

## **2006-388: THE ROLE OF NUMERICAL SIMULATION IN LABORATORY UNCERTAINTY ASSESSMENT AT THE INTRODUCTORY ENGINEERING PHYSICS LEVEL**

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# The role of numerical simulation in laboratory uncertainty assessment at the introductory engineering physics level

## Abstract

The teaching of laboratory uncertainty assessment via numerical simulation is used to supplement the pedagogy of discovery based learning at the introductory engineering level. The typical laboratory uncertainty assessment methods such as comparing the result to a directly measured quantity and assessing error bars on graph were extended to include the use of numerical simulation in more complex situations. In this paper, laboratory exercises in force-distance relationship, equipotential lines, RLC circuit with hysteresis loss, radiation, optics and null measurement were presented. Null measurement where the system approach to null is non-linear (the measured quantity is affected by the off-null condition) was also presented. The laboratory setups were analyzed using numerical stimulation in analogy to real situations where there are no closed form mathematical expressions. The effects on the numerical simulation given a range of input parameter changes were used to assess the laboratory exercise uncertainties. Besides the applicability to realistic situations in a workplace, the numerical simulation also has an advantage of setting the student's attitude to understand the difficulties of the associated inverse problems, which usually are the problems that demand attention in a workplace. The disadvantage of the time-consuming repetitious computational tasks in the numerical simulation could be offset with well-designed computer techniques, using spreadsheets or introductory Visual Basic. The pedagogy of discovery base learning with this numerical simulation feature was shown to have a positive effect through the students' questions as well as grade improvement.

## Introduction

One of the most important functions of an introductory engineering physics course is to provide a solid foundation for advancement into higher level courses in the various engineering disciplines. In fact, an engineering physics program, together with courses, could be viewed as a universal donor<sup>1</sup>. Fundamental concepts are usually covered in introductory engineering physics course. However, it was reported in a recent 2005 case study that some 10 to 15% of senior engineering students still have confusion about the difference of energy and temperature<sup>2</sup>. The finding is consistent with an earlier 2000 report that some students identify movement of electricity and energy transfer as material flows, completely missing the concepts of random diffusion and collision<sup>3</sup>. The concept of randomness is usually first taught as an uncertainty assessment in a laboratory setting. The typical laboratory uncertainty assessment methods such as comparing the result to a directly measured quantity (using a formula to express the deviation as percent difference) and assessing error bars on graph are usually taught in the first semester. Some aspects of uncertainty or error come from randomness. Randomness is more than a formula. Thus, sometimes a formula is not sufficient. We have extended these typical assessment methods to include the use of numerical simulation in more complex situations.

In this paper, laboratory exercises in force-distance relationship, equipotential lines, hysteresis loss, radiation, optics and null measurement are presented. Null measurement where the system approach to null is non-linear (the measured quantity is affected by the off-null condition) is also

presented. The laboratory setups were analyzed using numerical stimulation in analogy to real situations where there are no closed form mathematical expressions. The effects on the numerical simulation given a range of input parameter changes were used to assess the laboratory exercise uncertainties. The pedagogy of discovery base learning with this numerical simulation feature was shown to have a positive effect through the students' questions as well as grade improvement.

The use of spreadsheet programs like Excel in uncertainty assessment has been well documented in thermal engineering courses by various authors<sup>4,5,6,7</sup>. Combining Excel with Visual Basic in thermal engineering courses was also reported to be fruitful<sup>8,9</sup>. Therefore we also selected Excel for our students. Visual Basic was used by the instructors from time to time for verification of complex Excel programs.

## **Examples and Discussion**

### **Force-Distance relationship**

The fundamental gravitational and electrical force-distance relationship in introductory engineering physics is quite difficult to demonstrate. The standard use of a torsion balance to demonstrate the gravitational force requires an extremely stable environment. In the case of the electrical force, the high voltage involved may pose a safety issue. The essence is not for the students to re-discover the laws of physics, but rather to learn how to go about to verify and discover the relationships. Therefore we developed a laboratory exercise using permanent magnets and a balance. The idea is rather simple. A bar magnet sits horizontally on a balance and another vertical bar magnet is moved across, changing the balance reading. The difficulty is that the uncertainty is not directly obtainable from a formula.

