Session 2247

The Photon Marathon – Explaining Chromatic Dispersion to Engineering Technology Students

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Introduction

Dispersion or pulse spreading is a basic topic in an undergraduate engineering technology course in fiber optic communication systems. Students need to understand what causes dispersion and to be able to calculate pulse spread and determine how it limits the length and data rate of a communications system. Chromatic dispersion, which results from the wavelength dependency of the velocity of light in an optical fiber, is the primary source of pulse spread in modern communications systems. The standard formula for calculating pulse spread due to chromatic dispersion can be derived using basic calculus and the derivation is comprehensible by most engineering technology students. However, although this formula is quite useful, it does not quantify all of the effects of chromatic dispersion on optical pulses. To get the total picture, one can always solve the pulse propagation equation, but this kind of rigorous analysis is more suited for engineering graduate students. A reasonable compromise between these two extremes is to model an optical fiber as a linear system and many texts do so using a Gaussian impulse response and pointing to references to justify the choice of Gaussian.

A typical open foot race, which often has 1000 or more participants, provides a model of the dispersion process. At the start the runners are tightly packed and it takes only a few minutes for all of the runners to cross the start line. By the finish, however, the runners are widely dispersed and the gap between the winner and the last runner can be an hour or more for a long race. Moreover, a plot of finishing time versus place for such a race resembles a Gaussian cumulative distribution function. This foot race analogy can be used to model chromatic dispersion in terms of the fiber impulse response. In the "photon marathon", the runners are photons with wavelength-dependent paces. The photon marathon has a perfect start – N photons are launched into the fiber simultaneously, corresponding to an optical power impulse. The "race results" define the impulse response of the fiber. The model and the associated mathematics are well within the capability of third and fourth year engineering technology students.

This paper is organized as follows:

- Overview of chromatic dispersion
- Customary approaches to modeling chromatic dispersion
    - First-order pulse spread formula

    - Pulse propagation equation
    - Linear system model
- The photon marathon model
- Summary and conclusions

Chromatic Dispersion in Optical Fiber

When an optical pulse is launched into an optical fiber, different components of the pulse arrive at the far end at different points in time. This phenomenon is known as dispersion and results in a pulse at the far end that has a longer pulse period and lower amplitude than the launched pulse. Dispersion is the result of one or more of the following:

- Modal dispersion – this occurs when the pulse propagates through the fiber in more than one transmission mode. Modal dispersion does not occur in single-mode fiber at wavelengths above the cut-off wavelength.
- Chromatic dispersion – this occurs because the group velocity, which is the pulse propagation velocity, is wavelength-dependent. Any real optical pulse includes a range of wavelengths and components at different wavelengths propagate at different velocities.
- Polarization-mode dispersion – this occurs because the pulse propagation velocity is polarization-dependent. Polarization-mode dispersion is usually significantly less than chromatic or modal dispersion.

This paper focuses on chromatic dispersion, which is the most common type of dispersion encountered in optical fiber telecommunication.

Figure 1 shows the wavelength dependence of the group delay (the inverse of the group velocity) for a typical standard single-mode fiber. Although the variation in group delay is small, the cumulative effect of this variation over 100 km of fiber can be significant. Note that the group delay has a minimum at a wavelength just above 1300 nm. This wavelength is called the zero-dispersion wavelength.

An optical pulse has a finite spectral width, i.e., has its power spread over a range of wavelengths, for one or more of the following reasons:

