2006-657: THE HUBBERT CURVE: ENABLING STUDENTS TO MEANINGFULLY MODEL ENERGY RESOURCE DEPLETION

Mark Schumack, University of Detroit Mercy

Mark Schumack is Professor of Mechanical Engineering at the University of Detroit Mercy. He teaches courses in heat transfer, thermodynamics, fluid mechanics, and energy systems. His research interests include thermal/fluid modeling using computational techniques, with applications in the automotive, manufacturing, and energy fields. Dr. Schumack earned his BS, MS, and Ph.D. degrees in Mechanical Engineering from the University of Michigan.

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Abstract

Courses in Energy Systems (alternatively named "Applied Energy Conversion," "Energy Conversion Systems," or some variant) often discuss the idea of energy resource depletion in terms of the exponential growth model. A typical problem is: given the current growth rate of oil production, in what year will known reserves be depleted? The exponential growth model, although offering reasonable results initially, becomes less accurate in the later stages of resource exploitation as issues of scarcity, cost, and technological hurdles become important. It grossly under predicts how long a given resource will last. A better model introduced in some textbooks is the "Hubbert curve," a bell-shaped curve resulting from the solution to the logistic equation. Textbook coverage of the Hubbert model, however, is usually limited to a brief allusion and perhaps presentation of a graph of actual vs. predicated production a fossil fuel such as oil or natural gas. This paper describes how a thorough analytical treatment of the Hubbert curve was explored in one energy systems class. Coverage includes mathematical and physical bases for the exponential and Hubbert models, comparisons of exponential and Hubbert model results, and application of the Hubbert curve to various nonrenewable fuels. Through comparisons with actual production data, students are made aware of the uncertainties associated with energy production modeling. The topic is contextualized through inclass discussions regarding the current controversy over "Hubbert's peak" for world oil production.

Background

The term "peak oil" refers to the period in history when humankind reaches the point of maximum oil production. After that time, many experts and observers warn of economic and political turmoil as countries transition to an uncertain energy future. Geologist Kenneth Deffeyes states:

> There will be chaos in the oil industry, in governments, and in national economies. Even if governments and industries were to recognize the problems, it is too late to reverse the trend.[1]

James Howard Kunstler, author of *The Long Emergency* and other books predicting the gloom of a post-peak world, is arguably the most rabid, yet most eloquent, proponent of the peak oil crisis:

> Many of my readers, I sense, wonder why things aren't falling apart across America right now, given the hallucinatory nature of our economy. The answer is that Peak Oil is not the end of anything, it's the peak of everything. We're getting more oil now than ever before or ever again, and it is making us crazy. It makes it possible for me to succumb to the invitation to fly across North America for a one-day meeting. It keeps feeding the spreading tumors of suburbia. It supports the illusion that burning liquid hydrocarbons results in the creation of wealth[2].

Others scoff at the idea of calamity. Energy expert Vaclav Smil states

> Energy transitions—from biomass to coal, from coal to oil, from oil to natural gas, from direct use of fuels to electricity—have stimulated technical advances and driven our inventiveness. Inevitably, they bring enormous challenges for both producers and consumers, necessitate the scrapping or reorganization of extensive infrastructures, are costly and protracted, and cause major socioeconomic dislocations. But they have created more productive and richer economies, and modern societies will not collapse just because we face yet another of these grand transformations. Unless we believe, preposterously, that human inventiveness and adaptability will cease the year the world reaches the peak annual output of conventional crude oil, we should see that milestone (whenever it comes) as a challenging opportunity rather than as a reason for cult-like worries and paralyzing concerns.[3]

In spite of the controversy over the consequences of reaching the year of peak oil, most observers agree that the peak year is indeed upon us, give or take a few years. Moreover, the controversy is not limited to oil, but also includes the other two fossil fuels: natural gas and coal.

Some textbooks used in energy system courses address the problem of resource depletion by concentrating on the exponential model[4,5]. The exponential model assumes exponential growth in production up until total depletion, resulting in unrealistically early depletion times. Other texts introduce more realistic models, but only in a qualitative sense[6,7]. They usually refer to the analysis by M.K. Hubbert that correctly predicted the year of peak U.S. oil production[8]. Hubbert's model is based on the logistic equation used in population studies. It more realistically accounts for the effects of higher costs and decreasing demand as resources become more scarce, resulting in a bell-shaped production curve.

