The Question of Units: Bothersome Details or Keys to Understanding?

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Introduction

The proper treatment of physical units is a pervasive problem in engineering education. The sporadic and uneven coverage of units throughout the typical engineering student's undergraduate experience can result in graduates who, in spite of having learned key engineering principles, can still stumble over units conversions and cancellations. When faced with a new problem, even the best students can lose sight of conceptual issues as they become overwhelmed by concern about how units will be dealt with in the determination of a reasonable solution.

Rowland [1] described how units can be used to deduce how given quantities should be combined to give a particular result (for instance, how velocity and area can be used to determine volumetric flow rate). He also discussed how the units cancellation process can provide a check of the validity of equations, both in choice of governing equations and whether any algebraic manipulations were done properly. His study suggested that students accept governing equations without much thought about the need for dimensional homogeneity, supporting the finding by cognitive scientists that "humans tend to discriminate only as finely as necessary." He discussed the inclination of students to interpret constants in relationships as being dimensionless.

Reif [2] pointed out the difficulties students have with converting general relationships into relations for specific instances. He cited the need to provide students with plenty of opportunities to implement "concept-interpretation procedures" by trial and error, allowing them to "compile a repertoire of knowledge about special cases and common errors." Successful applications of general concepts to particular instances necessarily involves the correct managing of units.

Dealing with units in the international system (*le Systeme International d'Unites*, commonly abbreviated as SI) is relatively straightforward for students, but non-SI units are another matter. In spite of occasional optimism in the literature – such as the following statement from Mark's Handbook [3]: "It is expected that in time all units in the United States will be in SI form" – the use of non-SI units in the United States has persisted, particularly in industries such as HVAC and construction. Many engineering textbooks have acquiesced to this fact by presenting examples and homework problems expressed in both SI and non-SI units. Carr [4] suggested that the American resistance to SI units is due to "personnel retraining costs, fear of expensive mishaps during transitions, the large preponderance of legacy systems and equipment utilizing customary units on their gages and other instruments, and the intransigence of the American people." He claimed that we do our students a disservice by not training them in United States Customary System (USCS) units.

Adding to confusion about units is the fact that students are presented in textbooks with an array of different "non-SI" systems, including British Gravitational (BG) [5], USCS [6], the "inch-pound-second" or "foot-pound-second" systems [7], and the English Engineering system [8]. The ASTM Quick Guide [9] does not mince words when it states that "Non-SI units in the US are called Inch-Pound units (I-P units)—*not* conventional units, *not* U.S. customary units, *not*

English units, and *not* Imperial units." This may be true for ASTM, but clearly is not a view shared by textbook authors!

Comings [10] identified three dimensional systems: absolute, gravitational, and the "engineering" or $FML\theta$ system. All three systems use length (L) and time (θ) as two primary quantities. For the absolute system, mass (M) is the third primary dimension, force (F) is the primary dimension for the gravitational system, and both mass and force are considered primary for the engineering system. Defining four primary dimensions requires the use of the constant g_c in Newton's Second Law:

$$F = \frac{1}{g_c} ma$$

where the force unit is pound-force (lbf) and the mass unit is pound-mass (lbm). With the definition of a pound-force as the force necessary to accelerate a pound-mass at 32.174 ft/s^2, g_c can be expressed as 32.174 lbm ft/lbf s^2.

The remainder of the paper describes the types and sources of units confusion students can have, explores the ways in which units calculations are abused, analyzes the ways in which student attitudes about units intersect with instructor and industry attitudes, discusses strategies for motivating students to appreciate the importance of units, and makes the case for how the adoption of a methodical problem solving and units cancellation approach can remove confusion and establish a solid groundwork for understanding fundamental engineering physics.

Units in mechanical engineering disciplines

Mechanical engineering is typically broken down into three areas: mechanical design, manufacturing, and thermal-fluids. Usually one of the first "hard core" engineering courses is statics. Given its primary focus on calculating forces on particles and rigid bodies, there are not too many occasions for units complications. Free body diagrams are drawn, equations of equilibrium are written, and givens are substituted to give equations for one or two unknowns. Students can get by without rigorous attention to unit conversion factors because the governing equations consist of terms with a single dimension (force) or the product of two dimensions (force and length). If students take care to use a consistent length and force unit (i.e., not mixing feet and inches or kips and pounds), everything usually works out fine. A benefit of this approach is that students can focus on physical principles without the distracting complications of units conversions. Unfortunately, this lack of emphasis on units conversions can generate an impression among sophomores that units are not that important, or just something tacked on to the end of a problem solution to wrap things up.

