The Integration of Technology, Writing and Mathematics into an Introductory Matlab Course for Engineering Students

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Abstract

In this paper we will present material used in the Vaughn College Introduction to Matlab course. CSC 215 is offered to engineering students that have had a course in calculus and writing.

The objective of the first half of the course is to teach students basic programming. What is unique is that the programming techniques are motivated by problems seen in the traditional calculus course, such as Newton's root finding method, numerical integration and differentiation.

The second half of the course is the study of the numerical solution of differential equations. We first look at the physical model, then make the appropriate numerical scheme. Next an error analysis is done, code is written, and solutions are studied. Finally, we look at differences between linear and non linear equations and the onset of chaos.

Students are expected to write a final report interpreting their results, combining the techniques they have learned and give group presentations. Open ended research questions appropriate for undergraduates will also be discussed.

Introduction

Students in the Vaughn College engineering program, take in their first and second year, calculus, physics, writing and engineering. The prerequisite for CSC- 215 is precalculus. Because of the number of required courses in the first year, most of the students are in their second or third year. In practice, most students have had a year of calculus, a one year introduction to physics in addition to statics, dynamics and a vibrations course. What students often fail to see is the relationship between their courses. In designing a second year introductory programming course, we thought it would be a good place not only to teach programming, but to apply what they have learned in other courses, and to encourage higher order learning.

The basis for this course is several studies that claim that student's understanding can be enhanced when writing is incorporated into engineering courses [1 2,3]. The claim of Writing Across the Curriculum is that in order to write about a concept clearly you must have a good understanding. In addition to the writing component, we are connecting the physics and calculus courses that freshman take. Kumar and Jalko[4] make the claim that mathematics courses should be taught from an applications point of view. We also look to reinforce several efforts at Vaughn to improve retention rates of engineering students. Some of these efforts have included freshman orientation courses, learning communities and faculty mentoring programs.

Projects in the course require several components. Students are expected to formulate the problem, that is, they are expected to understand the physical or mathematical principle behind the problem. Once the underlying principle is understood, students can then write a program to numerically solve the problem. Different types of error are explored. Students are then required to write and present a final report. As an illustrative example, we will use the simple pendulum, which students study in detail. This example has the advantage that it is applicable to mechanical as well as electrical engineering students.

Physical Model

When discussing a simple pendulum, we consider a mass attached to a string as figure 1. below illustrates with the pertinent parameters illustrated.

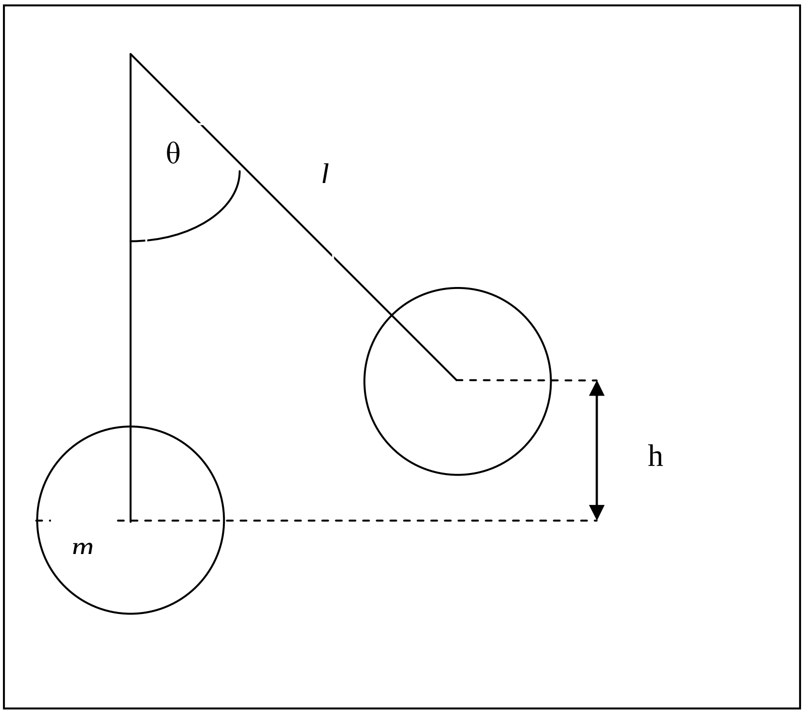

Figure 1. Diagram of a mass, m attached to a string of length l. The position of the mass is determined by the angle θ. The potential energy is determined by h.

