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The Inside Story: Revealing the Contents of CAD’s Black Box

Abstract

It is uncommon for CAD courses to delve into the underlying mathematics behind CAD models, however, a basic understanding of the functions performed by the CAD system are necessary for designers to utilize the software effectively. This paper will discuss several projects completed in a course on geometric modeling for senior/graduate level students in mechanical engineering that focus on developing this understanding. This exposure to the mathematical foundations within the CAD systems is intended to provide insight that will make the students better able to model complex shapes and build more robust solid models.

Background

There are many factors that have influenced recent changes in the engineering design graphics curriculum. Emphasis on theory and analysis in most engineering disciplines since the 1950s led to decrease in design abilities and less focus on practical skills needed in engineering practice. However, recent studies have identified a need for engineering graduates with better design skills such as teamwork and communication skills. Furthermore, the rapid pace of technology change has been a major influence in the area of engineering education, particularly graphics education.

Bertoline states that “approximately 92% of the typical design process is graphically based, and industry is utilizing visual modes of communication in greater amounts and variety, yet many engineering and technology curricula have seemingly let engineering graphical communication learning lapse.” ABET 2000 emphasizes the need for students to “communicate effectively”, but does not specifically include design graphics, as in earlier ABET accreditation programs. This has led to a reduction in the number of graphics courses in the typical engineering program, in spite of the overwhelming use of CAD in industry.

Studies done a decade ago aimed towards planning the curriculum for the 21st century include solid modeling and 3D CAD, along with spatial visualization, as the most important topics recognized by engineering graphics educators to be included in curriculum revisions, followed by teamwork and design activities, which ranked higher than conventional graphics topics. Crittenden lists mathematics as a general topic (not specific to geometry) and solid modeling (not constraint theory). Clark & Scales reported that 3D parametric modeling was the most important future trend identified by engineering design graphics educators. Cumberland & Miller reported that engineering graphics programs should include the topics of macro programming, data translation, file and data management, CAD standards, constraint-based solid modeling, web technologies, simulation and animation, internships, collaboration, and a study of current trends and issues. In a study by Branoff, Hartman, & Wiebe in 2003, of companies in Raleigh, North Carolina, the highest ranked topics were assembly modeling, constraint-based modeling, modeling strategies, 3D geometric primitives, and orthographic projection. However, these surveys do not distinguish between the need for understanding the basic theory vs. skill development.
Topics covered in introductory solid modeling courses typically include the creation of various features such as extrusions, revolves and pick-and-place features, modeling strategies, sketch constraint methods, assemblies, and creating and formatting drawings, amongst other topics from the conventional graphics curriculum such as creation of orthographic drawings, dimensioning and tolerancing, fasteners, etc.\textsuperscript{11} In response to the demands of industry and ABET, engineering graphics educators have also recently incorporated design projects into many introductory graphics courses\textsuperscript{12}.

Advanced CAD or computer graphics courses may also cover the creation of more complex swept and blended surfaces or solids\textsuperscript{13,14}; others include CAD tutorials within design courses which focus on manufacturing and analysis\textsuperscript{15,16}. Most of these courses appear to be primarily skill-based and focus on the use of the CAD tool in the context of design projects, with little concept or theory content. This trend seems to be driven by ABET criteria that require students to demonstrate the “ability to use techniques, skills, and modern engineering tools necessary for engineering practice.”\textsuperscript{5}

Is it important for students to understand the foundational mathematics that are used by the CAD systems to create and constrain complex geometries and parametric models? Does this understanding make them better able to model parts? CAD/CAE tutorials are widely used in introductory CAD or engineering graphics courses. However, these manuals do not teach basic CAD mathematics theory and concepts. Peng et al.\textsuperscript{17} claim to include theory as well as skill development in their CAx tutorials, but do not report on the content of these tutorials nor the influence of these tutorials on students’ understanding of basic engineering science fundamentals or CAD concepts. Their paper and others\textsuperscript{18} focus solely on the development of CAD skills.