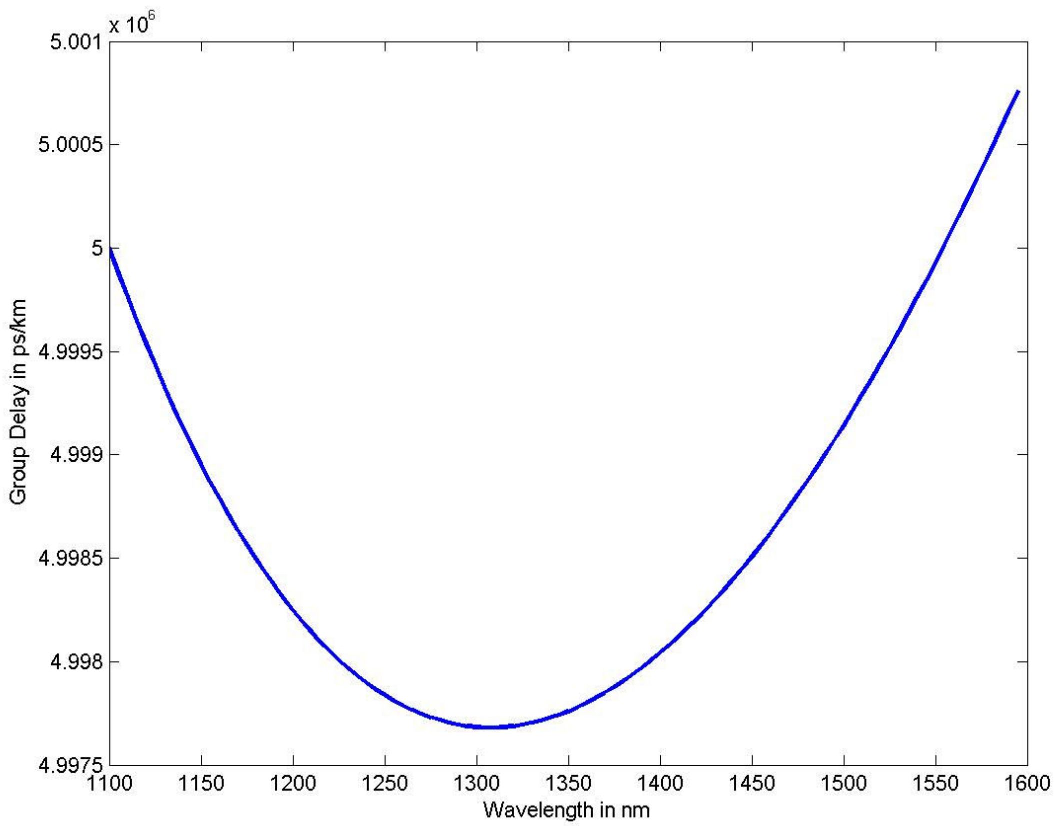
- Optical source spectral width – this is typically 50 – 100 nm or more for a light-emitting diode (LED) and 2 – 5 nm for a multi-mode laser diode. Single-mode laser diodes (e.g., distributed feedback (DFB) laser diodes) have spectral width much less than 1 nm.
- Chirp – this is a transient variation in the source wavelength that occurs when the source is directly-modulated (i.e., turned on and off rapidly). A directly-modulated single-mode laser has a spectral width on the order of 0.1 nm.
- Modulation – even if the source has a very narrow spectral width (e.g., DFB laser diode) and is indirectly modulated to reduce chirp significantly, the optical pulse will still have some spectral width because of the sidebands that are a consequence of modulation.

Thus an optical pulse has components at different wavelengths and these components require different amounts of time to propagate through a length of fiber. The pulse spreads, i.e., its period increases and its peak amplitude decreases, as it propagates. The spreading is small (theoretically zero) at the zero-dispersion wavelength and increases away from this wavelength (in either direction).

Pulse spreading eventually leads to inter-symbol interference (ISI), which happens when the pulses overlap significantly. Pulse spread also results in a power penalty due to the fact that not all of the pulse energy is received during a bit period. Both ISI and the power penalty contribute to an increase in the bit error rate (BER), which is a key measure of the performance of a telecommunication system.