With so much interest in how long the oil will last, it makes sense to address the idea of energy resource depletion modeling for students in a more thorough way. I decided to expand on the topic in my energy systems class, a technical elective taken by mechanical engineering students. The course is basically an applied thermodynamics class, covering conventional and unconventional power and refrigeration systems. Over the course of two class periods, I derived the equation resulting in the "Hubbert curve," and then had students use it to model the production rates of various fossil fuels, predicting the years of peak production rate and depletion.

<u>The exponential model</u>

The exponential model assumes that the instantaneous rate of production is proportional to the cumulative production, Q:

$$\frac{dQ}{dt} = aQ \qquad (1)$$

where a is the fractional growth rate, which is assumed constant for modeling purposes. This model takes its cue from population models that assume population growth is proportional to the population at any time[9]. Integration of equation (1) yields

$$Q = Q_0 e^{a(t-t_0)} \qquad (2)$$

where Q_0 is the cumulative production at time t_0. It is instructive to plot production rate dQ/dt vs. time using the differentiated form of (2)

$$\frac{dQ}{dt} = aQ_0 e^{a(t-t_0)}$$

or

$$\frac{dQ}{dt} = \left.\frac{dQ}{dt}\right|_0 e^{a(t-t_0)} \qquad (3)$$

where $\left.\frac{dQ}{dt}\right|_0$ is the production rate at time t_0, a quantity which is often easier to obtain from available data than Q_0. The area under the curve described by equation (3) represents the cumulative production up time t. The depletion time t_d occurs at the point where the ultimate cumulative production Q_∞ of a resource has been reached. Substituting $\frac{1}{a}\left.\frac{dQ}{dt}\right|_0$ for Q_0 in equation (2) leads to the following relationship for the depletion time:

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$$t_d = t_0 + \frac{1}{a}\ln\left(\frac{aQ_\infty}{\left.\frac{dQ}{dt}\right|_0}\right)$$

As an example, the United States produced 2×10^9 barrels of oil in 1950. At that time, the growth rate was 7% annually. Assuming an ultimate recovery of 2.14×10^{11} barrels leads to a depletion time of 1979. Figure 1 illustrates the scenario. Clearly, we did not run out of oil in 1979. This points out a fundamental flaw in the exponential model: it assumes exponential growth right up to the point where the resource vanishes and thus grossly underestimates depletion time.

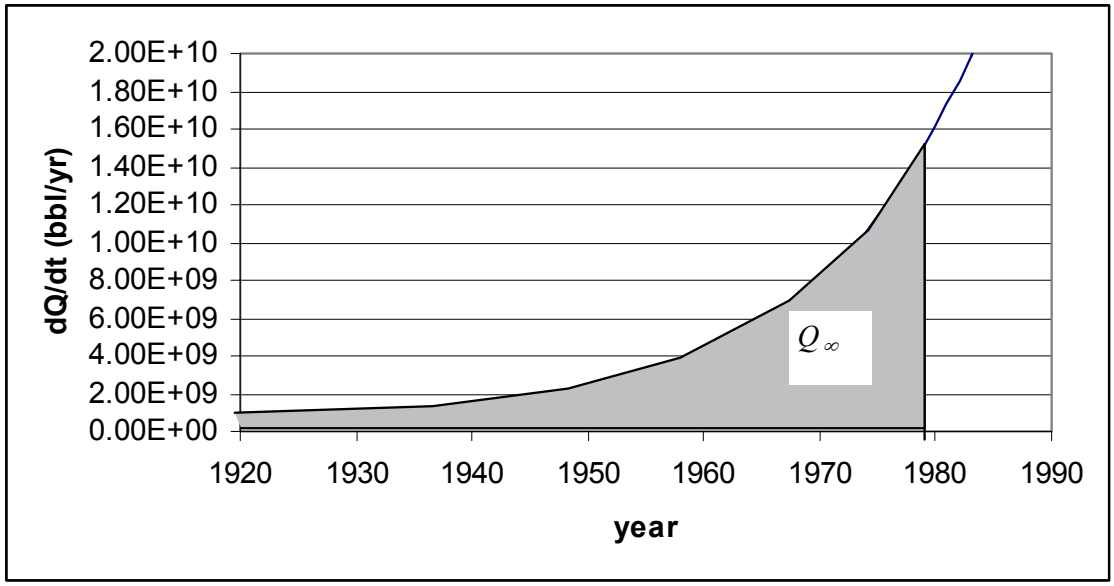

Figure 1. Results from the exponential model for U.S. oil production. The area under the curve represents the total cumulative production.