Hartman and Branoff\textsuperscript{19} observe that “to practice these various concepts, standards, skills, and techniques, engineering graphics students are often asked to create 3D models or technical drawings of objects within a classroom environment. While this possibly gives a view of the overall process of documenting the design and production of an object, it does not necessarily emphasize the strategic approach to the use of certain tools, particularly three-dimensional modeling tools in general and constraint-based CAD tools specifically.” Dankwort\textsuperscript{20} distinguishes between relevant mathematics and practical experience, and calls on universities to teach both. According to Dankwort, “the aim is to make students or future engineers able to judge how reliable a result calculated by a CAx-system is and which specific application problem might be caused by the mathematical representation.” In a survey by Ye et al.\textsuperscript{21} in 2004 of employees of CAD software companies, nearly three-quarters of the respondents indicated that CAD users should have a basic understanding of the fundamental mathematics of analytical curves and surfaces, while 42% felt that CAD users should also understand the fundamental mathematics of NURBS. The same study reported that respondents felt that CAD users should have a fundamental (29%) or strong (66%) understanding of feature-based modeling, while 86% felt that users need fundamental (39%) or strong (47%) understanding of parametric modeling, including constraints. The authors quoted respondents who felt that CAD education in the university levels focused too much on usage rather than theory.

Hamade\textsuperscript{22} found that students who were better CAD learners had stronger backgrounds in CAD-related mathematics in addition to strong graphics background. Other advanced mathematics did not seem to influence the students’ CAD modeling performance. In other studies, the same researchers\textsuperscript{23,24} show improvements in modeling time and model complexity based on students’
development of procedural knowledge (modeling strategy) as well as declarative knowledge (familiarity with software functions). Barr et al.\textsuperscript{12} observed that students in their introductory CAD course who used analysis tools such as mass properties and finite element analysis reported improvements in design and communication skills, but wanted more knowledge of the mathematical basis for the analysis. They were able to complete the assignments, but did not understand the results that were generated from these analyses. Lin et al.\textsuperscript{25} argue that including the mathematics of spline-based geometry in a CAD/CAM course helps to fulfill ABET requirement “(a) an ability to apply knowledge of mathematics, science, and engineering.”\textsuperscript{5} Thus, it would appear that including the foundational mathematics utilized in CAD systems in the engineering design graphics curriculum would be a worthwhile endeavor.

Course Description

The author has developed a two-credit senior/graduate level course for mechanical engineering students covering topics in computer aided design and geometric modeling. As stated in the syllabus, the course is not intended as a CAD training course, but includes learning the mathematical foundations for creating complex geometric objects, investigating how the geometric database may affect analytical or manufacturing applications, and learning how geometric objects are manipulated in typical CAD systems. The course is structured with two 2-hour lectures/week for seven weeks; there is no laboratory component.

The project-based course attracts full- and part-time graduate and undergraduate students with diverse academic and CAD backgrounds, and therefore the projects are not dependent upon the use of a specific CAD software package. The course description states that students should have a background in “calculus, linear algebra, introductory computer programming, and ability to utilize a solid modeling CAD system.” The students’ level of CAD experience in the most recent offering of the course is depicted in Figure 1 (n=16 students). Note that some students take the course with little or no CAD or solid modeling experience, in spite of the catalog prerequisites and the instructor’s statement of expectations at the beginning of the course.

![ME593 Students' CAD Experience](image)

Figure 1. CAD experience of Students in ME593 CAD and Geometric Modeling, Fall 2010
Projects are executed using a combination of CAD software, spreadsheets and equation solvers. In one project, students explore the mathematics used to define cubic and higher order Bezier curves. A second project requires students to derive and solve sketch constraint equations, and design a solid object using constraint conditions for variational solid models. A third project is focused on interoperability between CAD systems, and is not relevant to this paper. Each project is completed in approximately two weeks.

**Project 1: Mathematics of Complex Curves**

*Background:* The goal of this project is to develop the students’ understanding of the mathematics for basic curves and surfaces, and the concepts of continuity, tangency, and shape manipulation using control points. The use of splines (NURBS) in surface design is particularly important in the design of many products, which utilize “organic” freeform shapes for both aesthetic and functional reasons. Continuity and surface smoothness influence functional characteristics such as aerodynamics, kinematics of cam contours, and many other product performance factors. Thus, it is important for design engineers to understand how to manipulate curves and surfaces.

*Assignment:* Many computer graphics systems use Bézier curves as the basis for generating shapes for various fonts. Simple, cubic Bezier curves are mathematically similar to B-splines in many respects, such as the use of blending functions, control points, and continuity conditions. This project challenges the students to design a scalable, translatable D’Nealian cursive font based on cubic and higher order Bézier curves. Each student selects two lowercase letters of the alphabet and develops their designs in an Excel spreadsheet model. All letters must conform to specifications for size and continuity, and utilize a minimum number of curves necessary to reproduce the desired shape. In this project, students are required to implement the Bezier equations in their spreadsheet, manipulate the data points in order to create the desired shapes, with appropriate tangency and continuity constraints, and combine their results with others to form words.