The equation of motion is derived using the conservation of energy,

$$E = \frac{1}{2}mv^2 + mgh. \tag{1}$$

This approach is usually new to second year engineering students, in most physics and statics courses students are expected to resolve forces. Students are shown the advantage of using angular coordinates, i.e. $v = \frac{ds}{dt} = l\frac{d\theta}{dt}, h = l(1-\cos\theta)$. The above expression (1) for energy becomes,

$$E = \frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2 + mgl(1-\cos\theta). \tag{2}$$

Using the conservation of energy, $\frac{dE}{dt} = 0$, we obtain a second order nonlinear ordinary differential equation,

$$\frac{d^2\theta}{dt^2} + \omega_0^2 \sin\theta = 0. \tag{3}$$

The units of physical parameters are emphasized that is, angular frequency, frequency and period as summarized below in table 1.

Name of Parameter	Symbol	Units
Angular Frequency	$\omega_0 = \sqrt{\frac{g}{l}}$	rad/s
Frequency	$f = \frac{\omega_0}{2\pi}$	Hz
Period	$T = \frac{1}{f}$	s

Table 1. Summary of symbols.

Mathematical Approximations

Given that we are faced with a non linear equation from our physical model, the first topic of discussion is using a small angle approximation to derive a linear model. Students can use what they have learned in calculus about limits to obtain a linear equation. Using $\sin\theta \approx \theta$, (3) becomes a linear equation,

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0. \tag{4}$$

We now have two equations (3, 4) that we want to solve numerically. Students will have exposure to Taylor series; however they usually do not see an application. Forward, backward and central difference schemes are discussed along with their respective orders of convergence. We will use second order approximations to write two difference equations. Using x for θ, in the linear equation (4) and y for θ in the nonlinear equation (3), we obtain the following,

$$\begin{aligned} x_{i+1} &= 2x_i - h^2\omega_0^2 x_i - x_{i-1}, \\ y_{i+1} &= 2y_i - h^2\omega_0^2 \sin y_i - y_{i-1}. \end{aligned} \quad \text{with } i = 1,2,\ldots,N, \text{ and } h = \frac{T}{N}$$

Students are given the assignment to derive higher order schemes and explain their advantages and disadvantages. An expression for the relative error at each step is derived for the linear problem, students are then asked to pick an appropriate step size based on the physical parameters of the model and a given tolerance. We then discuss the relative and absolute error for the linear pendulum, and look to see how step size is related to the errors.

Programming

Once students have written the two differential equations as difference equations, they are then required to use what they have learned during the first half of the course to write a program. The program will begin by defining the physical and mathematical parameters, such as angular frequency, frequency, period, initial conditions and step size. A for loop is used to calculate the next value given the previous two steps. Although there are techniques used in Matlab to avoid for loops, we still use them to illustrate using an equal sign as an assignment, and to gain experience using control loops. A data file is created for the values of the positions of the two equations along with the respective time. At each step the relative error is calculated, for both the linear and non linear pendulum. Students will then graph, the solutions to the linear and nonlinear pendulum, along with the analytical solution to the linear pendulum.

Program 1

```
% This program will simulate the motion of a pendulum and compare the linear
% pendulum to the nonlinear pendulum.

% Set Parameters

% Physical Parameters

clear

g=9.8;
l=9.8;
om=sqrt(g/l);
f=om/(2*pi);
T=1/f;
```

```
%Mathematical Parameters

N=100;
h=T/N;
nu=2-h^2*om^2;
m=3;

% Set Initial Conditions

theta=pi/6;
x(1)=theta;
x(2)=x(1);
y(1)=x(1);
y(2)=x(2);

% Begin Iteration

for i=2:m*N-1

   x(i+1)=nu*x(i)-x(i-1);
   y(i+1)=2*y(i)-h^2*om^2*sin(y(i))-y(i-1);

end

t=[0:h:m*T-h];
x_min=min(x);
y_min=min(y);
s_min=min(x_min,y_min);

x_max=max(x);
y_max=max(y);
s_max=max(x_max,y_max);


plot(t,x,'k')
hold on
plot(t,y,'k:')
hold on
hold off
hold off
grid on
axis([min(t) max(t) s_min s_max])
title('Undamped Pendulum \theta_{o}=\pi/6')
xlabel('t')
ylabel('\theta')
legend('Linear','Nonlinear')
save nlpend t x y  N m h
```