Simulation could be used to calculate the expected result. There are four poles, and using the inversely square law, the balance reading (total vertical force) versus distance could be calculated easily in Excel. The Excel columns are essentially the columns that calculate the changing angles as the vertical magnet moves along horizontally. The Excel program is available from the authors upon request. Visual Basic is handy for checking the Excel column programming. The experimental result can be compared to the simulated result to assess the uncertainty. A log-log plot can be used to compute the power law index. This simulation has two main goals. First we show the students that uncertainty is not always given a formula, second if a student can understand the relationship between the data in Excel's columns in relationship to the changing angles, that student should know how to write down the integral when asked to do so in the lecture part of the class. In our experience, about 30 to 50% of our students who had already taken calculus could not write down the integral correctly and they found this Excel approach to be very helpful. An example of the Excel programming is illustrated in the Appendix.

## Equipotential line

The electrostatic equipotential line experiment is another example of simple laboratory procedure that involves tedious analysis. A carbon sheet with painted electrodes is used. The carbon sheet rests on top of an insulating sheet. The gap between the electrodes is painted with dots for x-y coordinate information. The voltage drop (V) from one electrode to each dot is measured. The dots with equal potential are connected to form equipotential lines. The 90-degree electrical lines could be traced by hand.

Simulation in Excel is straight forward in the sense that it can easily compute the  $dV/dx$  and  $dV/dy$  values after the grid point potential values are entered into the spreadsheet. Combining the components gives  $dV/dr$ . The angle between the nearest hand-traced equipotential line and the  $dV/dr$  vector should be 90 degrees and deviation is an indicator of uncertainty. Although the final uncertainty is a formula that calculates the deviation from 90 degrees, the analysis requires a fundamental understanding of the experiment and the simulation process. The uncertainty assessment is a process, unlike a statistical explanation of an uncertainly formula derivation.

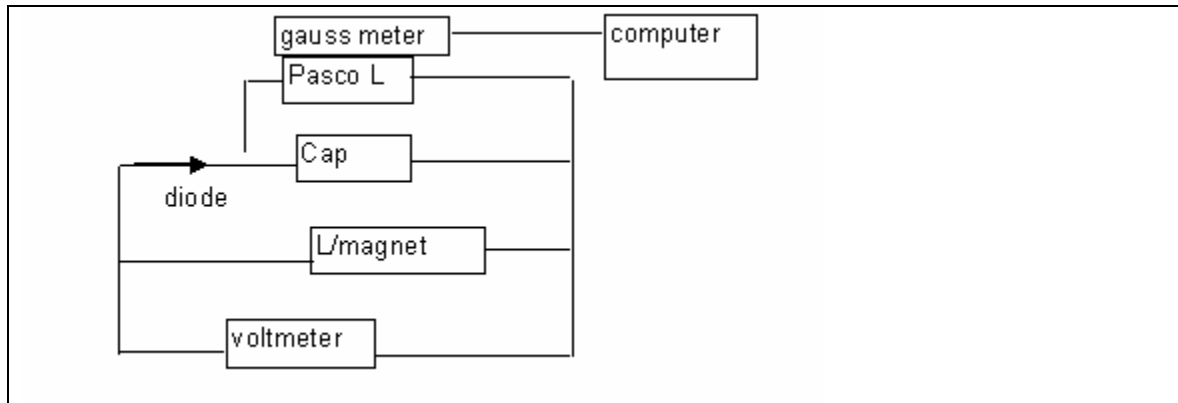
## RLC circuit with hysteresis loss

A series RLC circuit can be set up with a sinusoidal source. A dual trace scope is used to analyze the circuit. The scope usually has coaxial inputs so one side is always grounded. The voltage across (L + C) and C can be measured and data is entered into Excel. Sinusoidal data manipulation is easily implemented in Excel so the voltage across any two elements can be generated in Excel. Lissajous figures can also be generated by Excel. Lissajous figures are a family of curves described by the parametric equations  $x(t) = A \cos(at-b)$  and  $y(t) = B \cos(ct-d)$ . The figures generated by Excel can be compared to the scope data in x-y mode as well as the calculated data using the theoretical formulas. The uncertainty can be assessed via the distorted experimental figures and for the ranges of RLC values that match the data. Hysteresis is most commonly associated with materials that become magnetized when a changing magnetic field is applied. Also, an iron bar inside the coil can be used to demonstrate hysteresis loss.