Figure 1

Variation of Group Delay With Wavelength



Ideally, one would like to know exactly how dispersion affects the size and shape of an optical pulse. The goal of this paper, however, is to provide and reasonably justify a way to determine the effects of dispersion that can be explained to undergraduate engineering technology students. Most texts use one or more of the following approaches:

- First-order pulse spread formula
- Pulse propagation equation
- Linear system model

I will discuss all of these approaches briefly in the sections that follow and note some advantages and disadvantages. I will focus in particular on the trade-off between the level of detail provided by each approach and ability of engineering technology students to comprehend it. I will then introduce an analogy: a marathon footrace, which exhibits dispersion very dramatically. Finally, I will exploit this analogy to develop a straightforward linear system model for optical fiber. Although using a linear system to model optical fiber is not a new idea, the "photon marathon" approach appears to be unique.

Modeling Chromatic Dispersion in Optical Fiber

First-Order Pulse Spread Formula

The simplest way to characterize dispersion is in terms of the pulse spread ΔT, the increase in the pulse width due to dispersion. The pulse spread formula can be derived quite simply [1, 2].

Consider a fiber of length L (typically in km) and a wavelength-dependent group delay of $\tau_g(\lambda)$ (ps/km). The time T (ps) required for a pulse with wavelength λ to propagate through the fiber is

$$T = L\tau_g(\lambda)$$

If the spectral width of the pulse is $\Delta\lambda$, then the pulse spread is corresponding value of ΔT, which is given to first order by

$$\begin{aligned}\Delta T &= L\frac{d\tau_g(\lambda)}{d\lambda}\Delta\lambda \\ &= LD(\lambda)\Delta\lambda\end{aligned}$$

The parameter $D(\lambda)$, normally stated in ps/nm-km, is listed on the specification sheet for optical fibers used in telecommunication. The spectral width $\Delta\lambda$, normally stated in nm, is listed on the specification sheet for optical transmitters. Thus, given the fiber length and specifications for the fiber and the transmitter, one can use this formula to calculate the pulse spread.

This pulse spread formula is easy to derive and apply and is well within the grasp of engineering technology students. The formula is useful in practice since fiber optic systems are normally designed such that the pulse spread is no greater than some fraction (typically 0.25 – 0.5) of a bit period.

The disadvantage of the formula is that it provides only a one-dimensional view of the effect of dispersion on an optical pulse. How does dispersion affect pulse amplitude? Pulse shape? A more rigorous approach is needed to answer these questions.

Pulse Propagation Equation

The pulse propagation equation is based on the theory of propagation of electromagnetic waves through a guided medium, such as an optical fiber. The optical pulse is modeled as a modulated sinusoid propagating along the axis of the fiber. A lengthy derivation yields the following equation describing the propagation of an optical pulse through a fiber [2]:

$$\frac{\partial A}{\partial z}+\frac{j\beta_2}{2}\frac{\partial^2 A}{\partial t^2}-\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}=0$$

Where A(z,t) is the pulse envelop, β_n are terms in the Taylor expansion of the wave propagation constant and $j=\sqrt{-1}$. The beauty of this approach is that given the input pulse A(0,t), one can solve for A(z,t) and therefore have complete information about the size and shape of the pulse as it travels along the fiber. The beast is the mathematics involved. This approach is, in my view, inappropriate for undergraduate engineering technology students.

Optical Fiber Transfer Function

Linear system theory provides a compromise approach for describing the effect of dispersion on an optical pulse. According to the linear model, the output pulse $p_o(t)$ at the far end of the fiber is related to the input pulse $p_i(t)$ launched into the fiber by

$$p_o(t)=\int_{-\infty}^{\infty} h(t-u)p_i(u)du$$

or in the frequency domain

$$P_o(\omega)=H(\omega)P_i(\omega)$$

where h(t) and H(ω) are the impulse response and transfer function, respectively, of the optical fiber. According to [2], this model is approximately valid when the source spectrum is much larger than the signal spectrum. Moreover, again according to [2], the transfer function is approximately Gaussian if the operating wavelength is far away from the zero-dispersion wavelength of the fiber.

Like the pulse propagation equation approach, the linear system approach provides a means to determine the complete pulse shape, not just the pulse width. In addition, the linear system model is easy to apply, especially if both the input pulse and the transfer function are Gaussian – in this case the output pulse is also Gaussian. But the mathematical justification for this model, which is based on a special case of the pulse propagation model, is still a bit much for engineering technology students.

