The Relationship Between Spatial Skills and Solving Problems in Chemical Engineering

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Abstract

Through numerous studies, spatial skills have been shown to be important to overall success in engineering; however, most studies have focused on overall rather than specific areas of success. In this study, we examined the relationship between scores on a test of spatial visualization and a specific area of success—problem-solving in chemical engineering. Our quest included determining if a relationship exists between spatial ability and solving problems in chemical engineering; and if there is, what is the nature of such a relationship? In addition, we also wanted to know, how that relationship relates, to previous work on overall success in chemical engineering with respect to spatial skills. We tested students in a junior level chemical engineering thermodynamics course using a test of spatial visualization, the Mental Cutting Test (MCT). The test has twenty-five items and students were given 20 minutes to solve the test. Students were then presented with 13 problems from a sophomore-level chemical engineering course and were given 70 minutes to solve these over two class periods. All students had the same amount of time for solving problems with the assumption that those with better problem-solving skills would likely correctly solve a larger number of problems compared to the weaker students. Results indicate a strong correlation between the number of correct chemical engineering problems and the MCT results ($R^2 = 0.34435$, $p < 0.0001$). Additionally, there were indications that spatial skills may be more relevant in solving some types of problems compared to others, and problem representation may be a strong indicator of success in chemical engineering problem solving. That is, those students with high levels of spatial ability are better at problem representation, which enables them to be more successful problem solvers. In this paper, the project results are presented along with a detailed analysis of student performance on one of the problems.

Introduction

An essential component of engineering identity, problem solving is a skill whose development during an engineering education is required by accrediting bodies [1]. It is widely accepted that engineers are good problem solvers. Problem solving is not easy; it is a cognitively challenging process because a problem, by definition, is novel, has not been seen before, and stored information in long term memory is not readily available to easily identify a solution path. Problems can vary greatly in style from ill to well-structured, convergent to divergent and in other ways as summarised in a taxonomy provided by Jonassen [2]. Problem solving activities often expose large variations in individual performance and research in STEM (science, technology, engineering and mathematics) education has shown a significant portion of this variation to be shared with spatial ability in contexts such as chemistry [3], physics [4] and mathematics [5]. The purpose of this study was to examine the role of spatial ability in problem solving in the context of chemical engineering, a topic for which there are few, if any, reports in the literature.

Spatial ability – the ability to create and manipulate visualizations - is a key factor in several models of intelligence and working memory. In factorial models of intelligence its definitions are repeatedly tied to visualizing well-structured images and performing mental operations on these images such as rotating (e.g., Figure 1), cutting (e.g., Figure 2), folding or manipulating in other ways [6], [7], [8]. Wherever visualization is important to completing a task, spatial skills are needed and spatial tests will predict performance. Spatial ability also occupies a prominent role in memory models of cognition that differentiate between tasks that are routine or rehearsed and are supported by long term memory and tasks that are novel and are mediated by short term or working memory. In the model developed by Baddeley & Hitch [9], working memory is conceived to consist of three components – the central executive to manage attention, the phonological loop responsible for holding speech information and the visuospatial sketch pad responsible for the storage and manipulation of visuospatial information. Wherever working memory is key to completing a task, spatial skills are also needed. Hence, spatial ability should be highly relevant to tasks that are novel or non-routine such as problem solving.

