

# The Signals and Systems Concept Inventory

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## Abstract

This paper describes the development of continuous-time and discrete-time signals and systems concept inventory exams for undergraduate electrical engineering curricula. Both exams have twenty-five multiple choice questions to assess students' understanding of core concepts in these courses. The questions require little or no computation, and contain incorrect answers that capture common student misconceptions. The design of both exams is discussed, as are ongoing studies evaluating the exams at four campuses. Preliminary results from administering the continuous-time exam as a pre-test and post-test indicate a normalized gain of  $0.24 \pm 0.08$  for traditional lecture courses, consistent with reported results for the Force Concept Inventory exam in lecture courses for freshman physics.<sup>1</sup>

## 1 Introduction

Faculty have expressed a strong need to quantitatively assess the "amount" of learning achieved by students taking a particular class. Having the ability to do so would allow more reliable assessment and comparison of various teaching methods. One measurement technique is a concept inventory exam given at the start and end of a course (or sequence of courses).

This paper describes the design of concept inventory exams for undergraduate signals and systems classes. Several current pedagogical issues motivate the need for signals and systems concept inventory (SSCI) exams. First, the signal processing community is debating the appropriate order to present discrete-time (DT) and continuous-time (CT) signals and systems.<sup>2,3</sup> Second, many engineering educators are evaluating pedagogical choices such as lecture vs. studio classrooms, and collaborative vs. individual learning. Surveys and course grades are important assessment techniques for evaluating the benefit of these alternatives, but there is clearly a need for a quantitative measure of student performance that can be used for inter-campus comparisons. Third, the advent of ABET Engineering Criteria 2000,<sup>4</sup> which requires all programs to implement an assessment feedback cycle, further amplifies the need for objective assessment techniques throughout all undergraduate engineering curricula.

One of our models in developing the SSCI exams was the Force Concept Inventory (FCI), which spurred recent reform in and debate about the freshman physics curriculum.<sup>1,5</sup> The FCI is a multiple choice exam covering Newtonian physics. The exam includes carefully crafted incorrect answers that encompass many common student misconceptions about fundamental concepts. The questions incorporate visual diagrams and everyday situations to emphasize conceptual understanding over mathematical manipulation.

Inspired by all of these factors, as well as a grant from the NSF-funded Foundation Coalition, we set out to develop both CT and DT SSCI exams. We sought to write exams covering the core concepts of signals and systems in a manner emphasizing conceptual understanding over computational mechanics. This paper reports preliminary results from studies for both exams. The following section describes the development of the SSCI exams, including inventories of the core concepts assessed by each exam. Section 3 describes the design of the questions on the SSCI exams, including eight example questions taken directly from the exams. Section 4 outlines the studies now underway to evaluate the current versions of the exams. Section 5 presents the results from these studies with the data collected to date. Lastly, Section 6 summarizes our findings thus far and describes our future plans.

## 2 Exam Development

Signals and systems is typically taught in the late sophomore or early junior year. Representative texts for this subject include the books by Oppenheim and Willsky with Nawab,<sup>6</sup> and by Lathi.<sup>7</sup> As indicated in the introduction, there are several ways to organize the signals and systems material. One approach is to present continuous-time topics first, followed by discrete-time topics; a second approach reverses that order; and a third approach presents continuous- and discrete-time topics in parallel. Depending on the curriculum, signals and systems may be taught in one course or distributed over several courses.

Regardless of the pedagogical organization, introductory courses focus almost exclusively on linear, time-invariant (LTI) systems, with the primary application being filtering. LTI analysis techniques, such as convolution and Fourier, Laplace, and  $z$  transforms, constitute

much of the curriculum. Sampling is the key concept that links the continuous and discrete time domains. The study of all these topics requires a certain level of mathematical sophistication. At a minimum, students must be familiar with basic signals such as sinusoids and unit step functions, and must be capable of simple signal manipulations such as amplitude scaling, time shifting, and time reversal. In designing the SSCI exams, we grouped the core concepts in signals and systems into six categories: background mathematical concepts, linearity and time-invariance, convolution, transform representations, filtering, and sampling. Sections 2.1 and 2.2 below describe the development of the continuous and discrete-time exams, respectively, and document the concepts covered by each exam.

## 2.1 CT SSCI Development

We began developing the continuous-time version of the SSCI in late 2000, and produced an initial draft in January of 2001. Version 1.0 of the CT SSCI consisted of 30 multiple choice questions on the topics outlined above. During spring 2001 we administered the exam to 128 students at George Mason University and the University of Massachusetts Dartmouth. The test population consisted of undergraduate and graduate students from six courses in the areas of linear systems, signal processing, and communications. We had two primary goals for the alpha-testing phase: to examine the clarity and appropriateness of the questions and to investigate which alternate (distractor) answers were most attractive to the students. For each question we asked students to select one of five prescribed choices or to fill in a response of their own, allowing us to capture novel distractors caused by unanticipated conceptual confusions.

