AC 2010-390: THE NATURAL STRUCTURE OF ALGEBRA AND CALCULUS

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The Natural Structure of Algebra and Calculus

Preface

In every well-planned course, only one thing is studied.

In arithmetic, numbers are studied. After studying arithmetic, a student should know the various kinds, forms, operations, properties of and relations between numbers. In the end, a student should feel confident when working with numbers.

In geometry, shapes are studied. After studying geometry, a student should know the various kinds, operations, properties and relations of shapes. In the end, a student should feel confident when working with at least polygonal shapes and circles.

Then, what are the objects we study in College Algebra and Calculus?

Functions

The major concerns in every quantitative scientific area are the relationships between the variables of that area. Since these relationships in the classical quantitative sciences were continuous and smooth, the mathematicians of yesteryear developed the concept of the function and the techniques of using functions. The concept has proven to be extremely successful and deserves a place of honor in the study of mathematics along with numbers and shapes. Technical competency in our modern world requires a thorough understanding of functions. College Algebra and Calculus comprise a study of a particular class of relationships between variables, in particular, continuous, smooth (differentiable) functions.

Suppose we are interested in the length, L, of a man's shadow as he walks away from a lamppost at night. The variables that might influence his shadow length are: the height, h_l, of the lamppost, the man's height, h_m, and his distance, d_l, from the lamppost. Would increasing the man's height increase the length of his shadow? How would increasing the height of the lamppost affect his shadow? He observes his shadow lengthens as his distance from the lamppost increases. How can we think about or describe such relationships? The invention of the concept of a function provides a systematic way to study numerical relationships. Functions lie at heart of quantitative design and planning. What, then, are **functions** and what is there to learn about them?

The structure described in the title is depicted in Figure 9. The kinds and forms of functions are listed in tables 2 and 5. The properties of functions and some geometric objectives of calculus are listed in tables 3 and 4. After studying calculus, a student should understand of value of the summary provided by Figure 9 and Tables 2, 3, 4 and 5.

What are Functions?

A study of a simple quantitative system might be started by identifying the variables that describe the system. It is possible that there may be no relationships between these variables. However in special cases the values of some variables determine the values of other variables. When one or more variables determine the value or values of another variable the relationship between the variables is called a function. The relationship might describe variables that happen to move together, indicating co-variation. If the independent variable is time, the relationship describes the evolution of the dependent variable, a trend. The relatively fast changing voltage and current signals of electrical engineering are functions of time, whose graphs can be displayed on an oscilloscope. Often the relationship describes control. A change in the value of the controlling (independent) variable forces the value of the controlled (dependent) variable to change. These relationships, continuous functions, are the focus of Algebra and Calculus courses. Continuous functions are needed to describe co-variant or tracking variables, control, trends and signals. Variables track when they happen to move jointly with neither variable influencing the motion of the other.

Descartes invented the rectangular coordinate system, which enabled the trends contained in a discrete table of values for a function to be displayed as a set of dots or a connected set of line segments. The two dimensional coordinate system enabled an equation in two variables to be graphed as a continuous curve. The use of tables, curves and equations to represent continuous functions has become a necessity in modern science. Modern scientists, engineers and technicians must be able to recognize and use functional relationships, not only for their personal view of the deterministic continuous world, but also to be able to communicate with their associates about that world.

Functions are special relationships between variables. In the study of some physical systems, the value of one variable determines the value of another variable. However, most of the time, variables are not related. The length of a rectangle alone does not determine the area of the rectangle. The area of the rectangle is not a function of its length. A car's speed is not a function of the amount of fuel in the gas tank. However, the radius of a circle, indeed, does determine the area of the circle. This connection is special. The area of a circle is a function of its radius. Functions are related (or connected or linked) variables.

When one variable, say x, determines or controls the value of another variable, say y, we say that there exists a functional relationship between x and y. If we call this relationship f, than we write $y = f(x)$. This means that if we know or pick a value for x then, because of the functional relationship f, we can determine y. The controlling variable (in this case x) is called independent and the controlled variable (y) is called dependent. If the function is known, it can be displayed by a table, a graph, and sometimes an equation.

In the study of circles we might focus on the four variables: the radius, R, the diameter, D, the circumference, C and the area, A. It is known from geometry that the diameter, the circumference and the area are uniquely determined once the radius is known. These relationships are described by the equations:

$$D = 2 * R,$$
$$C = 2 \pi R \qquad \text{and}$$
$$A = \pi R^2 .$$

These equations describe in symbols the exact way in which the variables D, C and A are related to R. One is free to choose any positive number for the radius, but once the radius is selected there is absolutely no freedom at all for the values of the diameter, the circumference or the area.

