.9%) had completed a math course in the term before the one in which they were surveyed, whereas for 22.2% one term had elapsed, and for 55.0% two terms had elapsed. For 11.8% of these 442 students, there were ten or more terms between their last completed math course and the course in which they were surveyed.

Results

(1) On the definition of "function" item

Of the n = 661 persons surveyed which included the 23 LaSIP participants, only 11.3% (75/661) gave a correct definition of "function", 49.9% (330/661) gave an incorrect response, and 38.7% (256/661) didn't respond to the question. When the responses of the 23 LaSIP participants were omitted, the percent correct dropped to 10.3% (66/638).

It was expected that the percent correct on the definition item would vary depending on: (a) the course in which the survey was taken; (b) the number of completed math courses beyond College Algebra; (c) the number of completed math courses at the calculus level or above; and (d) the number of terms between the last completed math course and the math course in which the students were surveyed. The SPSS 8.0 Crosstabs procedure was used to sort the responses into the appropriate categories.

a. Since the completion of one calculus course is a prerequisite for enrollment in Elementary Statistics and many of the surveyed students had more than one course at the calculus level or above, it was expected that these students would do best on the definition item. This was not the case, however. The highest percent correct response was from Discrete Math students (16.7% or 23/138), followed by the Elementary Statistics students (13.7 % or 27/197), Elementary Applied Calculus students (5.7% or 6/106), Applied Statistics students (5.4% or 10/184), and Mathematical Concepts students (0% or 0/13). By comparison, 39.1% (9/23) of the LaSIP participants, all teachers of mathematics at the middle or high school level, correctly defined "function."

b. It seemed reasonable to expect that the percent correct on the definition item would be related to the number of completed math courses above College Algebra. However, for the n = 621 students who were surveyed on this item, the percent of correct responses exhibited an inconsistent behavior as the number of completed post-College Algebra courses increased. For example, for those with no courses beyond College Algebra, 10.7% (20/187) gave a correct response, whereas for those with one math course beyond College Algebra, the percent correct decreased to 5.5% (11/199). For those completing two math courses beyond College Algebra, the percent correct jumped to 15.0% (17/113), while for those with three completed math courses, the percent dropped to 10.3% (6/58). Of those with four or more completed math courses above College Algebra, 17.2% (11/64) gave a correct definition.

c. When the number of completed courses at the calculus level and above was considered, the results of the n = 621 students surveyed were somewhat different than (b) above. The percent of correct responses on the definition item increased as the number of completed courses at the calculus level and above increased. From a low of 7.7% (27/352) with no calculus work, the percent giving correct definitions increased to 12.5% (17/136) for those with one course in calculus, and to 21.4% (6/28) for those with three courses at the calculus level and above. Surprisingly, of the 29 students who had completed four or more courses at the calculus level and above, only 4 (13.8%) correctly defined

"function."

d. It was expected that as the number of terms between the last completed math course and the term in which the survey was conducted increased, the percent correct on the definition of "function" item would decrease. This was the case. Of the 442 students surveyed on the "number of terms between courses" item, those with no break between math courses did best – 15.8% (16/101) got the definition correct. Those with a one-term break followed closely behind -- 14.3% (14/98) gave a correct definition. When there were two or more terms intervening, the percent giving a correct response dropped considerably (6.6% when two terms intervened, 4.2% with three intervening terms, and 3.8% with a four-term break). Of the 442 students surveyed on this item, 85 had a seven-term or longer break between math courses. Only two of these 85 (2.4%) gave a correct definition.

(2) On the "example of a function" item

The example of a function item was asked of n = 579 students. An incorrect or absurd response was given by 14.9 % (86/579), and 34.7% (201/579) gave no response, leaving only 50.4% giving a correct example of a function. A linear function was the most frequent correct example given and was offered by 24% (139/579) of the students, followed by a quadratic function offered by 18.3% (106/579). The remaining 8.1% (47/579) gave a graph, a table, a listing of ordered pairs, a trigonometric function, or a polynomial of degree three or more as an example.