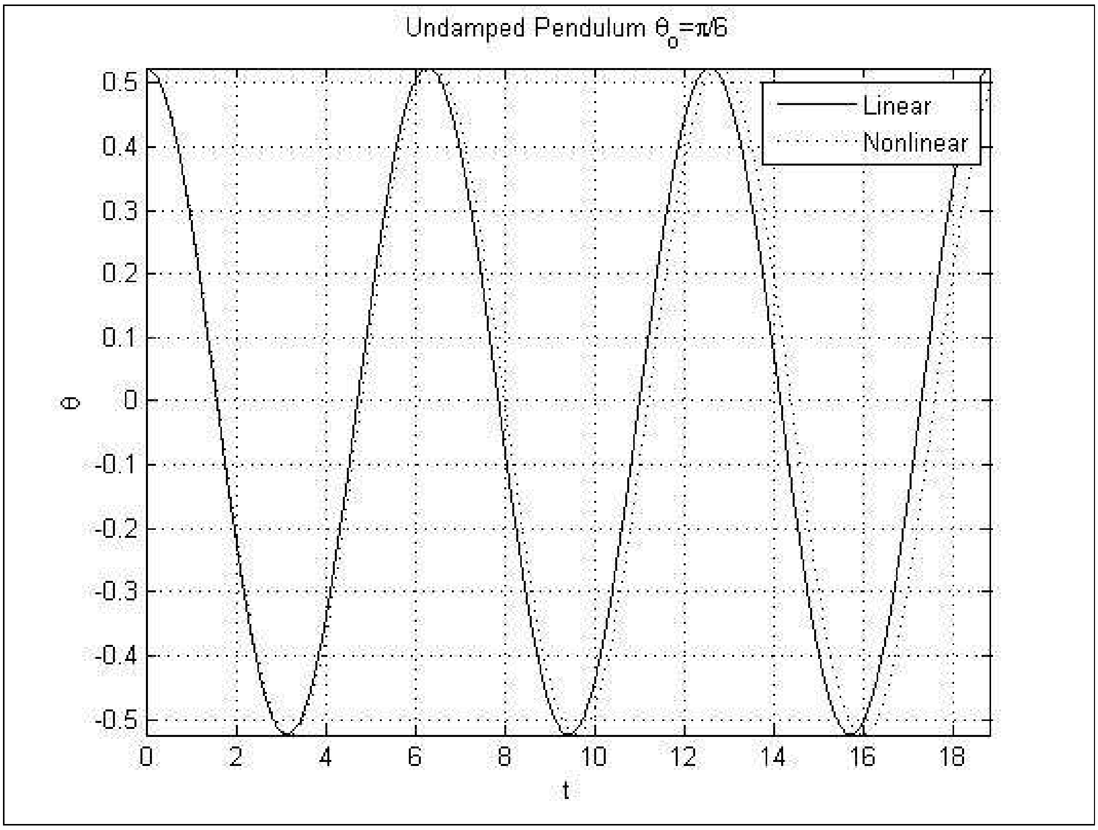

Figure 2. Output of the undamped linear and nonlinear pendulum.

$$\theta'' + \omega_0^2 \theta = 0, \theta'' + \omega_0^2 \sin\theta = 0, \theta(0) = \frac{\pi}{6}, \theta'(0) = 0, T = 2\pi, N = 1000$$

Writing

The last part of the project requires students to write a report about their project, explaining each step. At this step students are given the task of explaining the parameters developed in the physical model. In order to develop higher order thinking, students are given a set of interpretive questions to use in their narrative. For example, using Fig. 2 students would be expected to notice the differences between the curves for the linear and non linear pendulum. The two curves are virtually identical for the first period, but they begin to change during the second period. Comparing the curves for the second and third period we notice that the time for the nonlinear curve to obtain its peak is delayed. Once a student makes this observation, the next step would be to modify Program 1. to include more periods. Using the data file produced, we would then ask for a program to graph the time difference versus the period number. Using Matlab's logical search capabilities, students can then discuss quantitatively how results from the linear and nonlinear pendulum differ.

