

## **AC 2009-708: THE DSP OF AN UNSTABLE FINANCIAL ACCOUNT**

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# The DSP of an Unstable Financial Account

## Abstract

This paper discusses a simple framework that can be used to connect a significant number of the tools and techniques developed in a first course in either discrete-time signals and systems or digital signal processing. While this framework is not revolutionary, it allows for the rapid placement of new material into the course's context. Additionally, a simple model of an unstable financial system allows for a rapid introduction and overview of the course while observably increasing student interest and motivation. In these challenging economic times, it is somewhat reassuring to know that some unstable financial accounts are actually a good thing!

## 1 INTRODUCTION

It is widely known that even in the classroom “first impressions” are long lasting. Given this fact, we believe that the first impressions that our students develop related to either discrete-time signals and systems (DTSS) or digital signal processing (DSP) should be both memorable and inspirational. Pedagogically, it also makes considerable sense during a topic's introduction to give a complete overview of that material before continuing on into the details. This approach to teaching has wide ranging consequences. Specifically it can,

- Create a framework upon which the course is based
- Encourage and motivate the students regarding the importance of the new material
- Rapidly place the new material into context
- Provide the necessary structure to prevent “global learners” from becoming lost and frustrated

The first and probably most important of these bullets is to *create a framework upon which the course is based*. One example of this concept is illustrated in Fig. 1 in which the vast majority of the new tools, techniques, and mathematical procedures that an entry level student will study concerning DTSS or DSP are depicted. Specifically, the six *bubbles* of this figure represent major topics, if not entire chapters, of a textbook. At the end of a first DSP course, students should be able to comfortably move between the six *bubbles* shown. We call this figure “the DSP big picture,” in that we constantly refer back to it as we relate past, present, and future DSP topics. Once this framework is established, an appropriate and memorable example is necessary to quickly demonstrate these concepts.

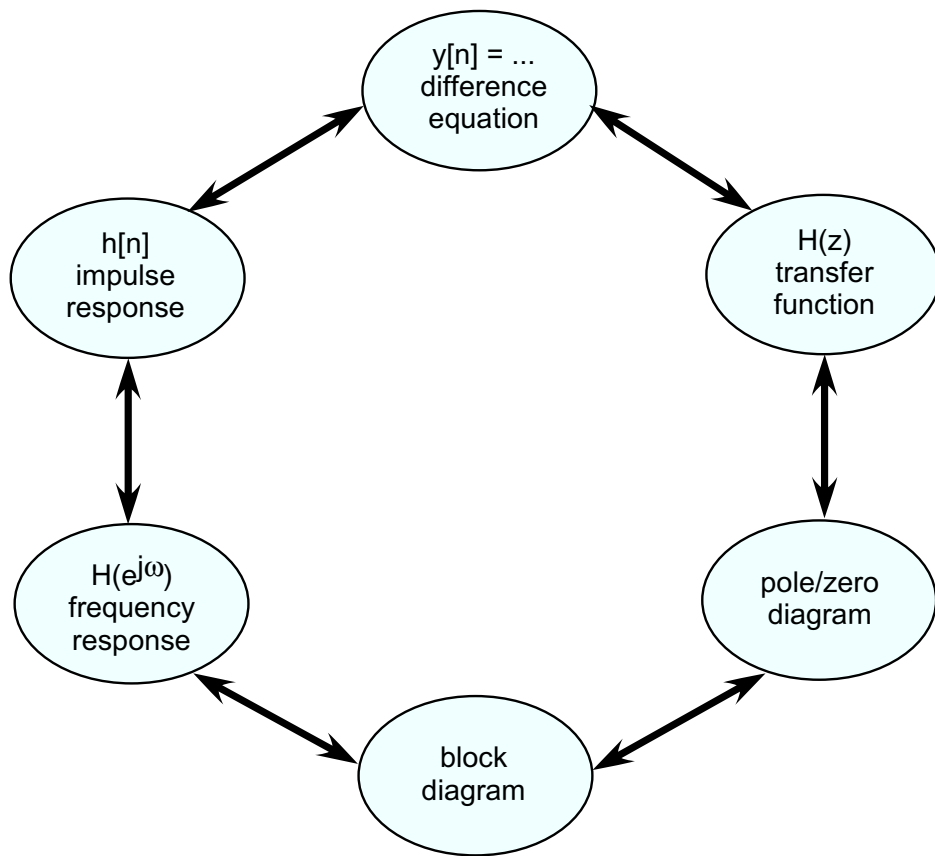


Figure 1: The DSP big picture.