In mechanics of materials, the consideration of units becomes more crucial because quantities such as stress, moment of inertia, and power begin to appear in relationships. For instance, consider the design problem to determine the required shaft diameter for a shaft transmitting 5 hp with a rotational speed, ω, of 1000 rpm and an allowable shear stress, τ, of 10 ksi. The equation for shear stress is

$$\tau = \frac{\mathrm{T}c}{J}$$

where c is radius and the polar moment of inertia, J, is $\frac{\pi}{2}c^4$. The torque is determined from the ratio of power to angular speed, P/ω. An approach followed by some textbook authors is exemplified by presenting the solution in two steps as

$$\mathrm{T} = \frac{5(550)}{1000\left(\frac{2\pi}{60}\right)} = 26.26 \text{ ft}\cdot\text{lb}$$

and

$$10(10^3) = \frac{c\ (26.26)(12)}{\frac{\pi}{2}\ c^4}$$

to get c = 0.272 inches, giving a diameter of 0.543 inches, or 5/8 inch rounded up to the nearest 1/8 in. Clearly, as a textbook example, this approach leaves much to be desired as the student tries to understand "where all the numbers come from." A lack of explicit units treatment can result in much hand wringing as the student becomes overwhelmed in imagining how she is ever to solve a new problem on her own. The problem can become compounded in subsequent design classes as students face shaft problems involving multiple diameters, stress concentrations, and combined bending and torsional loadings.

A course in manufacturing can give mixed messages to students about units. Equations given in textbooks include a mixture of relationships determined from first principles (such as the relationships for specific energy in metal cutting and grinding) and empirical relationships (such as that presented below). The equations derived from fundamental principles are dimensionally homogeneous, so units problems can be avoided by using unit conversion factors. Consider, however, the equation for material removal rate given in Groover [11] for electric discharge machining:

$$R_{MR} = \frac{KI}{T_m^{1.23}}$$

where I is electric current and T_m is the melting temperature. K is a constant given as 664 in SI units and 5.08 in U.S. customary units. Clearly, the equation is not dimensionally homogeneous if no units are assigned to K. The presentation of such an equation must *necessarily* be accompanied by a definition of the terms along with their units: R_{MR} is the metal removal rate in mm^3/s or in^3/min, I is the current in amps, and T_m is the melting temperature of the work metal in °C or °F.

Perhaps the most rigorous treatment of units is in the thermal sciences. Most thermodynamics, fluid mechanics, and heat transfer texts use unit conversion factors to demonstrate a rigorous cancellation process in example problems. Students are left with no doubts as to where numbers come from. In spite of the rigor with which units are treated, there is no common non-SI system used in thermal science texts. Fluid mechanics texts by Gerhardt [5] and Pritchard [12] use the BG system, with the derived mass unit being the slug. Thermodynamics textbook authors Cengel [13] and Moran and Shapiro [8] prefer the English system with pound-force and pound-mass. Both these texts advocate the unit conversion factor approach in which one pound-force is defined as the force necessary to accelerate one pound-mass at 32.1740 ft/s^2, leading to the unit conversion factor lbf s^2/32.1740 lbm ft = 1. This approach, helpfully, eliminates the need for the "archaic" [13] constant g_c in Newton's 2nd Law.

Student confusion regarding units

Student confusion about units takes many forms. One form involves the mistaken idea that one physical quantity can be "converted" into another physical quantity with different dimensions. For instance, a common question from students in fluid mechanics is "how can I convert m^3/s to kg/s?" This, of course, is not simply a units problem, but a confusion between physical quantities that are fundamentally different. Once the connection between mass flow rate and volumetric flow rate is established through $\dot{m} = \rho\dot{\forall}$, one sees that the mass flow rate is obtained by multiplying the volumetric flow by density, not that volumetric flow rate "is converted" into mass flow rate by multiplying by density. A similar conversion dilemma occurs when students need to convert "pressure" units of inches or millimeters of mercury to pounds per square inch or other genuine pressure units. In this case, the relationship between pressure difference and height comes from the fluid statics relation $\Delta p = \gamma h$. Height is not "converted" into pressure, but is multiplied by specific weight. Unfortunately, the "conversion" from inches Hg to standard pressure units appears in many conversion tables, serving to aggravate student misunderstanding.