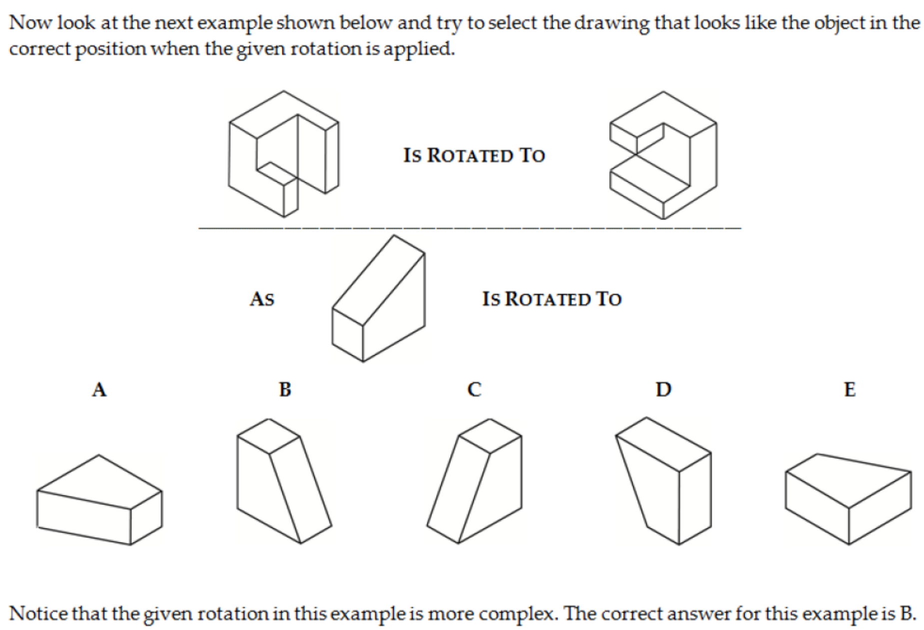

Figure 1. Sample question from the Revised Purdue Spatial Visualization Tests: Rotations [10].

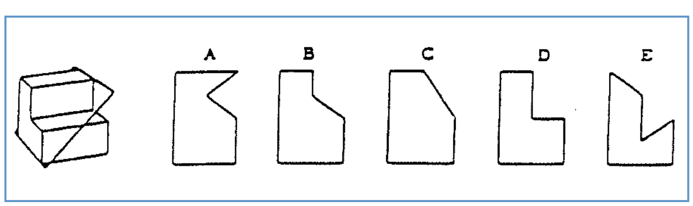

Figure 2. Sample question from the Mental Cutting Test [11].

The role of spatial ability in STEM learning has been shown to be very important in terms of interest [12] and overall achievement [13]. In the latter study, using data collected for Project TALENT, it was established that spatial ability measured in adolescence predicted career path and achievement in higher education: "spatial ability added incremental validity (accounted for a statistically significant amount of additional variance) beyond SAT-Mathematical (measuring mathematical reasoning ability) and SAT-Verbal (measuring verbal reasoning ability) in predicting these math–science criteria” [13]. With regard to particular aspects of the STEM curriculum, spatial ability has been shown to be significantly related to achievement in various aspects of mathematics [14], performance in reasoning about Newtonian mechanics [15], visualizing electric circuits [16] and several activities in chemistry [3].

Solving apparently simple problems in mathematics can be very challenging as illustrated in a study by Clement [17] who administered a number of word problems to engineering students. Just under one third of students were able to construct the correct algebraic equation from the following statement (using C to represent the number of cheesecakes and S the number of strudels): “At Mindy’s restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel.” Such high error rates in problem solving can be exacerbated by changing the problem phrasing. For example, Hegarty, Mayer & Green [18] found that rephrasing a simple word problem increased the error rate from 1% to 15% among undergraduate psychology students while Coquin-Viennot & Moreau [19] showed how different forms of phrasing the same problem evoked different solution strategies. To the human mind, the process of comprehending a word problem statement is not at all simple and straightforward but a very complex process.

Findings from such studies have led to the idea that problem solving consists of two phases – problem representation and problem solution. As explained by Mayer [20], representation draws on linguistic, semantic and schematic knowledge to understand the words being used in the problem, the context of the problem and the schema on which the solution phase will be based (e.g., distance = rate x time is a schema). The solution phase then follows, during which the problem solver chooses the order in which to perform the different mathematical procedures, i.e. a strategy, and for this phase to be successful, sufficient knowledge of the relevant mathematical procedures is required. Problems that require only basic procedural knowledge can be labelled as simple which is unfair when great difficulties can emerge in the preceding representation phase. In terms of cognition, representation is unfamiliar territory that draws heavily on working memory but solution is supported by familiar, rehearsed processes stored in long-term memory.

The role of spatial ability in problem solving has been investigated among elementary school children by examining the quality of visualizations produced during problem solving [5], [21]. These studies revealed how some participants provided ‘pictorial’ visualizations - images of the objects in the problem without relations between them - while others provided accurate, schematic visualizations that included both objects and relations. Success rates were significantly higher when solutions included schematic visualizations and the ability to produce these was significantly related to spatial ability. However, visualization quality did not account for all of the data as approximately two thirds of the solutions did not appear to be accompanied by any visualization yet the success rate for these was close to 50 % [5].

To examine the role of spatial ability in supporting the representation phase in problem solving in the context of engineering education, Duffy [22] administered several word problems in mathematics to a sample of first year engineering students. Consistent with the previous studies of 6th grade students, a significant relationship between spatial ability and problem solving was also found for this older sample. A test of the procedural knowledge required to complete the solution phase ruled out core competencies as a determining factor as the vast majority of the sample achieved a high score on a test measuring core competency in mathematics. However, many failed in the representation step and performance in this step was significantly related to spatial ability, as measured by the PSVT:R (Figure 1). Participants made several errors in applying linguistic and schematic knowledge to problem representation with low spatial ability students much more likely to make these errors. It is possible that working memory overload occurred sooner for the low spatial students compared with their high spatial ability peers.