## 2 The DSP Big Picture

The exponential weighted running average filter or alpha-tracker is an example of such a system.<sup>1</sup> This first-order, infinite impulse response (IIR) system has a single parameter,  $\alpha$ , upon which its characteristics can vary. The difference equation for this system is,  $y[n] = \alpha y[n - 1] + x[n]$ , where  $0 < \alpha < 1$ . This simple system can be used to filter noisy data and demonstrate the power of appropriately arranged multiplication and addition operations. It should be emphasized that typically the parameter  $\alpha$  must be kept between zero and one for stability reasons.

While a number of other excellent example systems exist, the popular press regularly points out in such articles as “Clues for the Clueless” by Daniel McGinn and Temma Ehrenfeld that most American adults do not understand the foundational concept of compound interest.<sup>2</sup> McGinn and Ehrenfeld go on to state that “. . . research shows that people who can answer questions like these (calculating compound interest) do better at planning for retirement, saving and managing their debts.”

So, given the need for more understanding of basic financial concepts, this important, yet very simple DSP system can be made to be much more motivational if we allow  $\alpha$  to be greater than one. This seeming contradiction to the stability requirements associated with the poles of a (causal) system becomes clear when instead of calling this an alpha-tracker we re-characterize this system as an interest earning savings account. Specifically,

$$\alpha = (1 + i)$$

where  $i$  is the interest rate (per unit sample),  $n$ . In this case,  $y[n]$  is the account’s current balance,  $\alpha$  is the multiplicative compounding term (e.g., 1.1 for 10 percent interest per sample interval),  $y[n - 1]$  is the previous account balance, and  $x[n]$  is the currently deposited amount. With this savings account example, we can now demonstrate the power, elegance, and simplicity of discrete mathematics and DSP.

For example, we can frame the problem as, “If you open a savings account that returns 10 percent interest per year with an initial deposit of 100 dollars, calculate the account balance at the end of one, two, and three years.” This is an opportunity to manually calculate the first three terms in a difference equation with zero initial conditions. Specifically,

$$y[n] = 1.1y[n - 1] + x[n]$$

$$\begin{aligned} n = 0, \quad y[0] &= 100 && \text{since } y[-1] = 0, \quad \text{and } x[0] = 100 \\ n = 1, \quad y[1] &= 110 && \text{since } y[0] = 100, \quad \text{and } x[1] = 0 \\ n = 2, \quad y[2] &= 121 && \text{since } y[1] = 110, \quad \text{and } x[2] = 0 \\ n = 3, \quad y[3] &= 133.1 && \text{since } y[2] = 121, \quad \text{and } x[3] = 0 \end{aligned}$$

The power of transform techniques can now be demonstrated. Specifically,

$$y[n] = 1.1y[n - 1] + x[n]$$

$$Y(z) = 1.1Y(z)z^{-1} + X(z)$$

$$Y(z) [1 - 1.1z^{-1}] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 1.1z^{-1}}$$

which allows us to confirm all of our previous calculations using MATLAB,<sup>3</sup> and its Signal Processing Toolbox by extracting the numerator coefficient,  $B = [1]$ , and the denominator coefficients,  $A = [1 \quad -1.1]$ . The account balance calculation is realized with the MATLAB command,

$$\text{balance} = 100 * \text{impz}(B, A, 4)$$

This command calculates the first four terms of the system's impulse response and then scales the result by 100. This scaling is required because we deposited 100 dollars into the account, not one dollar. In closed form, using the inverse  $z$ -transform (table lookup technique), the impulse response is calculated as,