The relationships describing this system provide examples of functions that would be symbolically described as:

$$D = f(R), \qquad C = g(R) \qquad \text{and } A = h(R).$$

In this system of four variables and three equations, R has been chosen to be independent. The remaining variables; D, C, and A that are determined by the equations are dependent. Usually one equation in one variable will determine the value of that variable. However, one equation in two variables will not determine the values of the variables. To determine the values of two variables, two equations are required. Loosely, **n equations are required to determine n variables**. If we had 4 equations with 6 variables we might choose 2 variables as independent. Now the 4 equations could be used to determine the remaining 4 remaining dependent variables. Let us examine the simple case of one equation in two variables. If one of the two variables is chosen as independent, the equation can be used to determine the value of the other variable. These equations describe the functions that are studied in Algebra and Calculus. Instead of describing functions with equations, functions may be described using a numerical table format. A table format may be convenient to use if the table is not too long and the desired values of the independent variable are listed. A table description of the function $A = \pi R^2$ is shown in Table 1.

R	$A = \pi R^2$
0	0
1	$A = \pi = 3.14159$
2	$A = 4\pi = 12.56637$
3	$A = 9\pi = 28.27433$
4	$A = 16\pi = 50.26548$
5	$A = 25\pi = 78.53982$
6	$A = 36\pi = 113.09734$

Table 1

Another format for describing functions is called a graph. In a graphical format the independent variable is usually plotted on a horizontal scale while the dependent variable is plotted vertically. In this format every equation in two variables appears as a curve. While a graphical format may lack precision, the format is invaluable for depicting trends and many of the features of functions.

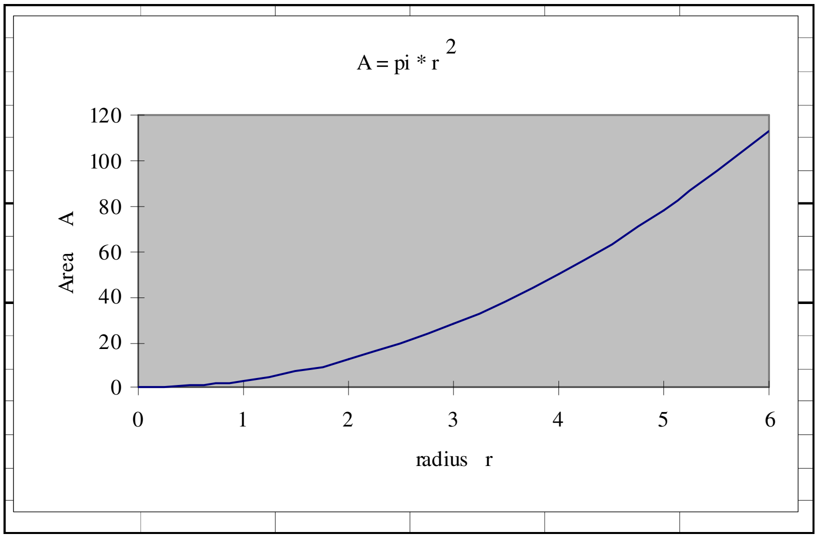

Figure 1

The properties of increasing, decreasing, rates of increase and decrease, maxima, minima, zero crossings and concavity become obvious in such a graph. The graph for the function $A = \pi R^2$ is shown in Figure 1.

All three formats,--- equations, graphs and tables have advantages and disadvantages but they are commonly used to describe quantitative relationships and a beginning algebra student must develop the ability to use any form and conceive of the problems that may occur in changing between the forms.

A curve that represents a function, where there is only one value for the dependent variable for every value of the independent variable, may cross a vertical line only once. Such a function is called single-valued. If the curve has no gaps or jumps, the function is called continuous. Some continuous curves exhibit sharp changes in direction called corners or “cusps”. Functions whose curves contain no cusps are called differentiable or smooth.

Properties of Functions

A short list of the properties of functions treated in algebra and calculus is provided in Table 3. These properties all have visual interpretations. Increasing means that as the independent variable increases, the dependent variable also increases; that is the points on the curve drift from lower left toward upper right. Decreasing means that as the independent variable increases, the dependent variable decreases; that is, the points on the curve descend from upper left toward lower right. Functions that are increasing or decreasing everywhere are called monotonic. Maxima are the peaks of the dependent variable and minima are the valleys of the dependent variable. Algebraic curves, like the circle and the tilted parabola, usually have more than one value for every value of x. In these cases mathematicians usually break the multi-valued curves into continuous single-valued segments called branches.

Kinds of Functions

The functions of algebra and calculus, called by mathematicians, the elementary functions[8] can be grouped, by their algebraic form or their method of construction into five categories:

1) Polynomials, 2) Rational functions, 3) Algebraic functions, 4) Transcendental functions and 5) Piecewise-defined functions.