The purpose of this study is to examine the same phenomenon in the context of chemical engineering problem solving. The hypothesis addressed is: There is no difference in level of procedural knowledge required for chemical engineering problem solving between low and high spatial ability students but high spatial students are more successful in the representation phase than low spatial students. The study also aimed to highlight qualitative differences in representation between the two groups. Knowing more about this phenomenon will provide chemical engineering instructors with greater insight into the ways students can approach problem solving and how this can vary with spatial ability level. Given that spatial ability can be easily measured, these instructors could potentially diagnose difficulties in problem solving in advance of students taking their courses, provide these students with support leading to increases in grades and retention rates.

Approach

Students enrolled in a 3rd year course in a Chemical Engineering program at University of Cincinnati were administered a test of spatial cognition, the Mental Cutting Test (MCT) [11]. University of Cincinnati is a large public university in the Midwest. A total of 63 students participated in the study (19 Female, 44 Male). A sample problem from the MCT was shown in Figure 2. Students then solved problems based on concepts learned in a prerequisite course. Some of the problems utilized typical chemical engineering concepts re-framed with "everyday" examples and other problems were taken directly from chemical engineering textbooks. Examples of these two types of problems are given in the following:

> One vegetable oil contains 8 % saturated fats and a second oil contains 26 % saturated fats. In making a salad dressing from these two oils how many ounces of the second must be added to 10 ounces of the first in order for the dressing to have 14 % saturated fats.
>
> One thousand kilograms per hour of a mixture of benzene and toluene containing 50% benzene by mass is separated by distillation into two fractions. The mass flow rate of benzene in the top stream is 450 kg/hour and that of toluene in the bottom stream is 475 kg/h. The operation is at steady state. Calculate the component flow rates in the output streams.

Problem sets were administered during a class period in Thermodynamics, a third-year course in the Chemical Engineering program, but the topics tested in the problems are typically covered in a second-year course in chemical engineering. Since the students were all given the same amount of time, those who were better at problem-solving would likely solve a larger number of problems compared to weaker students. Not all students attempted all problems and scores were only corrected based on the number of problems attempted. Figure 3 shows the correlation between the number of problems correctly solved and scores on the MCT.

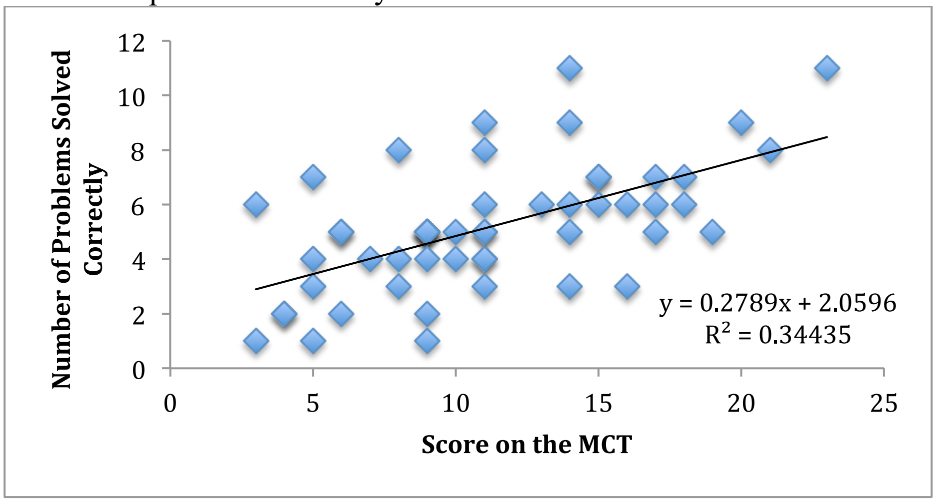

Figure 3: Scatter Plot with Results from Chemical Engineering Problems

A strong positive correlation (R=0.59, $p<0.00001$) between spatial skills test scores and the number of problems successfully solved by the students was found. In addition to the overall performance on the chemical engineering problem-solving, we also examined the individual problems to determine which specific ones were most reliant on spatial skills. In this analysis, the average score on the spatial skills test (MCT) for the students who answered the problem correctly was computed and compared to the average score for those who got the problem incorrect. Table 1 includes the analysis from this portion of the study for all of the problems administered. For example, for problem #1, the average MCT score for the twenty-eight students who answered this problem correctly was 10.11 and the average MCT score for the fourteen students who answered it incorrectly was 10.42 (out of 25 points possible on the MCT).