$$h[n] = 1.1^n u[n].$$

where  $u[n]$  is the unit step function. The account balance can now be calculated as 100 times the impulse response or,

$$\text{balance}[n] = 100h[n] = 100(1.1^n u[n]).$$

The roots of the numerator and denominator polynomials of the transfer function,  $H(z)$ , can now be calculated. This process can be simplified greatly by using the MATLAB command,

$$\text{zplane}(B, A)$$

to calculate and plot the location of the transfer function's pole and zero. The resulting plot is depicted in Fig. 2 and as expected, it clearly shows that the system's only pole is located *outside* of the unit-circle, at the location of 1.1 on the real axis of the  $z$  plane. This pole location is the classic indicator of an unstable system. Despite the fact that this system is unstable and has no valid frequency response, MATLAB will still calculate a frequency response for this system using the command,

$$\text{freqz}(B, A).$$

The results of this disturbing and clearly inappropriate frequency response calculation are shown in Fig. 3. This provides an opportunity to discuss with students how to always look critically at computational results, whether from a computer or from a calculator. Finally, we complete the last bubble of the DSP big picture by creating the block diagram of this system as shown in Fig. 4.

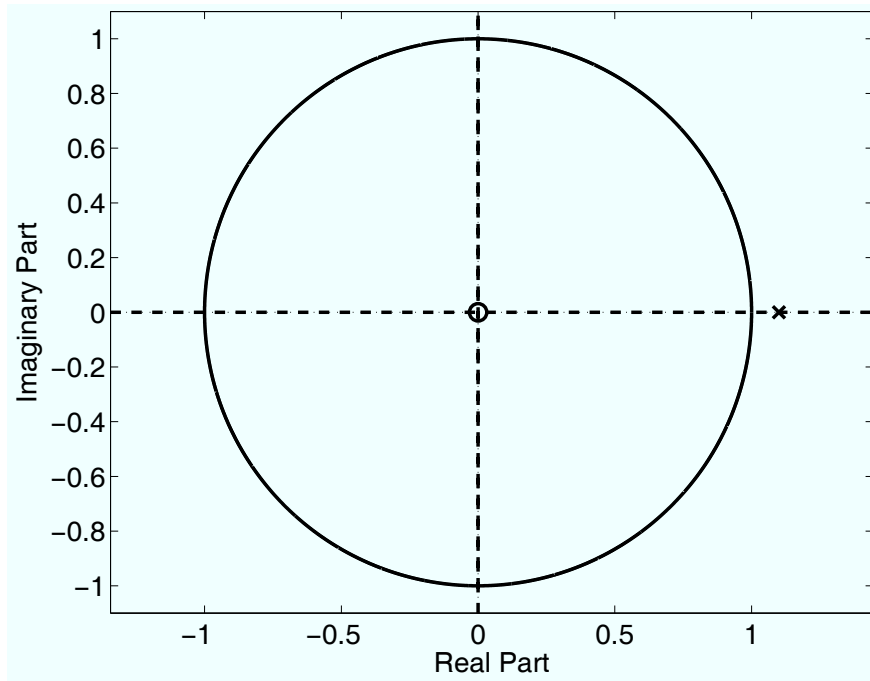


Figure 2: Pole/Zero diagram.