Descriptions of the graphs of each of the kinds of functions follow:

1) **Polynomials** are single valued, continuous, smooth curves, defined for all values of the independent variable (say x) whose vertical values approach either + or − infinity for large absolute values of x.

Polynomials have <u>no</u> point gaps, <u>no</u> excluded intervals, <u>no</u> jumps, <u>no</u> cusps, <u>no</u> multiple values, <u>no</u> horizontal or vertical asymptotes (poles), <u>no</u> linear asymptotes for degree greater than 1, and <u>no</u> periodicities. Adding, subtracting and multiplying polynomials, produces another polynomial. But dividing polynomials may not produce another polynomial. Division may produce a new kind of function called a **rational function**. The number of crossings that a polynomial can have with a straight line can not exceed its degree. See Figure 2.

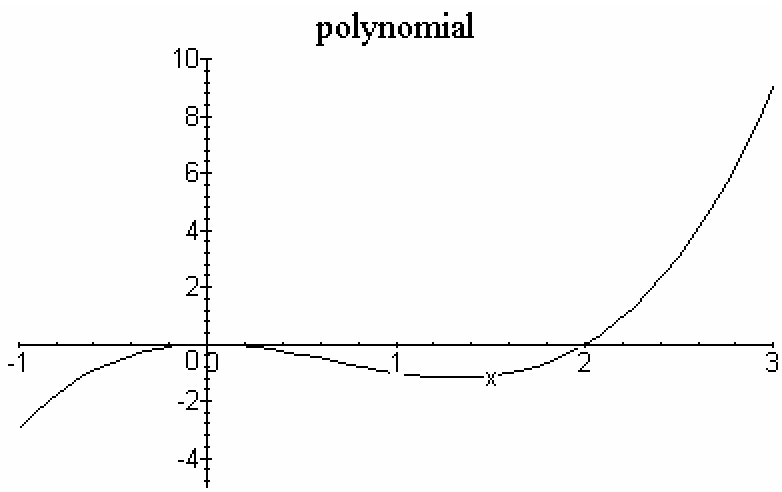

Figure 2 The polynomial $y = x^2 (x - 2)$

2) **Rational functions** are defined for all x, except at the finite number of points where there are zeroes in the denominator. Rational functions may have point gaps or vertical asymptotes where these zeroes occur in the denominator, but otherwise are represented by single-valued, continuous and smooth curves. These curves may behave at x = + and – infinity like polynomials but, perhaps, instead may have a horizontal asymptote. See Figures 3 and 4.

Rational functions cannot possess finite jumps, excluded intervals, cusps, loops, multiple-values, or periodicities.

Adding, subtracting, multiplying, dividing or composing rational functions produces another rational function. The set of rational functions is said to be “closed” under these operations.