The Marathon Approach

Anyone who has run in (or even observed) a heavily-subscribed long-distance foot race has observed that the burst of runners leaving the starting area in a dense pack disperses significantly by the end of the race, especially in a full marathon. However, only those who also teach fiber optic telecommunication may have drawn an analogy between this phenomenon and chromatic dispersion in optical fiber! And these days marathon organizers often post results on the web, making it easy to study this analogy in more detail.

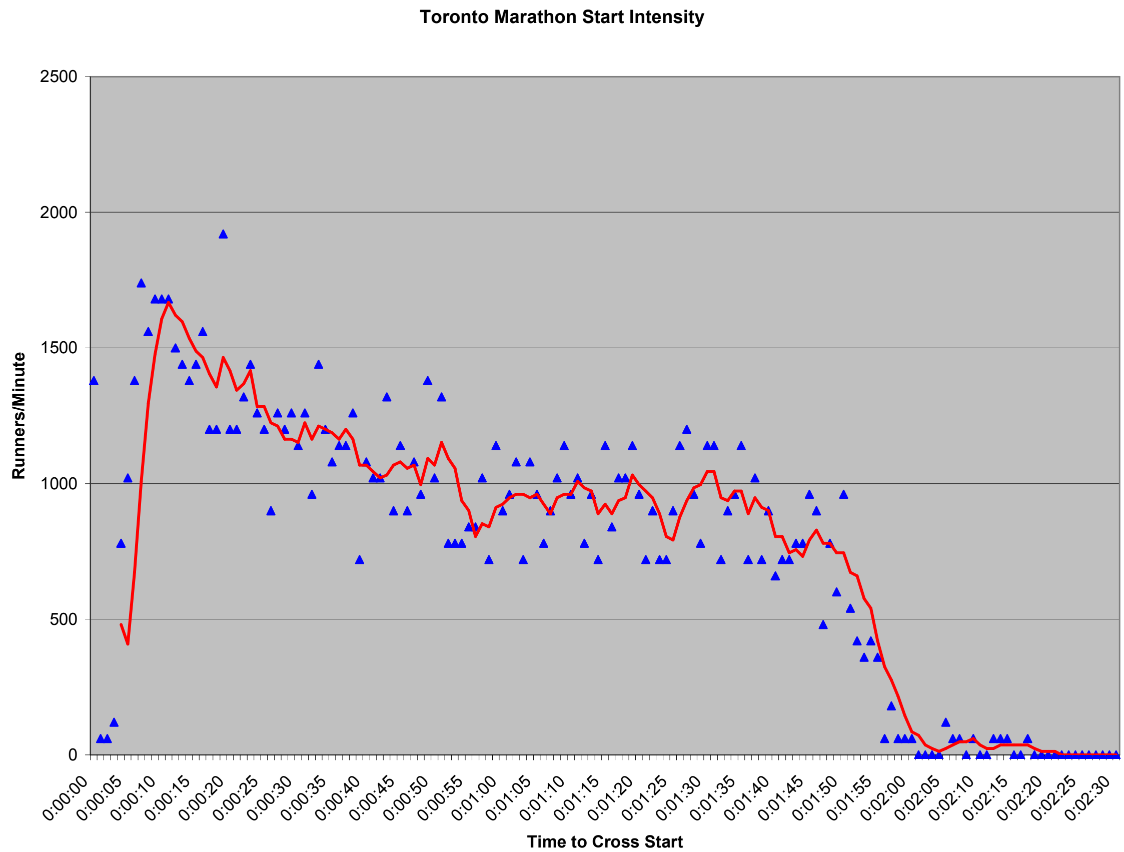
For example, the results of the 2004 Toronto Marathon are posted on the web [3] in a form that can be easily downloaded for analysis. The results data include, for each runner* who completed the race

- Place
- Official ("gun") time – finish time relative to start of race
- Elapsed ("chip") time – finish time relative to individual start time
- Other data (name, category, etc.)