Another confusion arises from abbreviations, particularly in the English system, which is rife with terms like gpm, kips, ksi, and cfm. Not only is there inconsistency in the abbreviations (why isn't "cfm" expressed as "cfpm?"), but the abbreviations must be replaced with explicit expressions before unit conversion factors can be formed ("psi" must be expressed as "lbf/in^2"). Some authors attempt to accommodate these abbreviations by including them in property expressions, such as listing the ideal gas constant for a specific gas in units of psia · ft^3/lbm · R instead of the more conventional Btu/lbm · R, but this is awkward and can increase confusion as students lose connection with the physics.

In thermal sciences the relationship between mechanical and thermal energy terms can cause trouble, as in the first law of thermodynamics where units of ft^2/s^2 must be converted to Btu/lbm before adding specific kinetic energy to specific enthalpy. By the time they are sophomores, most students can recite Newton's 2nd Law as $F = ma$, but are tripped up when confronted by a ***new*** equation $F = ma/g_c$ if English units are being used. And, in the thermal sciences, the sheer number of conversions that must take place in applications of the first law or the energy equation for pipe flow can be overwhelming for students.

Sources of student confusion regarding units are manifold. The variety of emphasis placed on units conversions and cancellations across the curriculum can lead students to believe that units difficulties are subject-specific, leading to a "units anxiety" that occurs fresh with every class. The emphasis can range from a consistent methodical treatment of units in class examples to a total absence of numerical computations. (The author took an undergraduate heat transfer class in which students needed to rely on classroom lectures because there was no assigned text. The instructor presented all material by deriving governing relationships in a purely symbolic fashion. Not a single example, homework problem, or exam problem contained a numerical calculation, leaving the students no opportunity for gaining experience with units common to the subject.)

Students are misled by some derivations from first principles that are accompanied by reference to specific units, giving the impression that only particular units are relevant to the equations being developed. For example, the specific energy for grinding in a manufacturing textbook is expressed as

$$U = \frac{F_c v}{v_w w d}$$

where U is specified as the specific energy in J/mm^3 or in lb/in^3, F_c is cutting force in N or lb, v is wheel speed in m/min or in/min, v_w is work speed in mm/min or in/min, w = width of cut in mm or in, and d is depth of cut in mm or in. The textbook presentation of definitions of parameters along with units may imply to the student that *only* those particular units should be used (as is the case in some empirical equations as described in the previous section). The equation for specific energy is derived from the ratio of power to rate of material removal. Specific energy dimensions are energy/volume, and *any* consistent units for the quantities on the right hand side are appropriate as long as they cancel to give energy/volume (or force/area). For instance, the wheel speed could be given in ft/min, resulting in units for U of ft lb/in^3. Or, if the desired units for U are in lb/in^3 and the wheel speed is given in ft/s, unit conversion factors of 60 s/min and 12 in/ft would be used in the actual numerical calculation.

In those institutions with co-op programs, students may return to academic terms having been exposed to shorthand equations, only to conclude that the “real world” has no patience with rigorous units treatments. One such example is the HVAC industry. A common expression for estimating the heat transfer rate in a system is q = cfm * 1.085 * ΔT, where q is in units of Btu/h, cfm is ft^3/min, and ΔT is in units of °F (in fact, 1.085 is often replaced with unity for “quick and dirty” results, leaving q = cfm * ΔT). The constant, 1.085, necessarily has the units appropriate for the equation to be dimensionally homogeneous. A student using this equation on a daily basis – or a seasoned HVAC engineer, for that matter – may be hard pressed to demonstrate that the equation actually is derived the first law of thermodynamics (heat transfer rate = mass flow rate times enthalpy difference) using standard values for air density.

In some classes there is rigorous treatment of units in the text or in examples during lecture, but posted worked-out homework solutions give numerical calculations without units, favoring expediency over rigor. This can give students the idea that the rigorous use of unit conversion factors is unnecessary once a certain familiarity with the governing equations is reached.