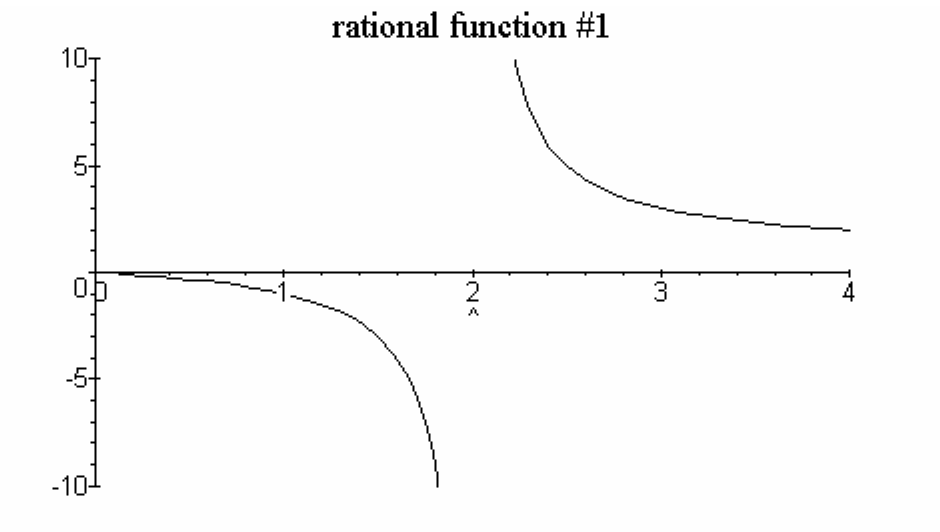

Figure 3 The discontinuous rational function $y = 1/(x - 2)$

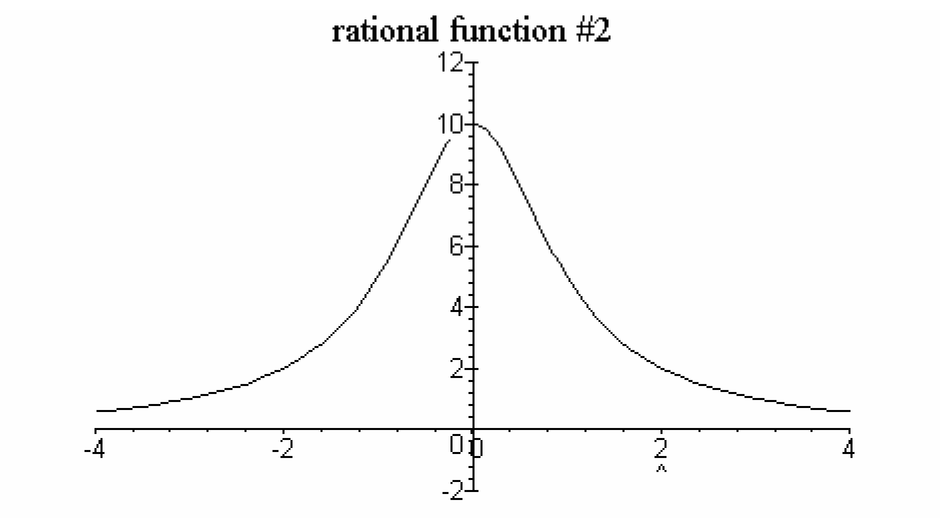

Figure 4 The continuous smooth rational function $y = 10/(1 + x^2)$

3) **Algebraic functions** are generally smooth continuous curves. These curves may be multiple-valued, may have cusps, loops, self-intersections and may have point gaps, excluded intervals and may be bounded or unbounded vertically and horizontally. Algebraic functions <u>cannot</u> be periodic. A circle is a simple example of an algebraic curve. See Figure 5.

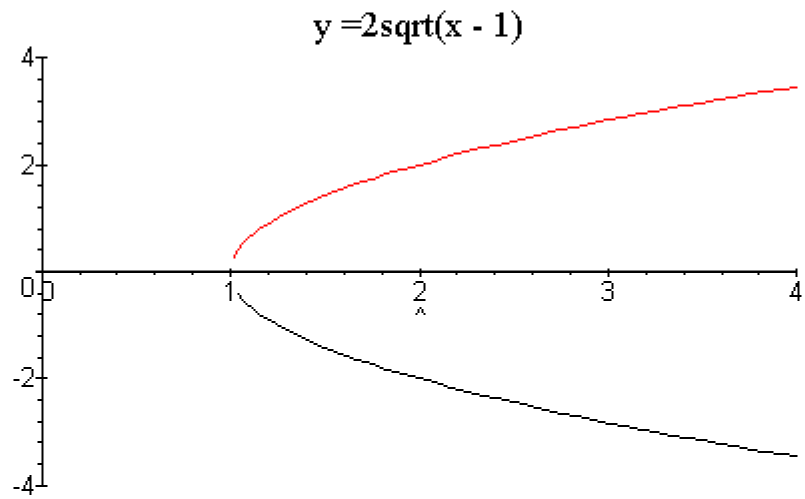

Figure 5 An algebraic function

It can be shown that the curves of polynomial, rational and algebraic functions can intersect a non-coincident straight line only at a finite number (called the degree of the curve) of points.

4) **Transcendental functions** like the trig functions have few elementary restrictions. They may be multiple valued, periodic and may intersect a straight line an infinite number of times. They may have an infinite number of asymptotes or may spiral. Also they may have point gaps, excluded intervals and cusps. See Figures 6. and 7