The "chip" time is measured from the time each runner crosses the start line, as recorded using an electronic device tied to the runners' shoes. The difference between the "gun" time and the "chip" time is the time required to reach the start line.

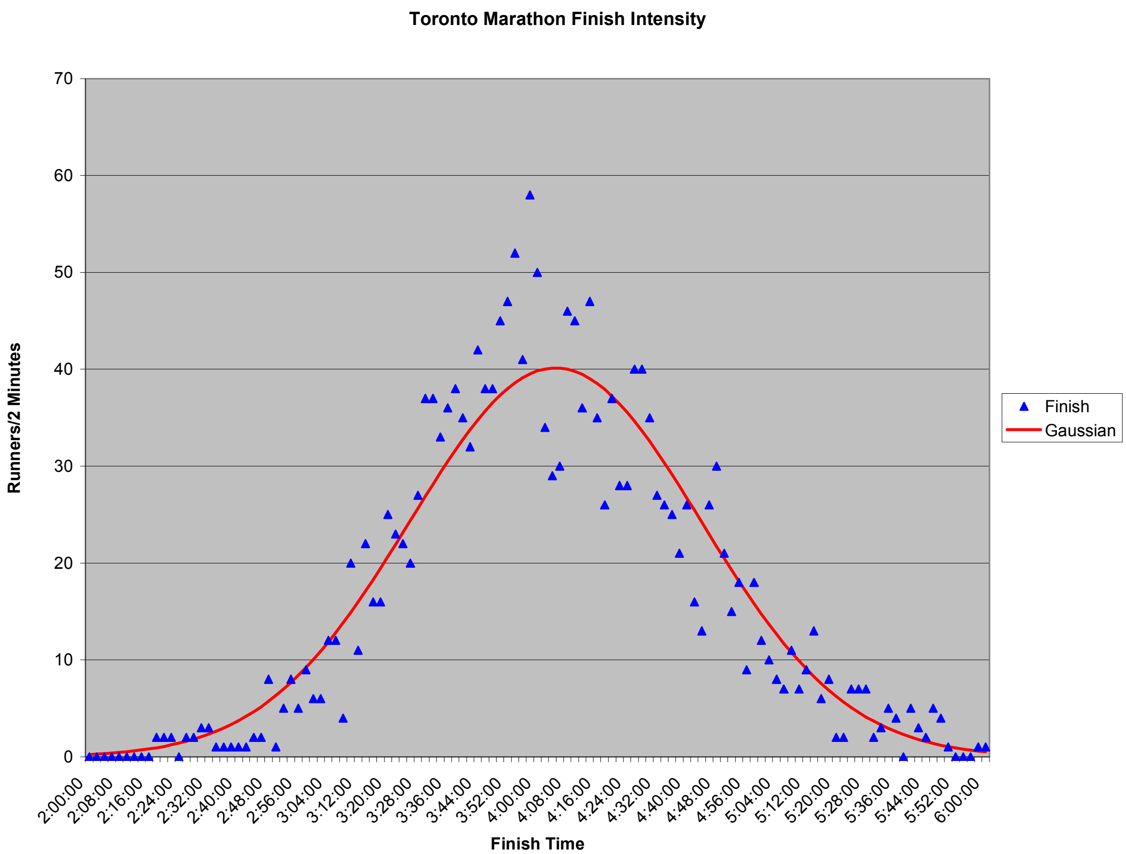
* Including the author!