The Hubbert curve

In 1956, geophysicist M.K. Hubbert made his famous prediction that U.S. oil production would peak in the early 1970s[8]. Historical data has proven him right. Hubbert provided further details of his prediction model in a 1982 paper[10]. Other authors such as Laherrere[11] have presented thorough analyses and critiques of his work. Although Hubbert's model was based on the logistic equation used in population studies, I will follow a common practice and refer to graphical predictions of resource production using the logistic equation as "Hubbert curves."

Hubbert provided the following basis for his model. We want a function $\frac{dQ}{dt}$ that approaches zero as Q approaches both zero and the ultimate production Q_∞. The simplest function is a parabola: $\frac{dQ}{dt} = c_1 + c_2 Q + c_3 Q^2$. Applying the conditions that $\frac{dQ}{dt} = 0$ for $Q = 0$ and $Q = Q_\infty$ yields

$$\frac{dQ}{dt} = a\left(Q - \frac{Q^2}{Q_\infty}\right) \qquad (4)$$

where a replaces the constant c_2 and can be thought of as a growth parameter that is unknown a priori. Equation (4) is the logistic equation[9]. Comparison of equation (4) with equation (1) indicates that the new equation is similar to that from the exponential model with a "correction factor" included which attempts to account for more realistic behavior (note that the constant "a" does not have the same significance in both equations).

Integration of equation (4) yields

$$-\ln\left(\frac{1-Q/Q_\infty}{Q}\right)=at+c \qquad (5)$$

where a and c are unknown constants. Since the year of peak production rate, t_m, is of interest, we use the known production at t_m to determine the constant c. The production at time t_m is determined by recognizing that $\frac{dQ}{dt}$ is a maximum at t_m, so setting $\frac{d^2Q}{dt^2}=0$ and differentiating equation (4) leads to the condition $Q=Q_\infty/2$ at t_m. Using this condition in equation (5) leads to

$$Q=\frac{Q_\infty e^{a(t-t_m)}}{1+e^{a(t-t_m)}}$$

which, upon differentiation, gives

$$\frac{dQ}{dt}=\frac{aQ_\infty e^{a(t-t_m)}}{\left[1+e^{a(t-t_m)}\right]^2}. \qquad (6)$$

A qualitative comparison of the curve resulting from the exponential model (equation 3) and the Hubbert curve (equation 6) can be seen in Figure 2. Although they agree in the early period of resource exploitation, the Hubbert model accounts for the leveling off and subsequent decline of production rate as the resource becomes more scarce.