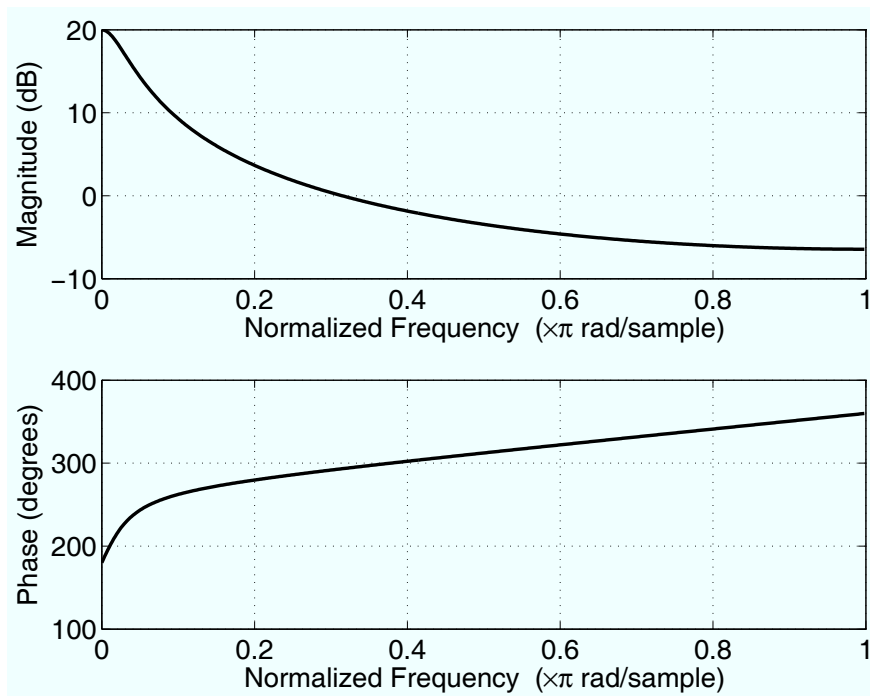


Figure 3: Frequency response.

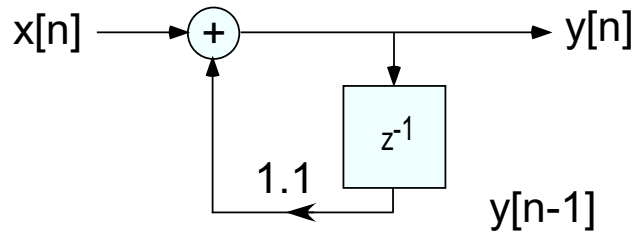


Figure 4: Block diagram.

Alternatively, a graphical user interface (GUI) version of the basic filter analysis tools can be accessed using the MATLAB command `fvtool(B, A)`. A slightly modified version of the resulting plot is shown in Fig. 5. The modifications included changing the displayed plot to the filter’s “Step Response” and using the “Analysis” pull-down menu to display only the first 41 terms of the step response. Students can thus observe the unstable nature of this system, which in this case is not a bad characteristic.

### 3 Connecting the Bubbles

These discussions, calculations, and subsequent numerical verifications in MATLAB are accomplished very quickly, and put in context a significant portion of the upcoming course, yet provide little of the underlying theory and depth required in a university DSP course. Depending on the presentation pace, follow-on discussions could include,

- similarity of the form of DT difference equations to continuous-time (CT) differential equations
- a few of the basics of using MATLAB for DSP
- similarity of transform theory between CT and DT systems
- relationship between depositing one dollar to the account, the impulse response and linearity which allows us to scale the result by any amount, e.g., 100 to account for a 100 dollar deposit
- step response, which is the cumulative sum, `cumsum(h)`, of the impulse response and is equivalent to depositing one dollar per sample period (e.g., per pay period, per month, or per year)
- unstable systems are a reality in the real world. In this example, the *last* thing that you want is a stable (damping or exponentially decaying) system determining your account balance
- in reality, all accounts are ultimately stable due to human mortality and the realization that the account will not grow for an unlimited amount of time
- numerical tools, e.g., MATLAB, are incredibly powerful, but a thorough understanding of the assumptions that underly these computations is critical to preventing the assignment of significance to meaningless results