## RLC circuit with pulse excitation

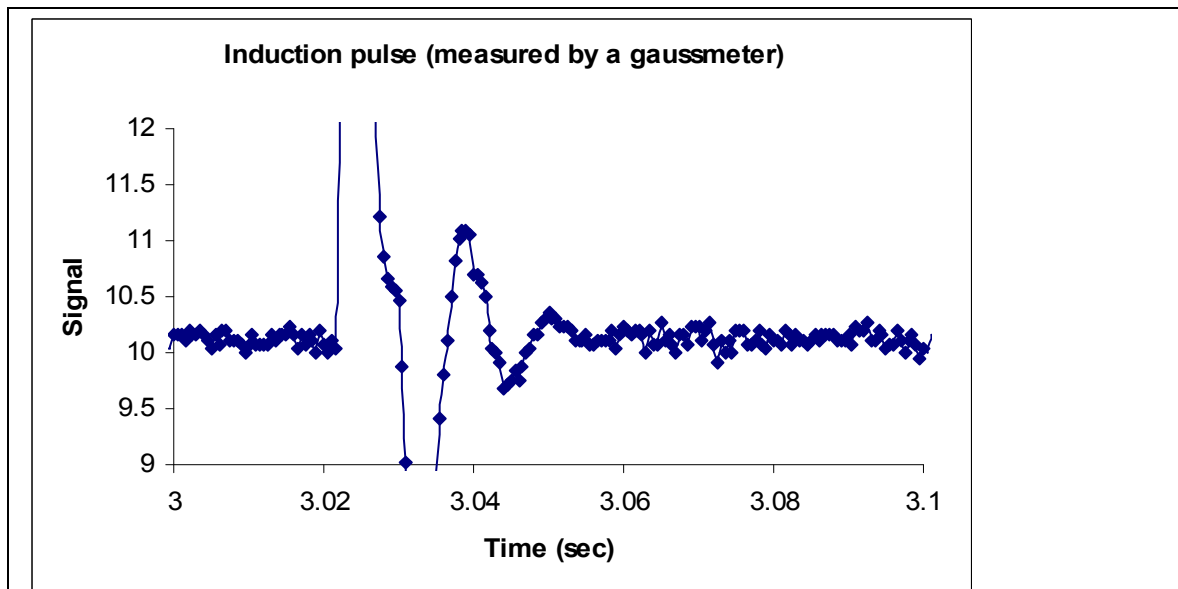
The pulse is a fundamental excitation for a RLC circuit, although the mathematical representation is easier for a steady sinusoidal source. The solution is given in standard introductory calculus physics textbook (for example, Halliday, Resnick, & Walker: *Fundamentals of Physics*). The envelope decays as  $\exp(-Rt/L)$  and the oscillation period is related to L and C. A series RLC circuit can be set up with a 1.5 V DC battery source and a pulse can be generated by disconnecting the battery after the coil is charged. We use a Gauss-meter supplied by Logger Pro with millisecond time resolution. Putting the Gauss-meter on the bulge of a 0.5 Henry transformer with a 10 micro-Farad capacitor gives a signal peak of about 20 Gauss (2 mT). The numerical solution can be generated in Excel using time as the Excel variable. Damping curves with different set of RLC values can be generated and compared to the data. The uncertainty can be assessed for the ranges of RLC values that match the data.

There are two extensions to this experimental setup. It could be modified to include Faraday induction by adding a diode, an open coil, and a magnet (about 2 mT), thus eliminating the battery (see below).



**Figure 1.** The Faraday induction configuration. The connection to the Pasco coil is open when charging the L-coil with a magnet. The connection is connected for dumping the capacitor energy to the Pasco coil (disconnect the charging L so the capacitor does not send charges toward the charging L).

The magnet going in and out of the charging coil would generate charges in the capacitor. A digital voltmeter could be added to monitor approximately how many times the magnet has to move in order to charge the 300 micro-Farad capacitor to 2 volts. Disconnect the charging coil so that the capacitor energy could be dumped into another open coil (from Pasco) with an iron bar (about 20 mH). We put the Gauss-meter at the end of iron bar and the peak signal was about 2 mT. A typical data trace is displayed in Figure 2.



**Figure 2.** A typical data trace of the experimental setup shown in Figure 1. The signal is measured in gauss (0.1 mT)

The second extension is to replace the capacitor with an open capacitor. An open capacitor can be built from two pie-pans. The Gauss-meter can be inserted into the gap and the  $dE/dt$  values can be measured via the Maxwell displacement current equation. The Gauss-meter position can be varied to demonstrate the spatial variation. The essence of this experiment is to demonstrate that energy flows through space, radiation, and to the pie-pans. The energy flow is described by the Poynting vector while some energy flows through the wire as Ohmic loss. Oscillation is difficult to measure since the pie-pan has low capacitance and Excel is not used for this extension.