Ways in which students abuse units

The term “abuse of units” refers to lack of respect for a rigorous treatment of units as part of a problem solution. For instance, students may use units calculations as a “crutch.” Faced with a daunting problem, students who do not know where to start may begin by converting square feet to square inches simply to give themselves the idea that they are making some kind of progress. They also may recognize the proper units for the answer, and take a short cut by manipulating the givens such that the units work out correctly, in so doing ignoring the governing physical relationships.

Sometimes students ignore units conversions completely, assuming that if they use a certain combination everything will work out. No units appear in the calculations until the final answer is obtained. This "hope for the best" approach betrays a "plug and chug" attitude that is a poor substitute for a solid understanding of the underlying physics.

Sometimes units in industrial settings are abbreviated, such as in the HVAC industry, where the term "Btus" is used when what is meant is "Btus per hour." This particular abuse serves to exacerbate student confusion between energy and power. Working engineers like to quickly (and rightly so) estimate feasibility by doing calculations in their heads, and if certain numerical conversion factors are used repeatedly for years, the actual reasoning behind the unit conversion factors can fade. This may serve a seasoned engineer in a traditional industry well, but problems arise when engineers fresh out of college arrive on the scene and struggle to sync their education with what they're hearing on the job, or when new opportunities with international partners necessitate a switch to SI from a non-SI system.

Engineering students, as engineers in the working world, commonly learn to use a spreadsheet tool such as Excel to perform repetitive calculations. The disadvantage of using a spreadsheet is that there is no straightforward accommodation for units consideration. The only way to resolutely handle units conversions is to ensure through a sample hand calculation that units cancel correctly, then make sure to apply the proper conversion factors in cell formulas. Short-circuiting the rigorous units cancellation process in this way can lead students into a debugging morass.

Attitudes towards units

Students can be cavalier about units. Common complaints upon receiving a graded exam include "He took off points because I forgot to divide by twelve!" or "I got the right answer; it was only the sign that was wrong!" But where does this dismissive attitude come from? It may originate in having been taught by instructors that themselves treated units rather loosely in class, or, for those students in co-op programs, by supervisors that never accompanied figures with units, probably because the scope of their operations dealt with one or two parameters for which the units were presumed. An experienced engineer can rely on her experience to catch any gross errors in calculations that result from units conversion problems, but a fresh engineer ignores units at her own risk.

Lessons from dimensional analysis

The principles of dimensional analysis are typically taught in fluid mechanics, and later reinforced in heat transfer. The idea of nondimensionalizing a problem before performing experiments or doing model tests is applicable to many fields, however, a fact lost on many students and practicing engineers who don't appreciate its universality. A key concept presented in any discussion of dimensional analysis is that ***dimensions*** must be the same for each term in any relationship derived from fundamental physical principles regardless of the particular units used. For instance, for the Bernoulli equation given in the following form:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

each term has dimensions of length, and ***any*** units can be used for each of the parameters as long as unit conversion factors are applied so that each term has the same length unit.

A study of dimensional analysis also highlights the difference between dimensions (*MLT* or *FLT*) and units. The formation of dimensionless groupings for particular problems and in laboratory experiences can give students valuable insight into unit conversions. And, as students learn the distinction between dimensions and units, their maturity level in dealing with units issues increases.

Motivating students to pay attention

On September 23, 1999 the $125 million Mars Climate Orbiter was presumably destroyed by intense heat resulting from traveling through the Martian atmosphere at an elevation much lower than anticipated [14]. Orbiting maneuvers were executed based on force calculations in a computer software subroutine that assumed SI units when in fact the input data was calculated in a subroutine whose output data was in English units. Force input data was low by a factor of 4.45, which is the numerical constant in the unit conversion factor 4.45 N/lb [15].   It was determined the software problem was due to lack of communication between software development teams and ignorance of organizational requirements for units standardization.

Another example of a near disaster stemming from units confusion is that of an Air Canada airplane in the year 1983 that ran out fuel in midflight (a disaster was averted as the pilot glided the plane 60 miles, landing the aircraft at an abnormally high speed of 180 knots). The cockpit gauges were malfunctioning, so fuel volume was determined using a "drip stick." Fuel mass in pounds was determined from the measured volume, and the numerical result was reported to the pilot, who believed that the report was in kilograms, meaning that the pilot believed he had over twice the amount of fuel that was actually present [16].