The initial round of testing indicated that, while the questions were generally clear\*, the exam was too long and too difficult. Most students struggled to finish within the one-hour proposed time limit, and the mean score was 29.5/100. The difficulty of the exam is further illustrated by the fact that 87% of students scored below 40/100. Regarding the distractor analysis, Version 1.0 appeared to capture almost all common misconceptions since few students gave a solution different from the prescribed alternatives.

Based on the alpha-test results, we revised the CT SSCI during summer 2001 and made several important changes. First, we added several new questions that address the mathematical background knowledge required for the study of signals and systems. Second, we used the results of the distractor analysis to eliminate the least common alternate answers, resulting in four choices for each question. Finally, we reduced the total number of questions to 25 by focusing the exam on the most basic concepts. Table 1 shows the core concepts covered by CT SSCI version 2.0. The lists in parentheses next to each of the five main topics contain the question numbers in this version of the exam that address that topic. Note that some questions cover more than one concept.

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\*Requests for clarification of the questions were minimal.

- Background mathematical concepts (1-4, 14)
  - basic signals, *e.g.*, sinusoids and unit step functions
  - basic signal manipulations, *e.g.*, amplitude scaling, time shifting, time reversal
  - forms of the solutions to linear, constant-coefficient differential equations
- Linearity and time invariance (5, 23, 24)
- Convolution (8, 12, 15, 23)
  - mechanics
  - commutative and distributive properties
  - relationship of impulse response and causality
- Fourier and Laplace transform representations (6, 7, 9-11, 13, 15-22, 25)
  - Fourier series
  - connection between time and frequency domain properties of a signal
  - Fourier transform properties and theorems including linearity, conjugate symmetry, delay theorem, and modulation theorem
  - effect of the poles and zeros of a system function on the frequency response, impulse response, and stability of causal systems
- Filtering with LTI systems (6, 25)
  - of infinite-extent sinusoids
  - of narrowband pulses

Table 1: CT Signals and Systems Concept Inventory list.

## 2.2 DT SSCI Development

Using the continuous-time exam as a model, we began developing a discrete-time version of the SSCI during spring 2001. We produced an initial draft by writing DT versions of each of the 30 questions on version 1.0 of the CT SSCI. As we revised the CT exam questions (based on the alpha-testing results), we made similar revisions to their discrete-time counterparts. In addition, we wrote three new questions that address concepts central to DT signals and systems that are not covered by the CT exam, such as sampling. From this pool of questions, we selected 25 for version 1.0 of the DT SSCI. Table 2 shows the list of fundamental concepts covered by the DT exam. The DT SSCI contains two sampling questions and an additional background mathematics question that the CT exam does not have. To include these questions without increasing the total number of questions beyond 25, we had to eliminate the DT counterparts of several CT questions. Specifically, we eliminated two

questions related to the modulation theorem and one question involving Bode plots, which we felt were relatively less important in discrete time than the new questions. We completed version 1.0 of the DT SSCI in time to begin alpha-testing in fall 2001. Similar to the alpha version of the CT exam, the alpha version of each DT question asks students to select one of five prescribed choices or to fill in their own answer. This provides the opportunity for students to provide unanticipated distractors.

### 3 Question Design

The goal of the SSCI is to assess conceptual understanding rather than computational skills. Several studies of pedagogical techniques in freshman physics courses found that courses focusing on conceptual understanding also improve students' problem solving skills.<sup>1,8</sup> In designing questions to probe the concepts outlined in the previous section, we considered several important issues: notational conventions, the relative merits of single-concept vs. synthesis questions, presentation modality (whether questions are posed using words, figures, and/or equations), and the types of reasoning required. The four following paragraphs briefly describe each of these issues, and Sections 3.1 and 3.2 illustrate how these issues were addressed during the development of the SSCI by presenting questions from the CT and DT exams, respectively.

Regarding notation, the most important variable is frequency. Some textbooks use radian frequency,<sup>6</sup> while others use hertz.<sup>9</sup> Our goal was to design an exam that could be used with either notational convention. The SSCI uses radian frequency, however the distractors do not distinguish between radians and Hz. In other words, incorrectly scaling frequency by a factor of  $2\pi$  should never cause the student to pick the wrong answer. The SSCI denotes the continuous-time Fourier transform by  $X(j\omega)$  and the discrete-time Fourier transform by  $X(e^{j\omega})$ , which is consistent with notation used in the text by Oppenheim, Willsky, with Nawab.<sup>6</sup> These notational conventions are clearly stated on the cover pages of both the CT and DT exams.