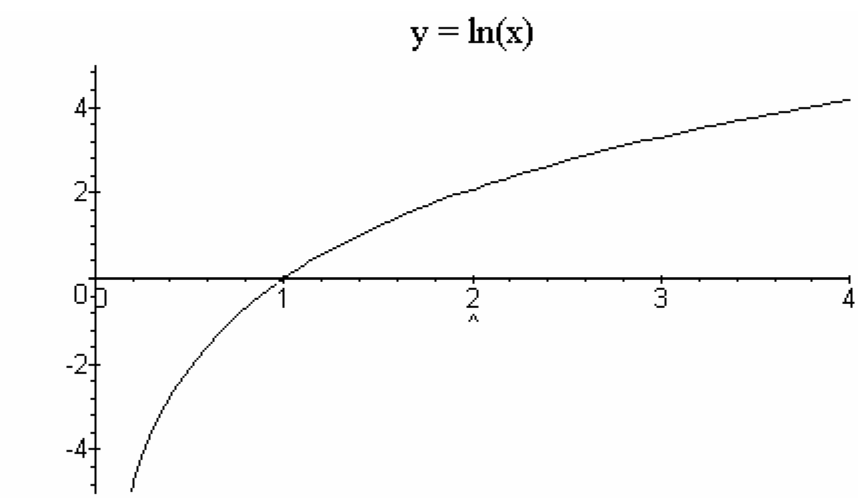

Figure 6 A Transcendental Function $y = \ln(x)$

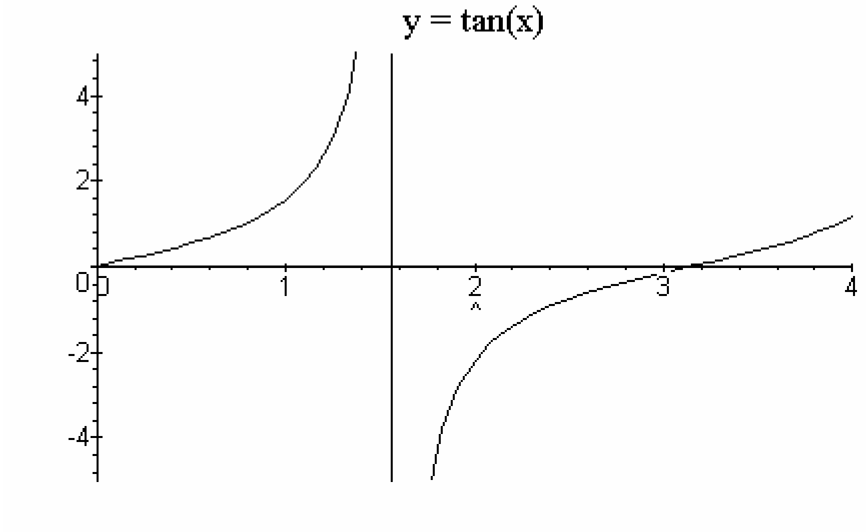

Figure 7 A Periodic Transcendental Function y = tan(x)

5) **Piece-wise defined functions** are made by piecing together segments of the above four categories. Examples are the step, pulse, staircase, absolute value and sawtooth functions. These may take on any of the properties of the curves of which they are made and may be discontinuous. See Figure 8.

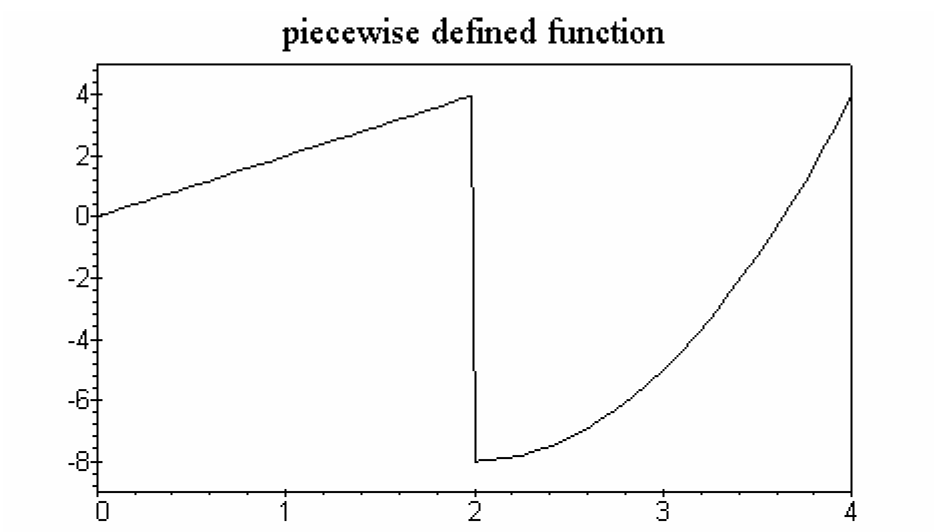

Figure 8

Operations on Functions

Many functions of algebra and calculus are obtained by performing operations on other elementary functions. The operations can be classified as unary or binary according to whether the operations require one or two operands (note: the operands are functions, not numbers as in arithmetic). Binary function operations, such as addition, subtraction, multiplication and division, take two functions as operands and produce a third function. Unary function operations, such as negation, reciprocation or squaring, take one function as an operand and produce a second function. The unary and binary function operations are conceived as performing arithmetic operations on the dependent variable for each value of the independent variable. Each of the following operations has a visual interpretation[8]. The student should find the pursuit of these visualizations intellectually rewarding.

unary function operations: $a\,f(x)$, $-f(x)$, $\frac{1}{f(x)}$, $(f(x))^2$, $(f(x))^n$ and $\sqrt[n]{f(x)}$

binary function operations: $+$, $-$, $*$ and $\div$

Calculus operations: $\frac{d}{dx} f(x)$ and $\int f(x)dx$

Following are some **important** theorems concerning the preservation of the properties of continuity, smoothness and monotonicity by some simple functional operations:

- Functions that are produced by adding, subtracting, or multiplying continuous/smooth functions will be continuous/smooth.