These two examples demonstrate to students that simple units confusion can lead to incredibly costly mistakes, both economically and in terms of human life. Most units problems do not lead to such catastrophic consequences, but, at the very least, a units conversion issue in the workplace can cause a significant amount of embarrassment.

As explained in the following section, a rigorous treatment of units can help a student determine if the chosen governing equation is correct. If substitution of givens along with their units and a conversion process results in the dimensions not being the same on either side of the equation, then this can indicate that the wrong equation was chosen or mathematical manipulations were erroneous. Thus, careful attention to units can provide a valuable check on a solution.

Furthermore, once a numerical answer with proper units is obtained, students can be confident that their answer is ready for the "reality check."

A methodical approach for handling units

In order to use units to check equations, the problem solution must first be performed symbolically. This often involves choosing the correct governing equation in its abstract, general form and then reducing it to a simplified form suitable for the particular problem at hand. A precursor to this simplification is often a diagram of the system of interest, showing state points in the case of a thermodynamics or fluids problem, or a free body diagram in the case of a mechanics problem. Indeed, this action of drawing a figure is, in and of itself, an exercise of the student's capacity for abstraction, because in order to complete the diagram and perform subsequent equation manipulations the student must replace any numerical givens with symbols of her own choosing.

After the equation has been solved and manipulated to isolate the unknown on the left hand side in terms of the givens on the right hand side (provided this is possible), the numbers ***along with their given units*** can be substituted. Finally, unit conversion factors can be applied and units can be cancelled to obtain the final solution and perform the reality check. It should be noted that units cancellations are the very last step. Following this approach forces mental energy into mathematical operations as opposed to intermediate units calculations.

The following example demonstrates advantages of this approach. Figure 1 shows the given parameters for the problem, which is formulated in BG units. The task is to find the pressure at the exit of the fitting.

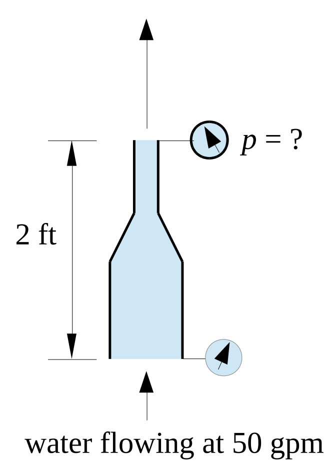

Figure 1. Bernoulli equation problem for a fluid fitting. Students are tasked with determining the exit pressure. The inlet diameter is 1.5 inches and the exit diameter is 1 inch.

When faced with such a problem, many students will write down the governing equation as follows:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

Once states 1 and 2 are identified on the diagram, a student may then attempt to solve for p_2 by prematurely substituting numbers into the equation as shown. This rush to substitute numbers is likely a result of the desire to start punching calculator buttons, which gives puzzled students the feeling that they're getting somewhere. It also provides a short cut to a numerical solution which can then be quickly checked against the solution manual. For this problem, the student rapidly becomes confused as she tries to reconcile inches with feet, gallons with cubic inches, and slugs with lbf. At this point, units anxiety sets in. The attentive student may eventually recognize that all terms must have the same units, and decide to convert everything to slugs, ft, s, and lbf before substituting. The speed at points 1 and 2 must be calculated first, so the student does a side calculation to determine that 50 gallons per minute is equivalent to 6.684 ft^3/min. The relationship between volumetric flowrate and speed, $V = Q/A$, is then applied. The student calculates areas in ft^2, and then after calculating V_1 and V_2 in ft/min divides the results by 60 to get V_1 = 9.08 ft/s and V_2 = 20.4 ft/s. After another intermediate calculation to determine pressure at point 1 in lbf/ft^2, the student substitutes numbers as follows:

$$2160 + \frac{1}{2}(1.94)(9.082)^2 = p_2 + \frac{1}{2}(1.94)(20.4)^2 + (62.4)(2)$$

Typically, the next line the student writes down is the answer, p_2 = 1711 lbf/ft^2 = 11.88 psi, implying that the student performed the necessary algebra in his head.