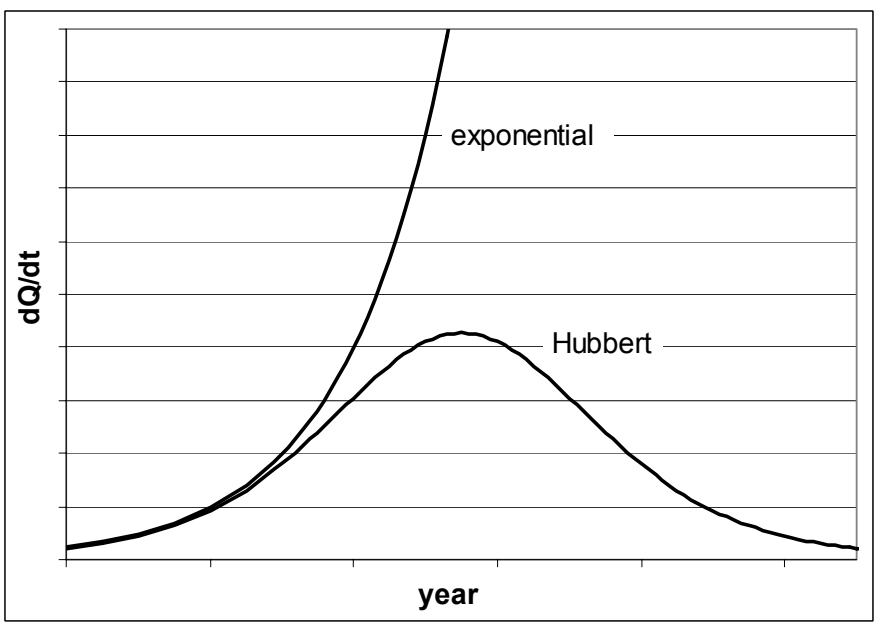

Figure 2. Qualitative behavior of the exponential and Hubbert models.

It should be noted that the value that must be input for ultimate recovery, Q_∞, is highly controversial among experts and can significantly affect predicted values for years of peak production and depletion time. Smil, for instance, presented a compilation of estimates for Q_∞ since 1990 for global oil that varied from 1.3×10^{12} bbl to 3.7×10^{12} bbl [12].

Predictions using the Hubbert Curve

There are two unknown constants in equation (6): the constant a and the time of peak production, t_m. The constants could be determined by using the known production rates for two different years. This procedure, however, can result in unrealistic results due to scatter in the historical data. I chose to have students, instead, match the curve produced from equation (6) to all data in a least squares sense. By using the Solver function in Excel, the constants a and t_m can be adjusted until the sum $\sum_{i=1}^{N}\left(\left.\frac{dQ}{dt}\right|_{Hubbert} - \left.\frac{dQ}{dt}\right|_{actual}\right)_i^2$ is minimized, where N is the number of years over which the comparison is made.

The depletion time, t_d, can be estimated as follows. Assume that t_d is the time when the production rate has decreased to one percent of the maximum production rate, $\left.\frac{dQ}{dt}\right|_m$. An expression for the maximum production rate can be obtained by setting $t = t_m$ in equation (6), resulting in

$\left.\frac{dQ}{dt}\right|_m = \frac{aQ_\infty}{4}$. Taking one percent of this value and substituting into equation (6) with $t = t_d$ leads to

$$0.0025 = \frac{e^{a(t_d - t_m)}}{\left(1 + e^{a(t_d - t_m)}\right)^2}$$

which can be solved to give $t_d = t_m + \frac{6}{a}$.

The Hubbert model is, of course, a gross simplification of a complex process. Real world events like political disturbances, development of new energy resources that replace existing ones, discovery of new reservoirs, and changes in extraction technologies all result in deviations from the smooth behavior predicted by the Hubbert curve.

Results

I asked all students in the class to model U.S. oil production. In addition, students were required to model another resource according to their last name:

First letter of last name	Resource
A - G	World oil
H - M	US natural gas
N - Z	US coal

I provided students with historical data for annual production $\left(\frac{dQ}{dt}\right)$ and ultimate resource recovery (Q_∞). Annual production data came from the Energy Information Agency website[13]. The values for Q_∞ were gleaned from a variety of sources, and as mentioned above, are by no means definitive. I asked students to plot the Hubbert curve along with actual data on dQ/dt vs. year plots, and predict values for t_m and t_d. Students were also required to discuss the validity of their predictions given the limitations outlined in the previous section. The comparisons discussed in the following paragraphs are typical of student results.

Figure 3 shows a typical comparison for U.S. oil production, for which the predicted year of peak production is 1975 and the year of depletion is 2072 – much different from the exponential model prediction of 1979. For the comparison shown in Figure 3, data was available from 1860 through 2003. The value for Q_∞ was 2.14×10^{11} bbl. The actual year of peak production was 1970. The figure clearly demonstrates the hazards of using a well-behaved equation to model the volatile behavior of actual data. The simple Hubbert model has no allowances that would account for the two actual peaks in production. Also, the figure shows results that have included post-peak production data in the least-squares fit. Interestingly, if only data up to 1955, the year before Hubbert made his famous prediction, is used in the least-squares fit, the results are virtually the same. This could be just coincidence, but it also adds ammunition to the claim that the Hubbert model, despite its simplicity, is at least as good as any other.