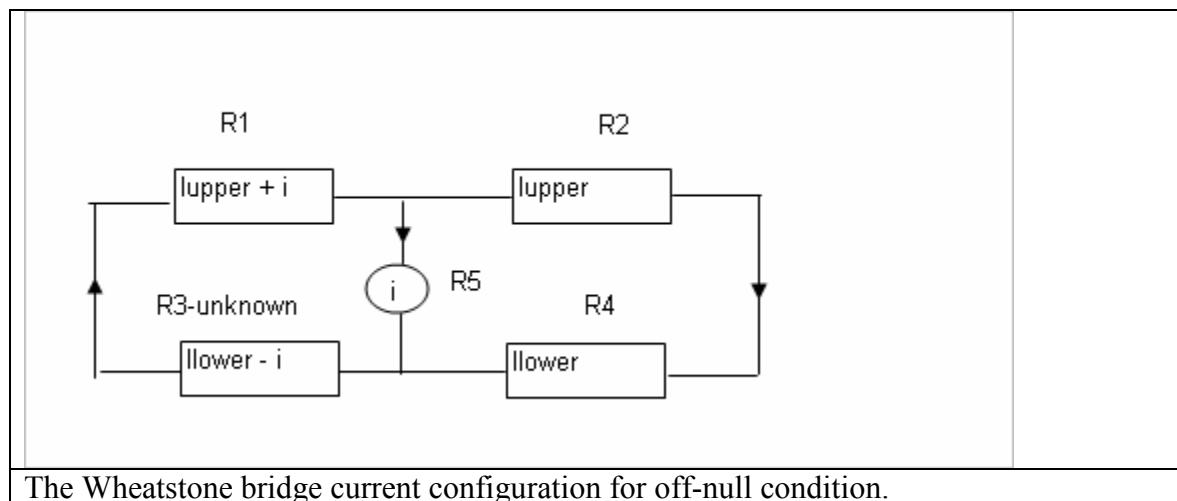
## Optics

The interference pattern caused by light passing through a double-slit can be modeled as fringes with approximately the same intensity when the slit width is very narrow as compared to the wavelength. For relatively wide slits, the interference of light from the double-slit is also modified by diffraction. The angular dependence in standard text books (Halliday, Resnick, & Walker: Fundamentals of Physics) is  $\cos^2(\beta) (\sin \alpha/\alpha)^2$ , where  $\beta$  is related to the slit separation and  $\alpha$  is related to the slit width. The angular dependence is similar to the RLC damping curve for pulse excitation. The Excel column programming technique can be used to generate a curve to compare with the data. The uncertainty is given by the ranges of the  $\alpha$  and  $\beta$  values that match the data. Laser and CCD detectors are readily available these days and the experimental procedure is straight forward. Data reduction from image to numerical data is more elaborate though. For the digitization process, we have used our own Visual Basic programs and also the NIH Image-J program that is available in the public domain.

## Null measurement method

One of the best ways to obtain better precision measurements is to use a null difference method instead of measuring a quantity directly. Recent examples include the null measurement for keeping the dark fringes on the photo-detector in the LIGO detection of gravitational waves<sup>10</sup>. The null measurement method principle is quite simple. The method measures the difference between two similar quantities, one of which is known very accurately and is adjustable. The adjustable reference quantity is varied until the difference is reduced to zero. The classical example is the Wheatstone bridge for resistance measurement. The uncertainty experienced when the Wheatstone bridge is at an off-null condition would be interesting for students and would enable them to learn about the fundamentals of null measurement.

A Wheatstone bridge has an upper arm and a lower arm. Let  $R_1$  on upper left arm,  $R_2$  on upper right arm,  $R_3$  the unknown on lower left arm, and  $R_4$  on lower right arm. Let the current through the null detector  $R_5$  be  $i$  for off-null condition. The current configuration is displayed below.



The left loop has an equation  $(lupper + i)R1 + iR5 = (llower - i) R3$ .