This approach has some serious drawbacks. First, there is no explicit units cancellation process that could help the student find errors, with many of the conversions taking place mentally and not on paper. Second, since there was no attempt to algebraically collect terms before performing calculations, there are redundant units conversions that must be performed. Finally, without a notation of units in the equation, there is more of a chance for mistakes as the student loses track of which units are being used in each term. The difficulty with such an approach is compounded when the pressure at state 2 is given and students are asked to find the volumetric flowrate, because now the velocity (or volumetric flowrate) appears as a squared term on each side of the equation, making mental algebra much more difficult.

In the more methodical approach all the algebra is done symbolically first, and the student avoids dealing with units until the end of the procedure. After algebraic manipulation, applying $V = Q/A$, and setting $z_1 = 0$, the equation becomes

$$p_2 = p_1 + \frac{1}{2}\rho Q^2 \left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) - \gamma z_2$$

The next step is to substitute numbers along with their units as given in the problem statement, without yet considering units conversions. The ***very last step*** is to identify the required units for the solution (lbf/in^2), and then insert appropriate units conversion factors. Only at this point are the calculators picked up. Following this procedure results in:

$$p_2 = 15\ \frac{\text{lbf}}{\text{in}^2} + \frac{1}{2}\left(1.94\ \frac{\text{slug}}{\text{ft}^3}\right)\left(50\ \frac{\text{gal}}{\text{min}}\right)^2 \left(\frac{1}{\{1.7671\ \text{in}^2\}^2} - \frac{1}{\{0.7854\ \text{in}^2\}^2}\right) \times \left[\frac{\text{lbf s}^2}{\text{slug ft}}\right] \times \left[\frac{\text{min}}{60\ \text{s}}\right]^2 \times \left[\frac{0.13368\ \text{ft}^3}{\text{gal}}\right]^2 \times \left[\frac{12\ \text{in}}{\text{ft}}\right]^2 - 62.4\ \frac{\text{lbf}}{\text{ft}^3}(2\ \text{ft}) \times \left[\frac{\text{ft}}{12\ \text{in}}\right]^2 = 11.88\ \frac{\text{lbf}}{\text{in}^2}$$

The benefits of this approach are manifold. All units cancellations are handled at the end through the straightforward application of unit conversion factors (shown above in square brackets, each of which has a value of unity and can thus be raised to any power). A successful cancellation of units into the desired result units (in this case, lbf/in^2) suggests that the correct equation was used and the algebra was done correctly. The equation when solved symbolically for p_2 makes it easy to see effects of parameter variation on the right hand side, giving a clearer understanding of the physics. Also, the result is easy to implement in a spreadsheet program, with the numerical conversions obtained simply by multiplying the unit conversion factors together for each term.

This approach, hereafter referred to as the "A1" (for "algebra first") approach, arguably involves more thought and careful use of calculator buttons, but gives students confidence as they see their correct solution materializing through the implementation of a logical process.

Assessment of student performance and opinion

An attempt to gauge the effectiveness of the A1 procedure included the assessment of student performance on three student exam or quiz problems. Student work for the first, a 15-minute quiz problem involving the use of the Bernoulli equation, and the second, an exam problem requiring the use of the momentum integral equation, was separated into two categories: those who performed the algebra first to symbolically isolate the needed value in terms of givens, and those who did not. The Bernoulli quiz was administered relatively early in the term, and the exam problem was given near the middle of the term. For the first, 18 of the 37 students, or 49 percent, performed the necessary algebraic manipulations before substituting in numbers. Of those who performed algebra first, 56 percent addressed units correctly, and the average score for this group was 8.6/10. For those who did not isolate the required variable first (51 percent), 53 percent addressed units correctly, and the average score for the group was 8.2/10. These results show that students who followed the process performed marginally better, in both their treatment of units and in the overall problem solution, than the students who did not. It should be noted that although the quiz score was adversely affected by improper units treatment, other conceptual misunderstandings could also result in points being taken off.

For the second assessment – the momentum equation problem – 66 percent of the 41 students taking the exam performed the algebra to isolate the required variable first. Of these, 63 percent addressed units correctly, and the average score was 7.6/10. For the 34 percent who did not do the algebraic work first, 43 percent addressed units correctly, and the average score was 6.2/10. These results suggest more strongly that there is some benefit to following the solution process suggested herein.