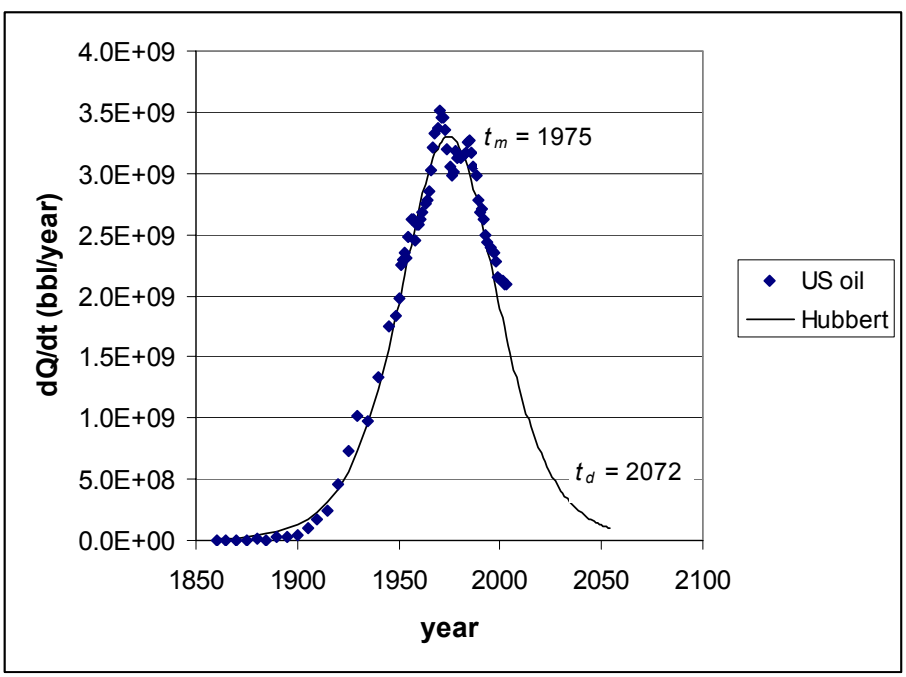

Figure 3. Actual data compared with Hubbert curve for U.S. oil production.

Figure 4 shows comparisons for world oil production. This comparison uses all the available data from 1960 to 2003. The value for Q_∞ was 3.01×10^{12} bbl. The calculated values for t_m and t_d are 2014 and 2184, respectively. Many experts would consider the year 2014 to be too late, believing that the peak has already occurred or will occur before 2010. Deffeyes believes the value of 3.01×10^{12} bbl for Q_∞ to be too high, and states that "there is nothing plausible that could postpone the peak until 2009.[14]" Smil, on the other hand, suggests the peak could be as late as 2030[15].