Table 1. Analysis of individual problems from chemical engineering

Problem	Avg MCT Incorrect	Avg MCT Correct	
1. You have been asked to provide 150 ml of hydrochloric acid at a 12% concentration. In the cabinet you find one bottle marked 10% HCl (10% hydrochloric acid) and another bottle marked 25% HCl (25% hydrochloric acid). How many ml of each should you use?	10.42 (n=14)	10.11 (n=28)	NS
2. How many pounds of M&Ms that cost $1.60 per pound must be mixed with 7 pounds of Smarties that cost $2.20 per pound to make a mixture of sweets that costs $2.00 per pound?	9.4 (n=17)	10.72 (n=25)	NS
3. One vegetable oil contains 8 % saturated fats and a second oil contains 26 % saturated fats. In making a salad dressing from these two oils how many ounces of the second must be added to 10 ounces of the first in order for the dressing to have 14 % saturated fats.	9.5 (n=4)	17.44 (n=9)	$p<0.0001$

Table 1 (continued). Analysis of individual problems from chemical engineering

4. One thousand kilograms per hour of a mixture of benzene (B) and toluene (T) containing 50% benzene by mass is separated by distillation into two fractions. The mass flow rate of benzene in the top stream is 450 kg B/h and that of toluene in the bottom stream is 475 kg T/h. The operation is at steady state. Calculate the unknown component flow rates in the output streams.	9.3 (n=21)	12.5 (n=34)	p=0.0169
5. Two methanol-water mixtures are contained in separate flasks. The first mixture contains 40.0 wt% methanol, and the second contains 70.0 wt% methanol. If 200 g of the first mixture is combined with 150 g of the second, calculate the mass and composition of the product.	4.8 (n=5)	11.88 (n=52)	p=0.0014
6. Two streams consisting of 3.0 kg/min of benzene (B) and 1.0 kg/min of toluene (T) are well mixed in a container. The mixed product flows from the container at a rate of m (kg/min) and has a composition of x (kg B/kg). Determine m (kg/min) and x.	7.71 (n=7)	11.94 (n=47)	p=0.0334
7. A liquid mixture containing 45.0% benzene (B) and 55.0% toluene (T) by mass is fed to a separation column. A product stream leaving the top of the column contains 95.0 mole % B, and a bottom product stream contains 8.0% of the benzene fed to the column. The volumetric flow rate of the feed stream is 2000 L/h and the specific gravity of the feed mixture is 0.872. Calculate the mass flow rate of the overhead stream and the mass flow rate and composition (mass fraction) of the bottom product stream.	11.54 (n=22)	15.17 (n=12)	p=0.0168
8. If the water-gas shift reaction, $CO(g) + H_2O(g) \leftrightarrow CO_2(g) + H_2(g)$ proceeds to equilibrium at a temperature T(K), the mole fractions of the four respective species satisfy the relation $K(T) = \frac{y_{CO_2} y_{H_2}}{y_{CO} y_{H_2O}}$ where K(T)is the reaction equilibrium const. K = 1at T = 1105 K. Suppose the feed to a reactor contains 1.00 mol of CO, 2.00 mol of H_2O, and no CO_2 or H_2, and the reaction mixture comes to equilibrium at 1105K. Calculate the equilibrium composition and the fractional conversion of the limiting reactant.	11.14 (n=22)	15.43 (n=6)	p=0.077
9. Given the dehydrogenation of ethane in a steady-state continuous reactor where the following reaction $C_2H_6 \rightarrow C_2H_4 + H_2$ occurs. If one hundred kmol/min of ethane is fed to the reactor, and the molar flow rate of H_2 in the product stream is 40 kmol/min, use an atomic balance to calculate the amounts of C_2H_6 and C_2H_4 in the product stream.	12.0 (n=13)	12.58 (n=19)	NS
10. A paint mixture containing 25.0% of a pigment and the balance water sells for $18.00/kg, and a mixture containing 12.0% pigment sells for $10.00/kg. If a paint retailer produces a blend containing 17% pigment, what should the sales price be in order to yield a 10% profit?	11.77 (n=13)	13.07 (n=14)	NS