The third assessment involved the administration of two quizzes covering the same pipe flow problem but differing in the way the question was posed. Group A consisting of 20 students was simply asked to solve the pipe flow problem without any guidance as to how the solution should proceed. For Group B consisting of 18 students, the question was posed in two steps: the first asked the students to solve algebraically for the unknown, and second asked the students to substitute in particular values along with their units to determine the solution. Forty percent of Group A had no units issues, while 44 percent of Group B had no units issues. There was virtually no difference in the average quiz score, with Group A averaging 7.7/10 and Group B averaging 7.6/10. These results show a slight improvement in units treatment for those "forced" to follow the procedure, but no improvement in overall quiz performance. It is notable, however, that for those given no direction on their solution process, 45 percent chose to follow the A1 procedure, with an average score of 8.2/10. If Groups A and B are combined, then the average score for those that followed the A1 procedure was 7.8/10.

The above results indicate that following the process of doing mathematical manipulations before substituting in numbers may have at least marginal success in improving the treatment of units. Near the end of the term students filled out a survey in order to gauge their opinion of the issues discussed in this paper. Each of the survey items consisted of a statement soliciting a response in the form of a Likert Scale. The questions and results are shown in Table 1. Figure 2 summarizes the percentage of respondents who either "strongly agreed" or "agreed" with the survey statements.

Table 1. Average student response to survey statements, with a value of 5 corresponding to "Strongly Agree,", 4 to "Agree,", 3 to "Neutral,", 2 to "Disagree," and 1 to "Strongly Disagree."

Statement	average	standard deviation	median
1) In my college engineering experience, all my instructors have strongly emphasized the importance of proper units cancellations.	4.11	0.86	4.0
2) I believe that the treatment of units varies greatly among the engineering courses I have taken.	3.11	1.20	3.0
3) In my co-op experience, it was expected that I pay careful attention to units in engineering calculations. (If your co-op involved no engineering calculations, circle "NA.")	3.86	0.94	4.0
4) Units are a major source of confusion in engineering calculations for me.	2.66	1.12	3.0
5) I find non-SI units confusing.	3.18	1.35	3.5
6) I find SI units confusing.	1.84	0.97	2.0
7) I believe that careful attention to units is less important in the "real world" than it is in academia.	1.97	0.82	2.0
8) I believe that units should be dealt with in a uniform manner throughout the engineering curriculum.	4.05	0.91	4.0
9) In engineering calculations, I consistently first perform the necessary mathematical manipulations to isolate the unknown variable in terms of givens before substituting in numbers.	3.53	1.18	4.0
10) In engineering calculations, I find that first performing the necessary mathematical manipulations to isolate the unknown variable in terms of givens helps with units cancellations.	3.79	0.78	4.0
11) In engineering calculations, I find that first performing the necessary mathematical manipulations to isolate the unknown variable in terms of givens helps me to better understand the physics behind the equations.	3.45	0.95	4.0