Once students understand the physical and mathematical principles and have their code working, we want them to apply what they have learned to a new application. Depending on the students interest they can pick any combination from below or they can discuss with me another application.

Applications

In what follows, we list applications in the form of questions that students can use to modify their existing code, and see if they can apply what they have learned to a new situation.

1. How does changing the length of the string affect the angular frequency and the period?
2. What effect does changing the sign of the initial velocity have on the maximum amplitude?
3. How can you interpret the solutions when both the initial position and velocity are not zero?
4. What do you notice about the difference in solution between the linear and nonlinear pendulum? Does changing the initial position affect them differently?
5. What happens to the difference in period between the linear and non linear pendulum as you graph for more periods.
6. How would damping change the solution to your differential equation?
7. How do you incorporate damping into your numerical scheme?
8. Describe the differences between under, critical and over damping.
9. How would your numerical scheme change if you introduce an external forcing term?
10. Pick several values of the strength of the forcing term and change the initial conditions by a small amount. What do you notice between the linear and non linear problems?
11. How would you use your data file to find the velocity of the pendulum?
12. Using your data to graph a phase plot, what differences do you notice between the linear and non linear case?
13. In an electrical circuit, how would the parameters mass, length and damping change?
14. What would be different from solving a second order initial value problem and solving a boundary value problem?

Assessment

The first part of the course is designed to introduce students the basic programming techniques. In order to do this we give the students several small projects to illustrate the concept and get them comfortable working with Matlab. This is done during the first six weeks of the semester and concludes with an in class written mid term. The exam is designed to assess a students' familiarity with basic syntax, control loops, variable assignment, matrix operations and logical indexing. Below we summarize the first several projects.

Project	Title	Purpose
1	Matrix Operations	1. Familiarity Matlab Environment 2. Creating Matrices 3. Using m files 4. Syntax
2	Summing Series	1. Assignment 2. Control Loops 3. Input Statement
3	Newton's Root Finding Method	1. Logical and Relational Operators 2. Relative Error 3. Absolute Error
4	Weather Data	1. Import Data File 2. ASCII Files 3. Logical Indexing
5	Numerical Integration	1. Construct Tables 2. Graph Results

Table 2. List of projects.

Feed back is given on the projects. Students are assessed on the content and the style of their program. They are expected to explain each part of the program and to make sure there is proper indenting, spacing and variable assignment makes sense.

The second part of the course will include a written report approximately three pages long, not including programs, graphs and tables. The items to be included in the report are given below.

1. **Introduction.** Explain what you are going to accomplish. Assume the reader know less than you.
2. **Physical Model.** Explain the physics behind the model you are using. This must include relevant parameters and correct units.
3. **Mathematical Approximations.** Explain any approximations are you using. Be sure to discuss any limitations.
4. **Program.** Include the program you wrote and any output, graphs or tables.
5. **Conclusions.** Explain what you have learned. You must give specific examples from data or graphs that were generated.

Students are broken into groups of at most three students. They will work together and take one part of their report expand upon it and give a fifteen minute power point presentation. Students are assessed using both their written report and their presentation. Below is a list used to help in the assessment.

Written Report
1. Organization of the report. Did the student follow the guidelines?
2. Is the spelling and the grammar correct?

3. Are conclusions backed by examples from other parts of the report?
4. Does the student explain the meaning of any equations, graphs or data used?

Oral Presentation
1. Is the presentation clear?
2. Are the slides informative?
3. Did the student speak clearly?
4. Did the student make eye contact with the audience?

Conclusions

Our goals are similar to the four thrusts described in the NSF coalition[5] on a local scale. That is we are hoping to integrate the curriculum, by offering a course that incorporates other courses in a meaningful way. By giving the students the tools and the flexibility to modify what they have learned, cooperative learning is being encouraged. Using the computer as a tool to do computations we are fostering technology enabled learning. The writing component offers students the opportunity to improve through self assessment. We think that when students apply tools from one course to another course we are fostering a higher order of learning.

Future Goals

There are discussions about introducing a physical experiment in the class. Students can collect data from a simple pendulum or an electrical circuit and use Matlab to model their data. Students can make comparisons between experimental, numerical and analytical results. There would be another element to the discussion about when a linear approximation breaks down.

Students in the upper level engineering courses use Simulink for control and design projects. It has been requested that we spend some time in our course introducing students to Simulink. Currently there are discussions about how this is to be done.

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