Table 1 (continued). Analysis of individual problems from chemical engineering

11. Ammonia is burned to form nitric oxide in the following reaction: $4NH_3 + 5O_2 \rightarrow 4NO + 6H_2O$ Calculate the ratio (kmol O_2 react/kmol NO formed). If ammonia is fed to a continuous reactor at a rate of 100.0 kmol NH_3/h, what oxygen feed rate (kmol/h) would correspond to 40% excess O_2?	8.87 (n=8)	13.0 (n=24)	p=0.0403
12. In the Deacon process for the manufacture of chlorine, HCl and O_2 react to form Cl_2 and H_2O. Sufficient air (21 mole%, 79% N_2) is fed to provide 35% excess oxygen and the fractional conversion of HCl is 85%. Calculate the mole fraction of the product stream components using atomic species balances.	12.11 (n=9)	14.33 (n=3)	NS

A thirteenth problem was also administered to the students but only six attempted to solve it. Despite this small sample size, we found statistically significant differences (p=0.02) between the spatial skills of those who solved the problem correctly (average=21.5) and those who were not correct (average=11.25); however due to the sample size, this problem was removed from the analysis.

As can be seen from the data presented in Table 1, a little less than half of the chosen problems seemed not to have a "spatial factor." For these problems (1, 2, 8, 9, 10, and 12) the spatial skills of the students who got the problems correct were not different from the spatial skill levels of the students who got them incorrect.

Further detailed analysis of problem number 4 was conducted to determine if there were differences in spatial skills observed at the problem conception stage (similar to Duffy's (2017)) findings with mathematics problems. For this particular problem, there were four "stages" in the problem solution:

1. Sketch the process flow diagram
2. Apply the given information to the diagram
3. Apply conservation of mass principles
4. Obtain the correct solution

Student responses were examined based on this solution scheme and the results shown in Table 2 were obtained. In this table, for each stage in the problem-solving process, the average MCT score of those who correctly implemented the stage were computed as were the average MCT scores of the students who did not implement the stage correctly. For example, for stage 1, sketching the process flow diagram, the average MCT score for those who correctly sketched the diagram was 11.18 and the average MCT score for those who did not properly sketch the diagram, was 11.00. It should be noted that a student could have completed stages 1 and 4 correctly but showed no evidence of stages 2 and 3 and thus would not have been given credit for doing those stages correctly.

Table 2. Analysis of individual stages in problem-solving process

Stage	Average MCT Correct	Average MCT Incorrect
1	11.18 (n=44)	11.00 (n=12)
2	11.97 (n=35)	9.81 (n=21)
3	11.47 (n=44)	10.50 (n=12)
4	12.56 (n=34)	9.27 (n=22)

None of the differences in MCT scores were statistically significant in the individual stages other than Stage 4—obtaining the correct solution. It is interesting to note that nearly everyone knew to draw a figure (stage 1) but not everyone labeled the figure correctly. The difference in average MCT scores for stage 2 was approaching significance (p=0.11) meaning that it does appear that the spatial skills are critical in the problem conception stage.

Figures 4-7 show representative samples of student work for this particular problem. Also given are genders and MCT scores.