![Figure 1. D’Nealian font (adapted from 26)](image)

*Results:* Students were provided with a variety of links to web-based applets which demonstrated the use of Bezier curves, as well as resources, including the course text²⁷, which described the relevant mathematics. Most students were able to complete this assignment with little difficulty. Those who were not familiar with the graphing functions in Excel had some problems, but these were not related to the mathematics. Examples of student output are shown in Figures 2-6. Note that some students were more successful than others in duplicating the desired shape with a minimum number of curve segments. In Figure 2a, the student used five curves to achieve the desired shape for the letter “h”, whereas another student, Figure 2b, used only three curves, but did not conform to the baseline tangency requirement and has a poorly shaped letter. Similar results are shown for the letter “m”, Figure 3. For the letter “k”, both designs show similar shapes, however, one student used four curves while the second student required five curves, as shown in Figure 4.
Figure 2. Student designs for the letter “h”

Figure 3. Student designs for the letter “m”
Figure 4. Student designs for the letter “k”

Figure 5. Additional examples of Bezier letters designed by students in Project #1
Feedback from students was generally positive. Sixteen out of 26 students responded to a post-course survey with the following comments about the Bezier project:

"At first glance, this project seemed much harder than it actually turned out to be. Once I was able to understand how the Bezier curves are intended to be used and the fundamental equations, the project became much easier... Prior to the class... I really had no understanding of how Bezier curves were even supposed to be used. I remember trying to use them in assorted programs and could never figure them out; I typically would just end up getting frustrated and giving up. However, I am now confident that I could use any program to tackle any task involving the use of Bezier curves."

"This project has definitely solidified my understanding of Bezier curves and their function in many real-world applications. It certainly made me realize the great amount of work that goes on ‘behind the scenes’ in any CAD modeling system..."

"While the project introduces interesting concepts, I thought it would be more fitting for an advanced mathematics project. It did illustrate the use of Bezier curves in modeling systems, but I was less interested in learning how modeling systems worked, and more interested in learning advanced modeling techniques."

"The first project was very interesting. I have always had trouble understanding how Bezier curves worked in graphics programs such as Adobe illustrator. The introduction to the curves from the course gave me a solid understanding of the mathematics behind them so that now I feel confident producing NURBS of my own design."

"I think this should still be included in the course, because students will definitely learn a great deal of the mathematical functions that go on ‘behind-the-scenes’; with fonts and sketching in CAD applications."

Project 2: Constraint Theory and Applications

Background: This project focuses on constraint theory. Solid modeling systems use a variety of constraint types such as ground constraints, geometric constraints, dimensional constraints, and algebraic constraints. Constraints are used to control 2D sketches as well as 3D features. While some solid modeling systems, such as SolidWorks, allow the user to generate solid features from under-constrained sketches, others, such as ProEngineer, require the user to create a fully constrained sketch. For any sketch, there may be many possible combinations of geometric and dimensional constraints that will fully define the sketch. However, each of these possible solutions has implications with respect to the design flexibility, robustness, and capturing design intent. An understanding of the mathematics used to solve the constraint conditions for 2D
sketches helps the designer to properly constrain sketches and debug problems with over- and under-constrained sketches. In the area of 3D modeling, dimensional and algebraic constraints may be used to capture design intent and ensure that variational models are robust. This is particularly important for models that will be used by someone other than the original designer, and may be subject to changes in design during the product life cycle.

Assignment: In this project, the students are asked to design an object that belongs to a family with multiple configurations. The object should have a specific feature that requires a 2D sketch profile with a unique design. The specific object used for this project was a chair with decorative cutouts on the back. The students were required to design the profile of the cut-out using both line segments and arcs, with a variety of geometric constraints and dimension types. Students were required to identify all of the characteristic points (vertices and arc centers) associated with the profile, determine the number of degrees of freedom for the profile, and develop a set of equations to solve for the locations of the characteristic points. Most of the students chose to solve this system of equations using MathCAD or MatLab.

The second part of the assignment is to create a robust, flexible solid model that could be used to fabricate variations of the chair such as a bar stool, children’s chair, bench, etc. with the same design features. Constraints were required to ensure that the model retained appropriate and aesthetically pleasing proportions and did not “crash” if inappropriate values were applied to the variable dimensions. These constraints were implemented primarily through the use of algebraic constraints (equations). Students used a variety of CAD systems, including SolidWorks, ProEngineer and Inventor.