The right loop has an equation  $lupperR2 = iR5 + llowerR4$

Excel column programming is particularly suitable for these two coupled equations. Using the resistance and current values in the lab, the students have shown no trouble in the numerical calculation column by column, although some students made simple mistakes in getting the algebraic expression for the unknown in the lecture. In the Excel simulation, the current  $i$  through the detector is taken as the minimum current that is measurable by the galvanometer detector. As the battery voltage is adjustable, the value of  $llower$  is set to an arbitrary value of 100 to 1000 times that of the detector current in the simulation. The  $lupper$  in the right loop is calculated first and is inserted in the left loop equation for the calculation of  $R3$ . Different Excel rows represent different detector currents (the amount of off-null). A plot of  $R3$  versus the off-null detector current illustrates the approach to the null condition. Changing the excitation current,  $llower$ , with respect to the minimum detectable current affects the linearity of the approach to the null condition. A typical student galvanometer has nominal values of 80 ohms and 20 micro-amp minimum detectable current. The Excel simulation using an excitation current,  $llower$ , of 2 mA, shows a nonlinear approach to the null condition. Higher excitation current of 20 mA shows a linear approach to the null condition. A current limiting resistor next to the battery or high resistance in the arms can be used to verify the simulation also.

## Conclusions

All the experiments discussed above are listed in our student lab manual<sup>11</sup>. Numerical simulation using Excel is practical in a laboratory setting. Our experience shows that our students can extend the Excel program readily when we give them the Excel design with limited columns and rows. The complexity of mathematical expression is replaced by many multiple Excel columns. The flexibility of modifying the input parameters reinforces the calculus concepts. Besides the applicability to realistic situations in a workplace, the numerical simulation also has an advantage of setting the student attitude to understand the difficulties of the associated inverse problems, which usually are the problems that demand attention in a workplace.

The pedagogy of discovery base learning with this numerical simulation feature was shown, to have a positive effect, as evidenced by the students' questions as well as grade improvement. If a student can follow the Excel columns, he or she can write down the mathematical equivalent, an integral or the solution to a set of coupled equations. The teaching of uncertainty as a process rather than a formula resonates well with the ultimate goal of education, the training of a critical thinker. We are not suggesting that engineering physics uncertainty assessment in a laboratory setting is similar to student life uncertainty assessment in a college counselor setting, but the training of a critical thinker does apply to every discipline.

Future work includes designing more Excel simulations for uncertainty assessment in laboratory exercises in such area as sound and thermal physics, incorporating Excel in the lecture components of the courses, and collaborating with instructors in other colleges to assess the effectiveness of this new approach.

### Acknowledgements

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### Appendix:

An Excel program is used to calculate the force in a 4-charge configuration.

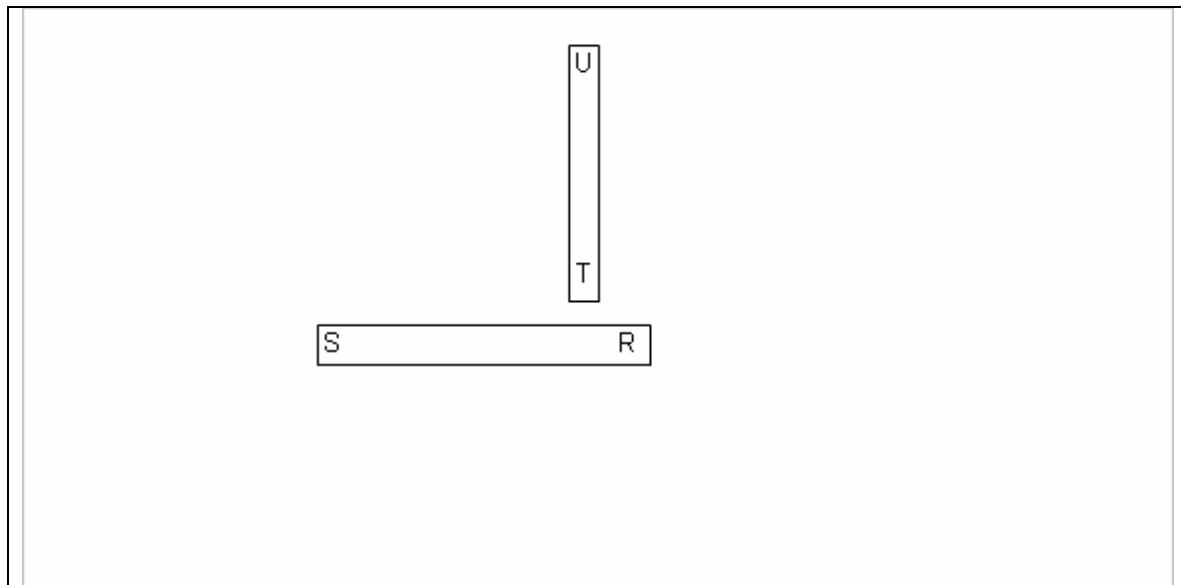


Figure A-1: The R, S, T, U charges are located on two bars. R-S and T-U are different polarities for the case of magnets. The T-U bar scans horizontally. The two bars are of equal length (15 cm). The vertical T-U bar is 1 cm above the horizontal R-S bar



The Excel program is displayed below.