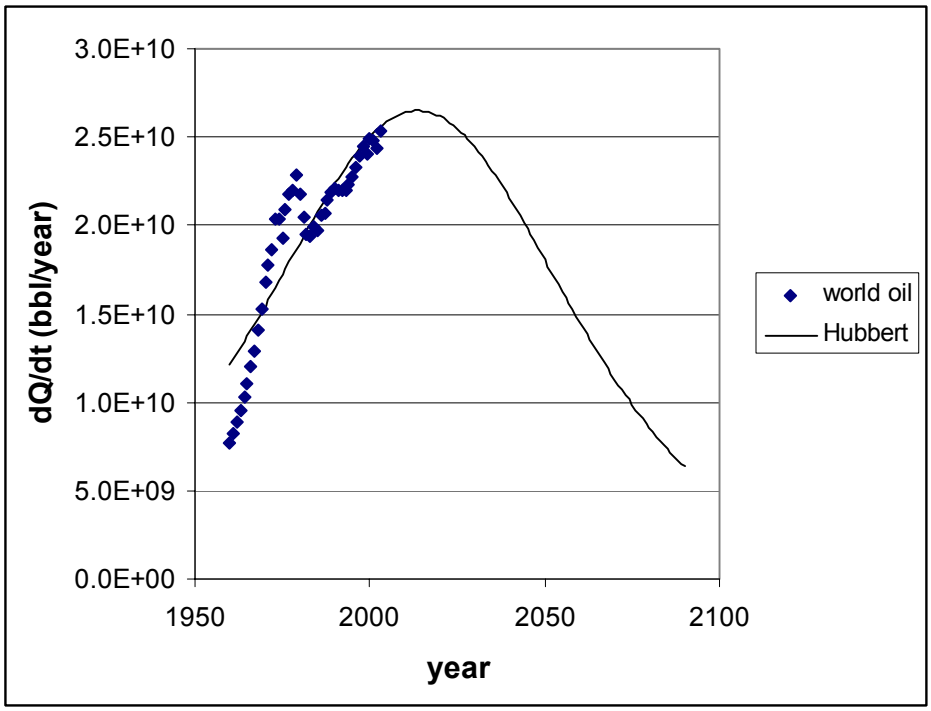

Figure 4. Actual data compared with Hubbert curve for world oil production using all data for the least squares fit. The calculated values for t_m and t_d are 2014 and 2184, respectively.

There is no reason why a subset of historical data could not be used for the best fit in an attempt to more accurately model historical behavior. (Disappointingly, none of the students thought to do this.) Using the data from 1960 to 1979, the year of the first peak in Figure 4, results in the comparison shown in Figure 5. Clearly, the Hubbert predictions shown in Figure 5 do not model the post-1979 data well. Another possibility is to use only the years from the bottom of the dip to the second peak, 1982 through 2003. This results in the curve shown in Figure 6. Here, the values for t_m and t_d are 2016 and 2187, respectively - only slightly higher than those determined from using the entire data set.

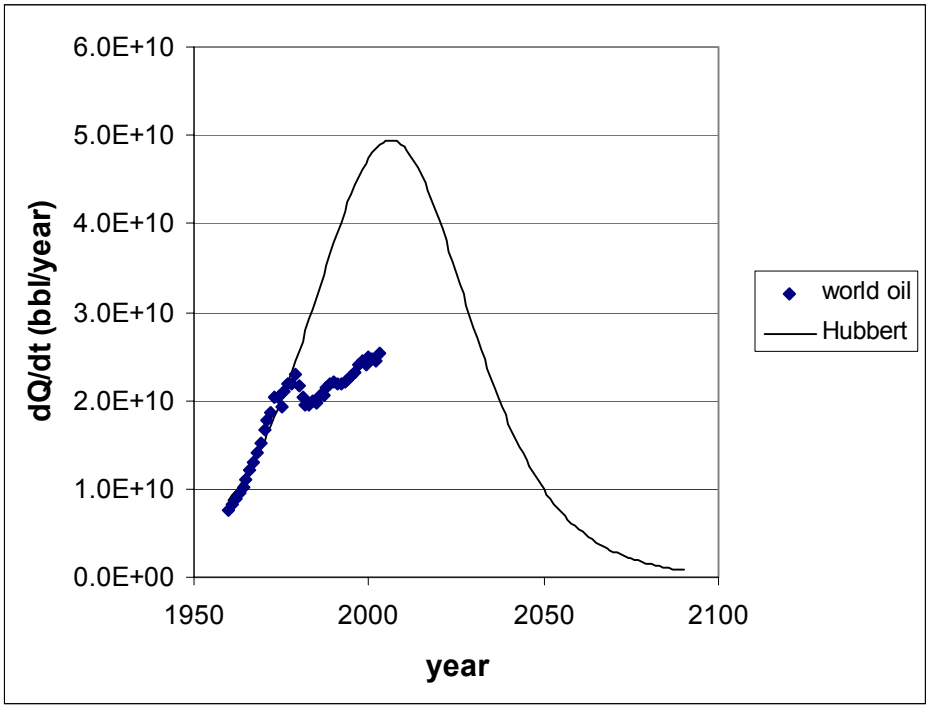

Figure 5. Actual data compared with Hubbert curve for world oil production using data from 1960 to 1979 for the least squares fit.