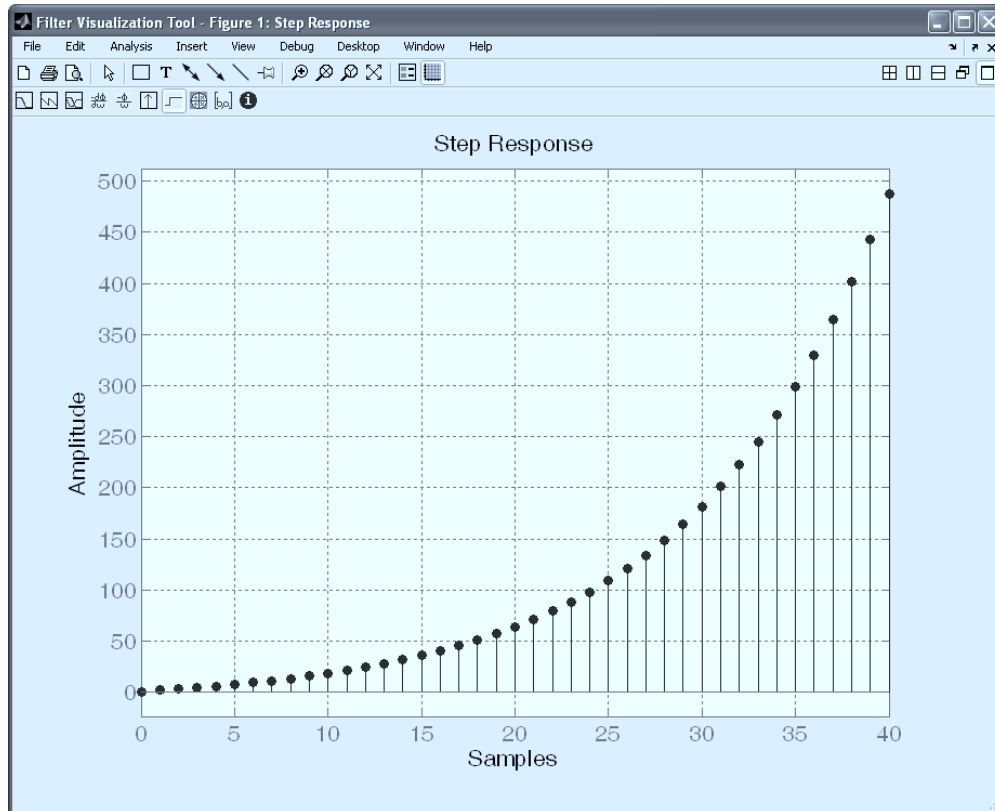


Figure 5: MATLAB’s fvtool.

Other financially-related topics can also be added, to include the “rule of 72,” which states that an investment will double in value when the product of interest rate and time equals 72. An example of this approximation is that 6 percent/year times 12 years equals 72. This implies that an investment returning 6 percent/year doubles in value in approximately 12 years. In reality, the investment increases by a factor of 2.0122 in 12 years.

If something other than, or in addition to, a financial model is desired for this “first impression” lecture, the M-th order moving average (MA) filter also works very well. Specifically,

$$y[n] = \frac{1}{M + 1} \sum_{k=0}^M x[n - k].$$

This is a filter that students have no difficulty understanding, and quickly understand why it is a type of low pass filter.

#### 4 CONCLUSIONS

We have discussed the concept of framing an entire course concerned with DTSS or DSP topics with “the DSP big picture,” and then using the real world example of an interest bearing savings



account to motivate our students. These techniques have been very well received in both a “Signal and Systems” course where only a very limited amount of course time was devoted to DT topics and in the “DSP” course which was devoted entirely to discrete time and DSP topics. We have tried using both the moving average model and the savings account model in the classroom and, while the vast majority of the discussions are identical, there is *considerably* more student interest in the savings account discussions.

We encourage educators to try this somewhat unconventional example as a first introduction in a course such as discrete-time signals and systems (DTSS) or digital signal processing (DSP).

## References

- [1] S. K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, McGraw-Hill, 3rd edition, 2006.
- [2] Daniel McGinn and Temma Ehrenfeld, “Clues for the clueless,” *Newsweek*, p. 38, April 14, 2008.
- [3] The MathWorks, Inc., *MATLAB: The Language of Technical Computing*, 2008.