- Functions that result from dividing continuous/smooth functions will be continuous/smooth except where the denominator contains zeroes.

- Functions that result from adding, monotonically increasing/decreasing functions will be monotonically increasing/decreasing.

- Functions that result from multiplying single-signed functions will be single-signed.
- Functions that result from multiplying positive monotonic functions will be positive monotonic.

- Functions formed by the composition of continuous/differentiable functions, that is, $\{y= f(u);\ u = g(x)\}$ will be continuous/differentiable.

Symbolic Forms of Functions

Just as numbers have many forms[3, 7], each of which is valuable for different reasons, functions also have different forms. Tables and graphs are commonly used to describe functions and there are several symbolic forms for describing functions that are listed below in Table 5. A large part of the traditional College Algebra course has been concerned with the study of algebraic forms, in particular, the techniques of changing from one form to another. The second order polynomial function in expanded form:

$$y = x^2 - 4$$

can also be described in the factored form as:

$$y = (x - 2)(x + 2).$$

For every value of x, squaring and subtracting 4 produces the exact same value as multiplying the sum and difference of x and 2. Both equations will produce the same tables; both will produce the same graphs; yet the forms of the equations appear different and the operations required to compute y, when x is known, are not the same. Each form has advantages and disadvantages. What a strange and interesting phenomenon. Equations, such as:

$$x^2 - 4 = (x - 2)(x + 2)\ ,$$

that relate apparently different forms of the same function are called identities. Since the completely factored form of a polynomial displays the roots or zeros of the polynomial, factoring becomes an important step in the strategy of graphing of polynomials. Since the poles of a rational function can only be located at the zeroes of the denominator, to locate these poles, the denominator must be factored.

The rational function identity below converts the rational function in improper form to mixed form, that is the sum of a polynomial and a proper rational function:

$$\frac{x^3 - x^2 + x + 2}{x - 1} = x^2 + 1 + \frac{3}{x-1}$$

As an identity every value of x produces the same value when substituted into either side; meaning that even though the computations on both sides are different, both sides produce the same table and the same graph. The polynomial on the right exhibits the asymptotic behavior, far from the origin, of the function. The proper rational function is simpler than the function on the left and approaches zero for large x but displays the vertical asymptotes.

Some students may think the function $y = x^2 - 3$ does not factor because it does not appear to be the difference in two squares, yet the following identity prevails:

$$x^2 - 3 = (\ x + \sqrt{3}\ )(\ x - \sqrt{3}\ )$$

Some students might think $x^2 + 4$ does not factor, yet surprisingly, the following algebraic identity prevails:

$$x^2 + 4 = (x + 2\sqrt{x} + 2)(x - 2\sqrt{x} + 2)$$

As to the trigonometric, exponential and logarithmic function identities, they are all marvelous and special. It is surprising to see the elementary trigonometric identity:

$$\sin^2 (x) = 1 - \cos^2 (x)$$

confirmed for some random 8 digit angle.

Inverse, Implicit and Multivariable Forms

Most students first encounter functions as a combination of operations on the independent variable, x which produces a value for the dependent variable, y. The equation, $y = f(x)$, is called the **explicit form** of the function. If this equation is solved for the independent variable x, the form $x = g(y)$ is produced. This equation, which describes the same curve, is called the **inverse form** of the original function. A more general form which includes both these forms is an equation which contains combinations of both variables x and y intermixed. This form, $F(x, y) = 0$, which favors neither of the two variables, is called the **implicit form**. This more general implicit form of a curve allows both values to be multi-valued.

As an example, examine an ellipse. Symbolically the equations of the three above forms appear as:

1) Implicit: $(x/5)^2 + (y/3)^2 - 1 = 0$,

2) Explicit: $y = 3\sqrt{1-\left(\frac{x}{5}\right)^2}$ and

3) Inverse: $x = 5\sqrt{1-\left(\frac{y}{3}\right)^2}$.

Interestingly, it may be easier to obtain the tangent line to a curve in implicit form than for the same curve expressed in explicit form. The solution curves found by the elementary techniques of the classic ordinary differential equations courses were usually described in implicit form.