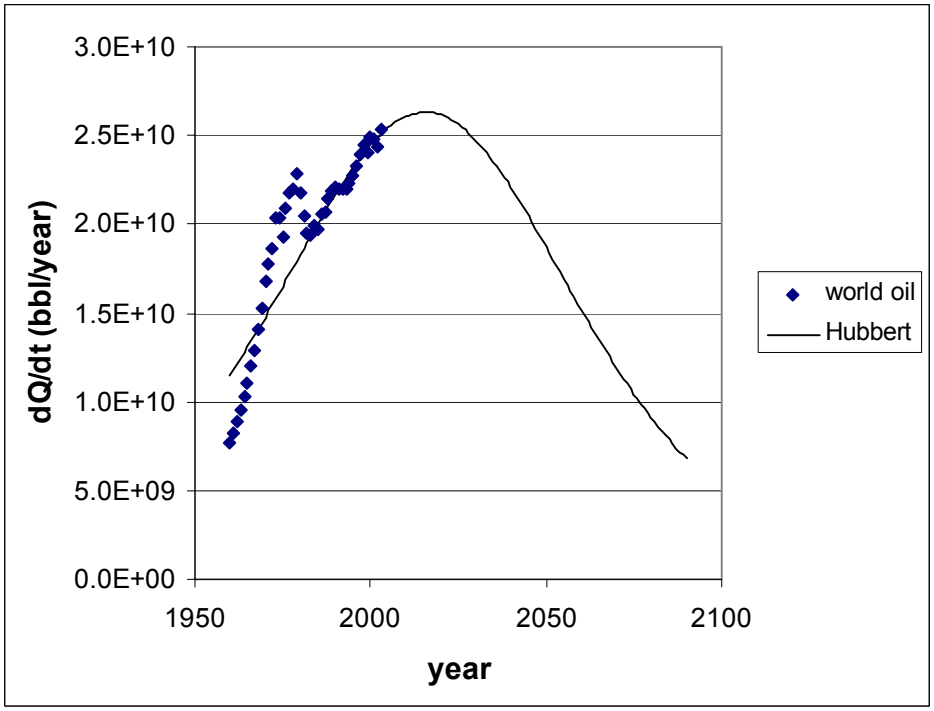

Figure 6. Actual data compared with Hubbert curve for world oil production using 1982 through 2003. The values for t_m and t_d are 2016 and 2187, respectively.

The comparison in Figure 7 is for U.S. natural gas. The values for t_m and t_d are 1983 and 2075, respectively. Since 1973 production has followed no discernable trend. And since 1999, production has seemingly leveled off. The future production of natural gas will be tied to our ability to extract methane from nonconventional sources such as gas hydrates and coalbeds. The increasing use of natural gas to power electric generating stations is also a factor. Whether U.S. natural gas production actually peaked in 1973 is yet to be seen as a variety of economic and technical factors play themselves out.

Results for U.S. coal are seen in Figure 8. The calculated values for t_m and t_d are 2062 and 2297, respectively. These predications are in line with other estimates for coal continuing to be available worldwide for another 300 years[16]. Although the model fit to the data implies that coal production is still in the exponential rise portion of the curve, coal's future production is dependent on a variety of factors. There is currently an increasing trend for using natural gas to fuel electrical power plants, thus diminishing the demand for coal. This could change, however, as pollution-preventing technologies improve and natural gas resources become scarcer.