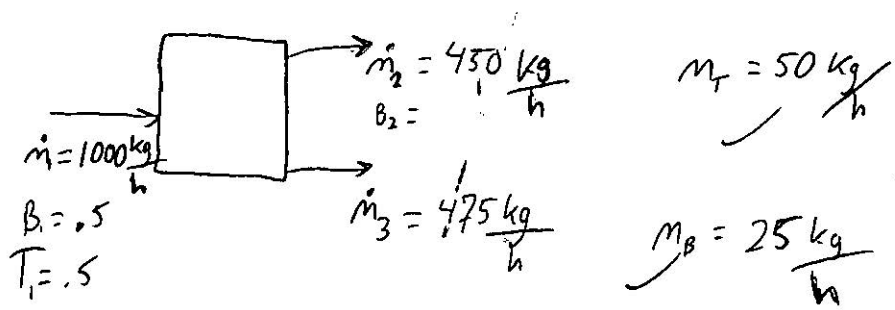

Figure 4. Student Example 1, Male, MCT=9

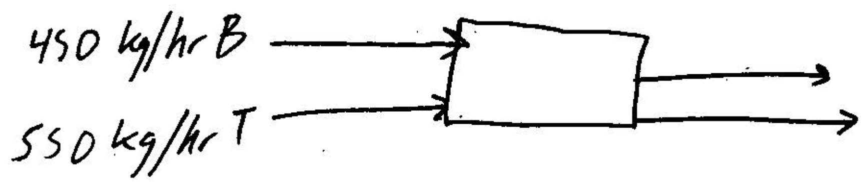

Figure 5. Student Example 2, Female, MCT=5

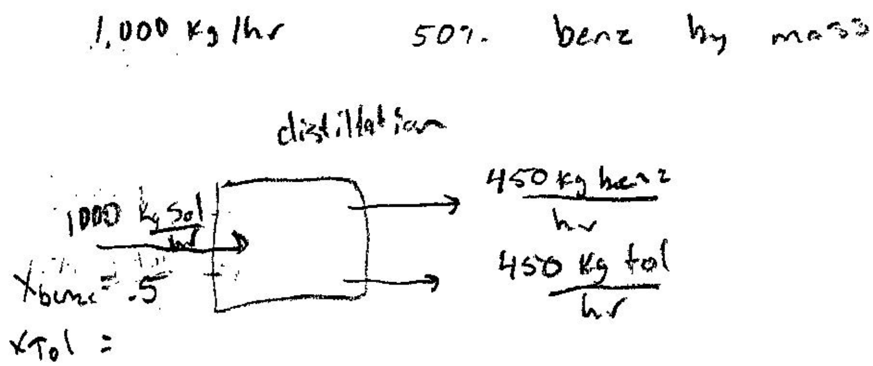

Figure 6. Student Example 3, Male, MCT=7

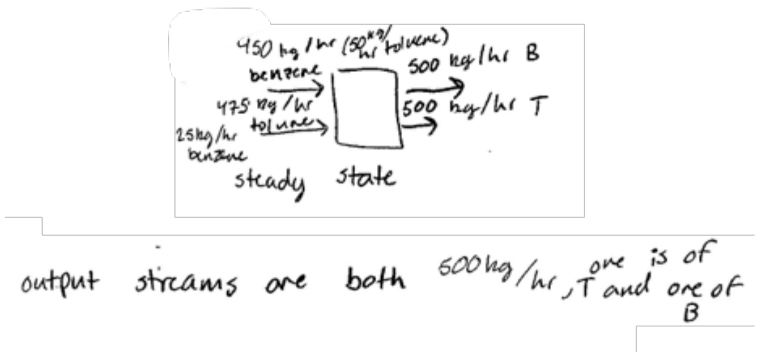

Figure 7. Student Example 4, Female, MCT=20

The solution shown in Figure 4 (Student Example 1) is correct. This was produced by a male student with average spatial skills. This is in contrast with the solutions given in Figures 5-7 where the solutions were all incorrect. Figures 5 and 6 show the work of low visualizers in the class. Notice that for Student Example 2 (low visualizer, female), the figure is drawn, but the labeling is incorrect and no solution is obtained. For Student Example 3 (low visualizer male), the figure is drawn with correct labels, but the correct solution is not obtained. The most puzzling solution is shown in Figure 7. This female student has one of the highest MCT scores of all students in the class (the highest female of all) and she has constructed the picture backwards. She actually has the correct solution shown, but did not identify the correct answers from her figure in her response (output streams are both 500 kg/hr, one of T and one of B). Gender differences in correct/incorrect solutions was not a focus of the current study but will be examined in future analyses of the data.

Conclusions

From the results presented here, it appears that spatial skills play a role in solving certain types of problems from Chemical Engineering. Similar to results obtained in a study of problem-solving in mathematics among engineering students [22] it appears that spatial skills are critical to success in the problem conception stage. The findings also resonate with the studies of the role of visualization quality in word problem solving even though these were conducted with 6th grade children [5]: there is a qualitative difference in the sketches provided by students 1 and 2 with the latter being somewhat pictorial in contrast with the accurate schematic representation of the former. As represented in sketch form, the visualization of the same problem by these two students is very different. Future plans include conducting a detailed analysis of the remaining problems to analyse the problem representation step in greater detail and determine the relationship between variation in the application of linguistic, semantic and schematic knowledge during representation and variation in spatial ability. The data will also be analysed to better describe why some problems reveal a relationship with spatial ability while others do not. Based on the working memory view of spatial ability, it is possible that performance on some problems is largely determined by having a core competency or not, i.e. either a critical core competency is available in long term memory or it isn't and without this the problem cannot be solved. In prior studies of math problems, the required core competencies were effectively common knowledge, but that may not be the case for the problems administered to this sample. These two approaches – quality of visualization and core competency availability – should account for the role of spatial ability in problem solving.

Acknowledgements

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