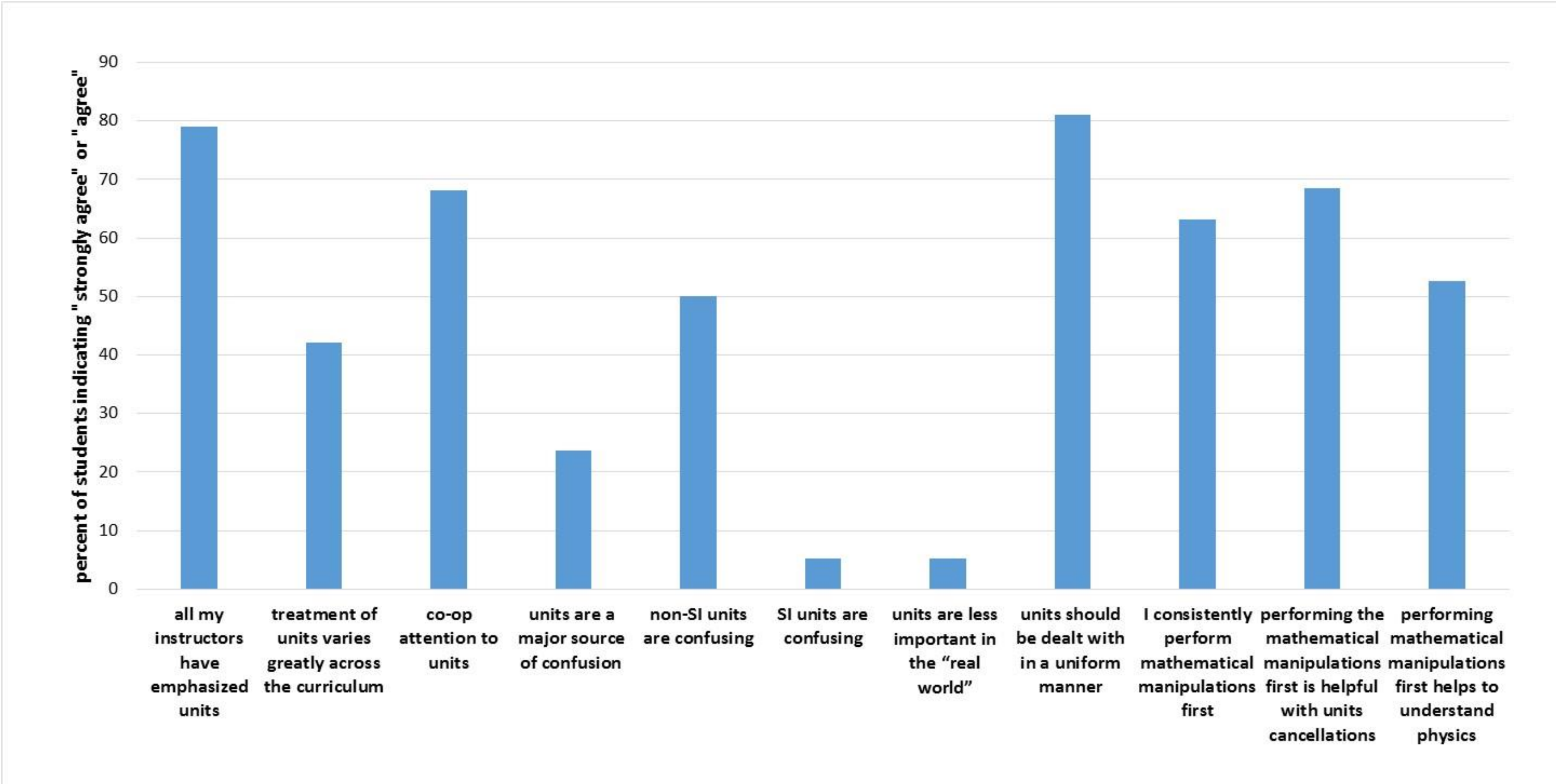

Figure 2. Percent of respondents agreeing with questionnaire statements

A few observations from the survey results are in order. The majority of students agree that all their instructors emphasize units in one way or another, and also agree that units should be dealt with in a consistent manner. The way in which units are treated from instructor to instructor was not explored in this survey, and would be a good direction for future research. Also, co-op employers in a majority of cases stressed the importance of units, although, again, the nature of this emphasis was not explored. (Detroit Mercy has a mandatory co-op program. It is interesting to note that 40 percent of the co-op experiences involved no engineering calculations.) More students found non-SI units systems more confusing than the SI system, as expected. A majority of students say they follow the process of algebraic manipulation before number substitution and find it helpful with cancellations and understanding the physics. Students overwhelmingly support the notion that units are just as important in the "real world" as they are in academia. It should be noted that this survey was administered at the end of a fluid mechanics course in which the solution procedure promoted here was encouraged, so student attitudes may have changed over the course of the semester.

Conclusions

Struggles with units are a persistent problem for engineering students throughout their years of study. Given the varieties of attitudes, coverage, and units systems that students have to deal with, it is no surprise that this is the case. A methodical approach to units adhered to throughout the curriculum would go a long way to building confidence in students, helping them to understand underlying physical principles, and preventing the kinds of mistakes that can have costly consequences in the workplace. Implementing a common thread like this across the curriculum is not a trivial task, but a discussion among departmental faculty regarding how their students address the problem of units is a good place to start.

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