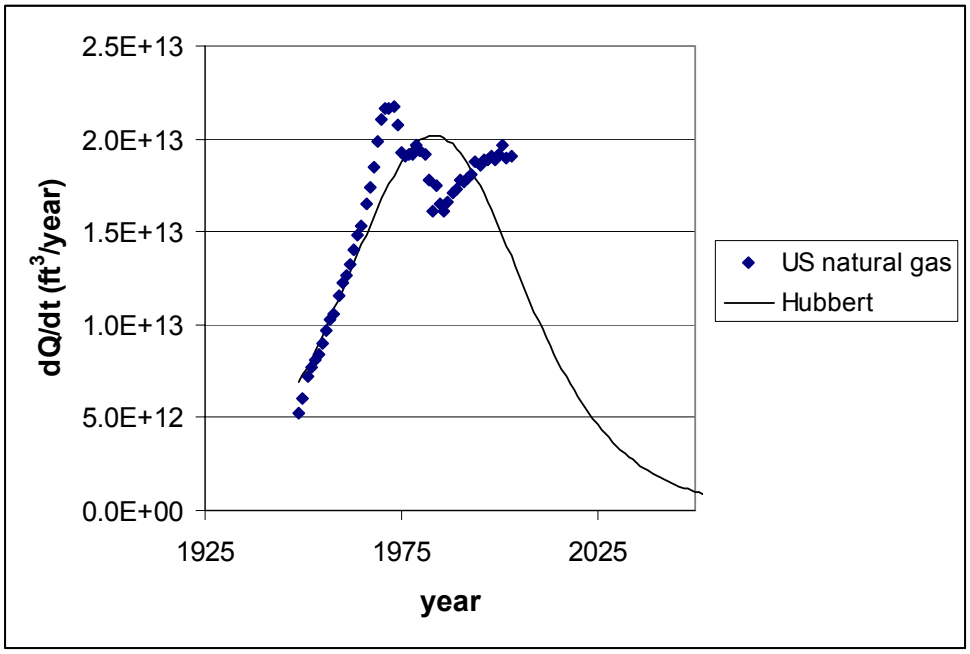

Figure 7. Actual data compared with Hubbert curve for U.S. natural gas production. The values for t_m and t_d are 1983 and 2075, respectively.

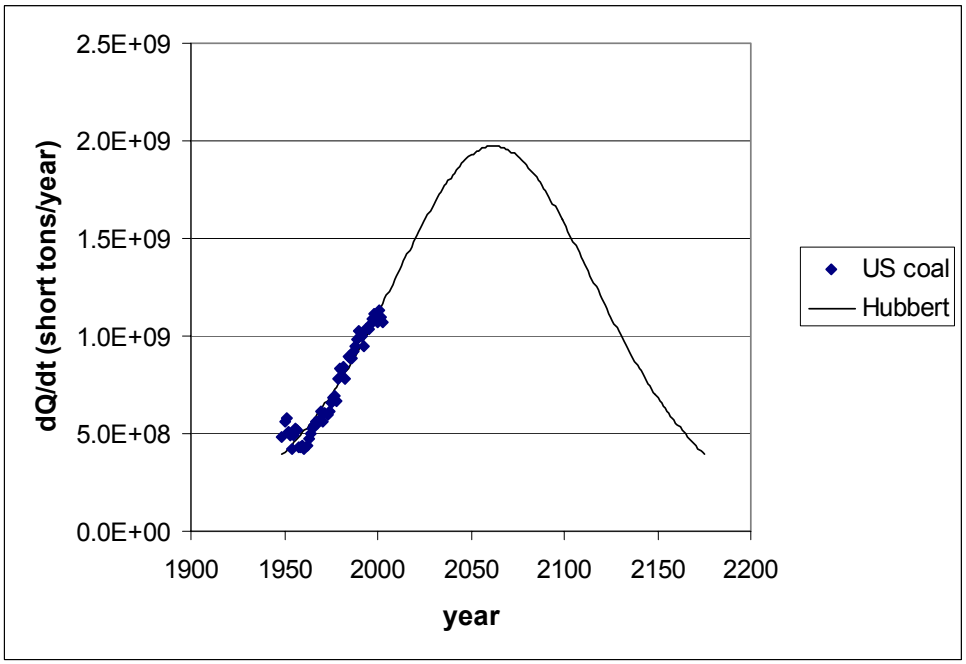

Figure 8. Actual data compared with Hubbert curve for U.S. coal production. The calculated values for t_m and t_d are 2062 and 2297, respectively.

Conclusion

Students successfully used the Hubbert model to predict years of peak production and depletion times for oil, natural gas, and coal. Values they obtained agreed reasonably with other published predictions. The Hubbert model gives students a much more powerful predictive tool than the exponential model. In classroom discussions subsequent to the Hubbert curve assignment, students were aware of the inevitable decline of fossil fuel sources, and were thus more sensitive to the importance of pursuing alternate forms of energy and designing systems to be as energy-efficient as possible.

One limitation of the assignment was that students simply utilized data sets that I provided to them. The assignment would have been more meaningful if students had been required to find the data for dQ/dt and Q_∞ themselves. This is not always an easy process – the data for Q_∞ must sometimes be generated by adding yearly production figures and recoverable resource estimates – but would give students an even deeper understanding of energy resource modeling.

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14) Deffeyes, chapter 8.
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