In studying a three-variable form of a two dimensional curve, where x controls y, another equation must be introduced along with the new variable. In the **composite or chain form**, the variable, x controls an intermediate variable, u, which in turn controls the dependent variable, y. This form appears as:

4) $y = f(u);$ $u = g(x)$

In composite form the above ellipse may appear as $y = 3\sqrt{u}$; $u = 1 - (x/5)^2$.
If the intermediate variable, u, is eliminated from these two equations, the explicit form is recovered. It is this chain form to which the chain rule of differential calculus applies.
If a device such as a gear train, lever, thermostat or amplifier accepts an input variable, x, and produces an output variable $u = g(x)$ and in turn this mechanism drives a similar mechanism $y = f(u)$ then the entire mechanism can be described by the composite form, $y = f\{u\}$; $u = g(x)$ which would have the explicit form $y = f\{g(x)\}$. In calculus courses, the computational techniques of related rates follow directly from the properties of composite forms.

Another three-variable form of a two dimensional curve introduces a variable called a parameter which controls both x and y. This form, called the **parametric form,** with parameter, t, appears as:

5)
$$y = f(t)$$
$$x = g(t)$$

The parametric form of the above ellipse is;
$$y = 3 \sin(t)$$
$$x = 5 \cos(t) \ .$$

Applying the trigonometric identity $\sin^2(t) + \cos^2(t) = 1$ will eliminate the parameter, t, from these two equations and recover the implicit form of the ellipse. As the parameter, t, varies between 0 and 2π, the point $P\{x(t), y(t)\}$ starts at the point 5,0 when $t = 0$, traverses the ellipse counterclockwise and returns to that point when $t = 2\pi$.

Curves such as the cycloids and the Lissajous figures have relatively simple parametric descriptions while their implicit forms appear horrendous. Computing the tangent lines for curves described in parametric form is relatively easy.

The Strategy of Solving Equations

One of the major concerns of algebra is finding values of variables that meet given specifications or conditions. The equations that describe the specifications are called conditional equations and the process of finding the values of the variables is called solving the equations. The values of the symbols that satisfy the equations should now be called unknowns, since they are fixed by the conditions and are no longer varying. As noted before, n equations are generally needed to find n unknowns. In the simplest case one equation is needed to solve for one unknown. The study of systematic methods of solving equations evolved over centuries.

Say we have one algebraic equation in one unknown; that unknown being allowed to appear in more than one place in the equation. We must change the form of the equation and the expressions in the equation so that the unknown appears only once and is isolated on one side of the equation. The rules for changing the forms of the conditions without changing the values of the unknowns are well known. If $a = b$ and $c = d$, then $a + c = b + d$, $ac = bd$, $a - c = b - d$, and if c and $d \neq 0$ then $a/c = b/d$. In addition, identities can be used to change the forms of expressions.

I have no fixed order for performing the form changing manipulations. I examine each equation in order to select a course of action, which will bring me closest to obtaining the solution. Acquiring the ability to follow fluently the mathematical calculations by either a professor or a text will require some practice on the part of the student but the strategy should always be clearly stated.

As an example, let us find the value for x that satisfies the condition:

$$\frac{12x - 36}{2x - 9} = 5$$

Which x should be isolated, the x in the numerator or the x in the denominator? The unknown, x, has only one value. Somehow a way must be found to combine both the x's into a single symbol. Multiply both sides of the equation by the left hand side denominator, $2x - 9$ to get

$$12x - 36 = 5(2x - 9) = 10x - 45$$

And then after adding 36 to both sides of the equation and subtracting 10x from both sides:

$$12x - 10x = 36 - 45$$

$$2x = -9$$

Now there is only one x. Divide both sides of the equation by 2 to get the solution,

$$x = -\frac{9}{2} = -4\frac{1}{2}.$$

Perform a check of the solution to ensure the calculation was executed correctly.

Let us consider the following example. Say we seek the coordinates of the intersections of the two curves with the equations: $y = x^2 + 3$ and $y = 4x$. Since at the intersections of the curves, the vertical values are equal, set the two y's equal to get the conditional equation: $x^2 + 3 = 4x$ which has the same solutions as the zeroes of the function,

$$f(x) = x^2 - 4x + 3 = 0.$$

Change this expanded form of our function f(x) to its factored form:

$$f(x) = (x - 1)(x - 3)$$

which then provides the horizontal coordinates of the desired intersections, $x = 1$ and $x = 3$. When $x = 1$, $y = 4$ and when $x = 3$, $y = 12$. These values when substituted into the original equations are seen to be the coordinates of the desired intersections. This strategy could be applied to finding the intersections of any two polynomial curves, providing the factoring could be performed.

Objectives

Functions have major importance in every quantitative area of study and the study of functions will significantly impact a student's understanding of these areas. In particular, a student who understands what a function is and who understands the kinds, forms, properties and the basic principles of working with functions will be better positioned to study other quantitative sciences. A number of applications of mathematical functions to the geometry of curves are listed in Table 4.