Figure 2



Toronto Marathon Start Intensity
Runners/Minute
2500
2000
1500
1000
500
0
0:00:00
0:00:05
0:00:10
0:00:15
0:00:20
0:00:25
0:00:30
0:00:35
0:00:40
0:00:45
0:00:50
0:00:55
0:01:00
0:01:05
0:01:10
0:01:15
0:01:20
0:01:25
0:01:30
0:01:35
0:01:40
0:01:45
0:01:50
0:01:55
0:02:00
0:02:05
0:02:10
0:02:15
0:02:20
0:02:25
0:02:30
Time to Cross Start

Figure 3



Figures 2 and 3 show the start and finish time distributions for the marathon. The "start pulse" is roughly rectangular with a pulse width of about 2 minutes. The "finish pulse" is roughly Gaussian with a pulse width (based on standard deviation) of about 2 hours. Also, the peak intensity of the finish pulse (20 runners/minute) is significantly smaller than that of the start pulse (1000 runners/minute). Clearly there was significant dispersion in the Toronto Marathon!

Now consider a photon marathon. A large number of photons line up at one end of a fiber. This group of photons has a total energy E depending on the number of photons and their wavelengths. At a certain moment, the photons are launched simultaneously into the fiber and race toward the far end. In terms of optical energy entering the fiber, the simultaneous launch of the photons is a step function. In terms of optical power, it is an impulse. Thus the pulse observed at the finish line will be identical to the impulse response of the fiber.

Let e(λ) be the energy spectrum of the optical pulse so that the total energy in the pulse is given by

$$E = \int_{-\infty}^{\infty} e(\lambda) d\lambda$$

Normally, $e(\lambda)$ is a narrow function centered around the nominal operating wavelength λ_c. If the "pace" of a photon is equal to the group delay (defined earlier), then the propagation time t (i.e., the "finish" time) for a photon with wavelength λ is

$$t = L\tau_g(\lambda)$$

An increment of energy $\Delta E = p(t)\Delta t$ arriving at the far end of the fiber corresponds to the increment of energy $\Delta E = e(\lambda)\Delta\lambda$ launched at the input where t and λ are related by the above equation (p(t) is the output pulse in terms of power). The energy increment can be viewed as a pack of photons running at the same pace. Equating the two expressions for the energy increment and taking the limit yields

$$\begin{aligned} p(t) &= e(\lambda)\frac{d\lambda}{dt} \\ &= \frac{e(\lambda)}{LD(\lambda)} \end{aligned}$$

This equation is further simplified using a linear approximation for the propagation time, i.e.,

$$t \approx t_c + LD(\lambda_c)(\lambda - \lambda_c)$$

where $t_c = L\tau_g(\lambda_c)$. Applying the linear relationship between t and λ yields the following expression for the output pulse

$$p(t') = \frac{e\left(\dfrac{t'}{LD(\lambda_c)} + \lambda_c\right)}{LD(\lambda_c)} = \frac{e'\left(\dfrac{t'}{LD(\lambda_c)}\right)}{LD(\lambda_c)}$$

where $t' = t - t_c$ is the time shifted by the nominal propagation time and e'(x) is e(x) centered about the nominal wavelength . This result states that the output pulse has the same mathematical form as the energy spectrum of the input pulse. Recall that since the input pulse is an impulse, then p(t) (dropping the prime) is the fiber impulse response h(t).

Thus the photon marathon approach leads to a relatively straightforward formulation of the fiber impulse response. The fiber impulse response can be used to determine the impact of the fiber on any input pulse.

Summary and Conclusions

I developed a course in fiber optic telecommunication technology when I came to RIT in the fall of 2000 and have taught it at least once per year since then. When it came to chromatic dispersion, I initially covered only the first-order pulse spread model. I was troubled, however, by the limitations of this model. Realizing that most students who take this upper-level undergraduate course have taken a course in linear system theory, I added the linear system model to the course material. Unfortunately, I was not able to offer much justification for any particular impulse response function – I simply followed the custom of assuming a Gaussian function.

The photon marathon, which occurred to me while I was training for the Toronto Marathon, turns out to provide a reasonable way to characterize the impulse response of an optical fiber. What it may lack in mathematical rigor is compensated by entertainment value, judging by the reaction of my students. The first-order pulse spread formula is still the best starting point for a discussion of chromatic dispersion and it is a useful tool in practice. Addition of the photon marathon takes the students a step further in understanding the technology and provides them with a more powerful tool.

Bibliography

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