Results: For the 2D sketch constraint portion of the project, some students had difficulty understanding the concepts of characteristic points and were unable to develop equations for geometric constraints that were not presented in class. Others were able to develop all of the equations, but were not able to demonstrate a solution. Only a few of the students in the class were able to solve their systems of equations to find the x-y coordinates of the characteristic points in their sketches. Several factors that contributed to these difficulties included the students’ lack of familiarity with equation solving software (MathCAD) as well as the complexity of the sketched profile. Students with overly complex cut-outs generally had difficulty with this part of the project. Inspection of the sketch profiles within the actual part files revealed that all students were able to fully constrain their cut-outs, and most students used appropriate geometric constraints to maintain symmetry and other characteristics related to design intent. Examples of student sketches are shown in Figure 7.

Figure 7. Examples of constrained sketches for chair cut-outs, project #2
Student comments pertaining to sketch constraints:

“This should be included in the course since the mathematical magic is made manifest with constraints.”

“I enjoyed this project as well because the sketcher constraint portion helped to improve my modeling skills and I felt it connected to my other coursework.”

“This project was an interesting experiment trying to recreate the equations that govern parametric modeling in modern CAD systems. Simple linear dimensions and ground constraints are easy to understand and model, but more complex relations like tangency and arc centers can be difficult to understand. By setting up these systems and solving as an entire system, the parametric adjustability of the sketches can also be easily calculated on the fly... The operation of these systems does require highly robust equations and smart thinking when defining relations by the user. It also requires that systems are neither under nor over defined for the system to come to a solution. This was a problem initially when setting up the relations on my chair back cutout. However by studying the system and thinking of each entities individual DOF’s, it is often easy to find the over defining relation.”

For the solid modeling portion of the project, students were enthused with the opportunity to be creative and come up with their own original designs. Examples of student chair designs and variations are shown in Figures 8 and 9. Student #1 chose to permit variations in seat depth, seat height and back height using family tables to control user options (Figure 8). Notice that the number of rungs as well as scaling and position of the cutout automatically change when the seat and back dimensions are altered. Student #2 chose to vary the seat width, leg height and depth of the grooves in the seat, while maintaining compatible dimensions of the dependent features.

Figure 8. Variations of chair design, student #1
Students demonstrated mixed results in constraining their models under variational conditions, and in general, the instructor was usually able to crash the models. Nonetheless, most students came up with at least some equations (algebraic constraints) to control their models. Students with more experience and better CAD skills were usually more successful. Table 1 lists and example of equations used by one student to maintain geometric integrity and control proportions and sizes of features in the model.

Table 1. Equations to control geometry of a variational chair model.
Student comments about variational modeling:

“This was very helpful in forcing me to learn about inputting equations and analytical constraints in CAD. I had never used those features before and became more experienced with them. This also helped with understanding better ways to approach designing a part or assembly.”

“... I was able to create a robust and adjustable chair model that could be very useful to a designer. These strategies are very important to understand when using CAD systems, as they can make modeling of anything that requires scaling or frequent adjustments much simpler and more robust.”

Overall Course Outcomes

A post-course survey was administered to determine whether the students were satisfied with their learning in the course and meeting the course objectives. Sixteen out of 26 students completed the survey. Results are shown in Figures 10 and 11.

General course comments:

“I learned a lot about the inner workings of CAD systems which I believe will help me with designs in the future. I did not know very much about CAD before I started the course, but I am confident enough now that I would be able to beat the learning curve in any on the job training.”

“I get more than what I expected. I really did get a deeper appreciation for CAD applications and all the work ‘behind the scenes’. It is definitely a great deal of work and mathematical functions that really go into sketching and modeling.”

“I think that the course description was accurate and that I learned about how CAD systems work and that gave me a better understanding to improve my modeling skills.”

“Yes, I feel like I have better understanding of the internal workings of the CAD system.”

**Course Improved Solid Modeling Skills**

![Pie chart showing survey results for course improved solid modeling skills](image)

**Figure 10. Did the course improve your solid modeling skills?**
Conclusion

The author has presented the outcomes of a course in geometric modeling for upperclass undergraduate and graduate students which includes a focus on fundamental CAD mathematics and constraint theory as a basis for design projects. The course did not include any CAD instruction, and students used a variety of CAD software as well as equation solvers and spreadsheets to complete the projects. Results suggest that including more mathematics and discussion of modeling concepts helps students to understand the internal workings of the CAD systems and supports the development of better CAD and design skills.

References