	A	B	C	D	E	F	G	H
1	scan	angleT-R	angle T-S	y-force from T	angleU-R	angleU-S	y-force from U	force total
2	0.5	$1/((A2*A2+16)*0.5)$	$1/(((15-A2)*(15-A2)+1)*0.5)$	$(-B2/(A2*A2+1))+C2/((15-A2)*(15-A2)+1)$	$16/((A2*A2+16)*0.5)$	$16/(((15-A2)*(15-A2)+16)*0.5)$	$(E2/(A2*A2+16*16))+(-F2/(((15-A2)*(15-A2)+16*16))$	D2+G2
3	0.5	0.89442719	0.068802092	-0.715216062	0.999512076	0.740986951	0.002311287	-0.712905
4	1	0.70710678	0.07124705	-0.35319173	0.998052578	0.752576695	0.00221848	-0.350973
5	1.5	0.5547002	0.073871686	-0.170273864	0.995634226	0.764291484	0.002111349	-0.168163
6	2	0.4472136	0.076696499	-0.088991563	0.992277877	0.776114	0.001990303	-0.087001
7	2.5	0.37139068	0.079745222	-0.050719176	0.988012034	0.788024374	0.001855923	-0.048863
8	3	0.31622777	0.08304548	-0.031050049	0.982872187	0.8	0.001708952	-0.029341
9	3.5	0.27472113	0.086629616	-0.020083542	0.976900017	0.812015371	0.001550277	-0.018533
10	4	0.24253563	0.090535746	-0.013524705	0.9701425	0.824041924	0.001360913	-0.012144
11	4.5	0.21693046	0.094809093	-0.009356276	0.96265094	0.836047911	0.001201985	-0.008154
12	5	0.19611614	0.099503719	-0.006557743	0.954479978	0.847998304	0.001014708	-0.005543
13	5.5	0.17888544	0.104684785	-0.004577104	0.945686599	0.859854744	0.000820374	-0.003757
14	6	0.16439899	0.110431526	-0.00309649	0.936329178	0.871575537	0.000620329	-0.002476

The above Excel columns are used to compute the vertical forces on the horizontal bar as exerted by the vertical bar. The vertical bar scan across the horizontal bar and a plot of force versus scan distance can be generated. A log-log plot would extract the power law dependence of force versus distance. The simulation result can be used to compare to the experimental data when the force is measured by a balance. Furthermore, students who understand the Excel columns usually have no trouble in constructing the integral for a continuous charge configuration.

## Bibliography

1. J. A. McNeil, "Engineering physics: the universal donor program", Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition
2. R. L. Miller, R. A. Streveler, M. A. Nelson, M. R. Geist and B. M. Olds "Concept inventories meet cognitive psychology: using beta testing as a mechanism for identifying engineering student misconceptions, " Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition
3. M. Reiner, J. D. Slotta, M. Chi and L. B. Resnick "Naïve physics reasoning: a commitment to substance-based conceptions," Cognition and Instruction, Vol. 18, pp1-43, 2000
4. S. M. Jeter, "A hand tool for convenient error propagation analysis: a user form for error influence coefficient", Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition

5. F. A. Di Bella and A. Chamarro, III, "Optimizing thermal energy storage for cogeneration applications: a faculty and engineering technology student collaboration using Excel", Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition
6. N.N. Sarker, and M. A. Ketkar, "Developing Excel macros for solving heat diffusion Problems", Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition
7. Sheldon Jeter "Application of error propagation analysis to the uncertainties of regression models in experimental thermal and fluids engineering", Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition
8. D. Elizandro and J. Matson, "Discrete event simulation using Excel/VBA", Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition
9. M. A. Collura, B. Aliane, S. Daniels, and J. Nocito-Gobel, "Learning the methods of engineering analysis using case studies, Excel and VBA- course design", Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition
10. Web address: [ligo.caltech.edu](http://ligo.caltech.edu)
11. Queensborough Community College Physics Department Engineering Physics Lab Manual 2005.