Summary

This main purpose of this paper was to provide the Algebra/Calculus structure chart, Figure 9, which depicts the natural algebraic structure which may not be evident in our conventional thick and heavy textbooks. This chart provides an organization to the disorganized topics usually found in pre-calculus and calculus textbooks. The chart focuses attention on the important concept of mathematical function by placing it at the top. The kinds, properties, and forms of functions are detailed in the accompanying lists, Tables 2, 3 and 5. The structure of the kinds of functions is seen to parallel the algebraic structure of the kinds of numbers in the real number system. Limitations are seen to be placed on the properties and possible shapes of the graphs of each of the kinds of functions. The list of properties, in one table, supports the visual aspects of the graphs of functions without algebraic clutter. Students are encouraged to research the algebraic tests that correspond to each of the properties. A brief discussion of the strategy of solving polynomial and rational equations is included. It is hoped that Figure 9 and the accompanying lists will aid students in organizing their analytical studies.

Algebra/Calculus Structure Diagram

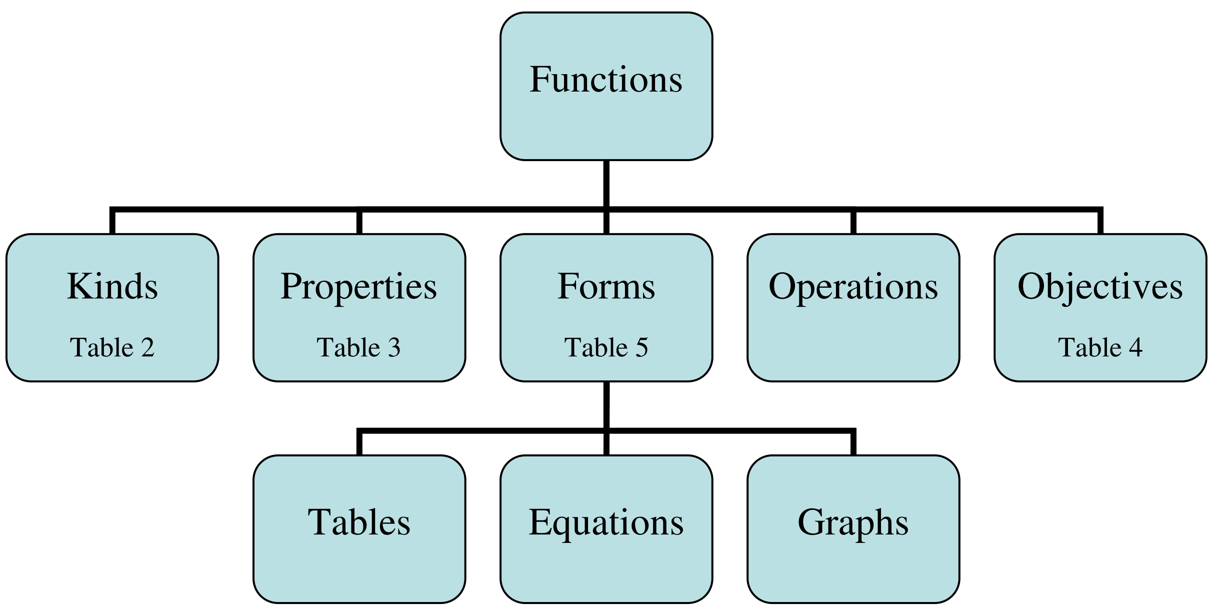

Figure 9

Kinds of Functions

polynomials
rational functions
algebraic functions
transcendental functions
piece-wise defined functions

Table 2

Properties of Functions and Curves

(Distinguish point properties
from region properties)

1. intercepts,
2. symmetries; even, odd, periodic, etc.
3. excluded intervals,
4. bounds,
5. asymptotes,
6. multi-valued,
7. continuity,
8. discontinuity
   - isolated discontinuities
     - finite jumps
     - infinite jumps, poles gaps
     - oscillatory discontinuities
9. differentiability,
10. slopes,
11. monotonicity,
12. extrema,
13. concavity,
14. points of inflection,
15. cusps,
16. radii of curvature,
17. arc length,
18. area
19. connected
20. closed,

Table 3

Symbolic Forms of Functions

explicit	y = f(x)
inverse	x = g(y)
implicit	F(x, y) = 0
composite (chain)	y = u(w); w = v(x)
parametric	$y = g(t)$ $x = f(t)$
polar	r = f(θ)
series	Taylor Fourier

Table 5

Objectives of Calculus

Evaluation:
* evaluating functions of a single variable
* graphing functions
* finding roots
* finding intersections
* evaluating inverse functions

Differentiation:
*finding extrema and points of inflection
* finding tangent lines
* describing direction
* describing rate of change
* computing incremental changes

Integration:
* finding the areas of closed regions with curved boundaries
* finding the arc lengths of curves
* finding surface areas and volumes of revolution

Table 4

Acknowledgement

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