It was expected that the students in Elementary Statistics courses would provide the more sophisticated function examples, and this was the case. Quadratic function examples were given by 37.3% (66/177) of these students,15.9% (17/107) of the Discrete Math students, and 13% (3/23) of the LaSIP workshop students. Percentage non-response on this item was greatest among the Mathematical Concepts students – 92.3% (12/13) and least for the Elementary Statistics students -- 13.6% (24/177).

(3) On the "radius of the sphere" item

To determine if students could solve a two variable literal equation for one of the variables as a function of the other, they were given the formula for the volume of a sphere and asked to write the radius as a function of the volume. Since this survey item was introduced in the 1998 summer term, the number of students surveyed (n = 434) was somewhat lower than for the other items. Of those surveyed, 34.6% (150/434) correctly solved for the radius, 29.3% (127/434) obtained an incorrect answer, and 36.2% (157/434) did not attempt to solve for the radius.

The low overall percent correct on this item was quite unexpected, as the level of mathematical sophistication required to work such a problem is quite low. College students usually encounter this material early in Intermediate Algebra, a prerequisite for College Algebra.

a. The Elementary Statistics students did best in this category, which was to be expected, since they have encountered this activity in related rate problems in a prerequisite calculus course. Of the 126 students in this group, 68 or 54.0% correctly solved for r, compared with 33.3% (28/84) of the Discrete Math students, 31.6% (43/136) for the Applied Statistics students, 14.7% (11/75) for the Elementary Calculus students, and 0% (0/13) for the Mathematical Concepts students. The fact that

none of the Mathematical Concepts students correctly solved for r may seem surprising; however, the sample size (n = 13) was small, and only two of the 13 students attempted to solve for the radius. The remaining 11 did not attempt the problem.

b. As expected, the percent of students who correctly solved for the radius increased as the number of completed courses above College Algebra increased -- the percent correct increasing from 18.9% (27/143) for those with no math work above College Algebra to 71.4% (20/28) for those who had completed four math courses beyond College Algebra.

c. When the number of completed courses at the calculus level and above was considered, the percentage of students who correctly solved for the radius increased from 19.1% (50/262) for those with no calculus work to 73.3% (11/15) for those students who had completed three courses at the calculus level and above.

What was unexpected in (b) and (c) was that it was only after a student had five or more courses above College Algebra or three or more courses at the calculus level and above before the percent correct reached 83.3% and 73.3% respectively.

d. It was expected that as the number of terms between the last completed math course and the term in which the survey was conducted increased, the percent correct on the solution for "r" would decrease. This is not the case, however. For those with breaks ranging from zero to five terms, there was little variability in percent correct – for those who had no break between math courses, 38.8% (26/67) correctly solved for r, compared with 45.2% (33/73) for those with a one-term break, 43.5% (20/46) for those with a two-term break, 45.5% (5/11) for those with a three-term break, 42.1% (8/19) for those with a four-term break, and 46.7% (7/15) for those with a five-term break. For those with 10 or more terms without math, only 21.9% (7/32) correctly solved for r.

Summary

The findings in this study suggest that while post-College Algebra students at LSUS are exposed to functions quite often in their subsequent math courses, a sizable fraction of them do not seem to have a good grasp of the concept. The opportunity to solve for a specified variable occurs frequently in courses at the calculus level and above, yet only 34.6% of the students surveyed on the "radius of the sphere" item correctly solved for the radius. Their ability to correctly solve for the radius increased as the number of completed courses at the calculus level and above increased.

As these students progress through their math courses, they will likely encounter an increasing number of situations in which "non-traditional" examples of functions are used, including random variables, barcoding, identification numbers, and encoding functions. These examples require a deeper understanding of the function concept, yet only 10.3 % of the students surveyed could correctly define "function." The number of completed math courses at the calculus level and above was certainly a factor in the student's ability to correctly define "function", but not to the extent it was with the "radius of the sphere" item. Perhaps a more thorough grounding in the basics of functions would be beneficial to these students.

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