The second issue we considered in designing the SSCI is whether to include questions that probe multiple concepts. Single-concept questions have the advantage that they provide a clear measure of what a student does and does not understand. Questions involving multiple concepts are more difficult to interpret, but need to be included because some core signals and systems topics inherently require the synthesis of several concepts. For example, to understand frequency-selective filtering, students must understand sinusoidal signals, LTI processing, and the relationship between the time and frequency domains. We designed four linked questions, including one synthesis question, to explore these filtering-related concepts. The linked questions are discussed further in Section 3.1 below. In addition to the filtering synthesis question, each of the SSCI exams has two other synthesis questions. The first of these deals with impulse responses, parallel and cascade system interconnections, and causality, and the second question examines the concepts of linearity and time-invariance.

- Background mathematical concepts (1-5, 8, 16)
  - basic signals, *e.g.*, sinusoids and unit step sequences
  - basic signal manipulations, *e.g.*, amplitude scaling, time shifting, time reversal
  - forms of the solutions to linear, constant-coefficient difference equations
- Linearity and time invariance (6, 23, 24)
- Convolution (11, 14, 23)
  - mechanics
  - commutative and distributive properties
  - relationship of impulse response and causality
- Fourier and  $z$  transform representations (9, 10, 12, 13, 15, 17-22, 25)
  - Fourier series
  - connection between time and frequency domain properties of a signal
  - Fourier transform properties and theorems including linearity, conjugate symmetry, and delay theorem
  - effect of the poles and zeros of a system function on the frequency response, impulse response, and stability of causal systems
- Filtering with LTI systems (9, 25)
  - of infinite-extent sinusoids
  - of narrowband pulses
- Sampling (7, 8)
  - mechanics
  - Nyquist criteria

Table 2: DT Signals and Systems Concept Inventory list.

The third issue is the format used to present information in the question statements. In designing these exams, we used three modalities to convey information: words, figures, and equations. Twenty-four of the 25 questions on both the CT and DT exams use graphs, block diagrams, and other plots as a part of the question statement. Twelve of 25 questions on the CT exam and 13 of 25 questions on the DT exam use equations or other mathematical expressions. The SSCI relies more heavily on figures than equations because it emphasizes conceptual understanding over computation.

Finally, the fourth issue is the type of reasoning students must use to answer the questions. We classified questions on the SSCI as requiring one of two types of reasoning skills: “forward” and “reverse.” Consider a simple example with a relationship among three pieces of information  $a$ ,  $b$ , and  $c$ . Suppose that the typical textbook or classroom presentation of the relationship provides  $a$  and  $b$  then demonstrates how to find  $c$ . We would consider a problem providing  $a$  and  $b$  and asking the students to find  $c$  an example of a problem requiring forward reasoning. In a reverse reasoning problem, the students might be given  $a$  and  $c$  and be asked to find  $b$ . This relationship might be as simple as an equation  $ab = c$ , or might be more general where  $a$ ,  $b$  and  $c$  are figures or facts describing a signal or system. We often found that students who do not understand a concept can correctly answer a forward question by rote, but that they reveal their lack of understanding when they try to answer a permuted version of that same question. Reverse reasoning skills are very important in engineering because they are often required to solve design problems, *e.g.*, to design the appropriate frequency response for a filter that will produce a desired output for a given input. Three questions on the CT SSCI and four questions on the DT SSCI require reverse reasoning.

### 3.1 CT SSCI Questions

To illustrate the exam design issues and to highlight what we learned during the SSCI’s development, this section and the next provide example questions from the CT and DT exams. For the CT exam, we consider the four linked questions mentioned above. Figure 1 shows the filtering synthesis question. In this question, students are asked to determine what happens to a signal  $x(t)$  containing two narrowband pulses as it passes through a lowpass filter. The students are given a plot of  $x(t)$ , its corresponding Fourier transform magnitude  $|X(j\omega)|$ , and a plot of the Fourier transform magnitude  $|H(j\omega)|$  of the filter. The question asks which of the plots in Figure 1(d) could be the output of the filter. To answer this question, students must: (1) be able to distinguish between high and low sinusoidal frequencies, (2) understand that LTI processing corresponds to a multiplication of transforms, and (3) be able to relate the time- and frequency-domain representations of a signal.

The filtering question described above was the first question on version 1.0 of the CT SSCI.<sup>†</sup> Alpha-test results for this question were very poor: less than 28% of the 128 students tested

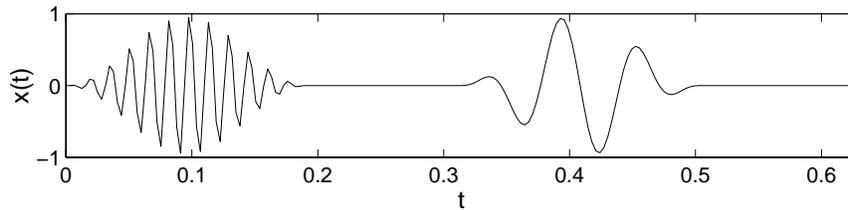
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<sup>†</sup>In version 1.0 this question had two additional distractors,  $y = 0$  and  $y = 0.5y_a(t)$ , which were subsequently deleted due to low response rates.

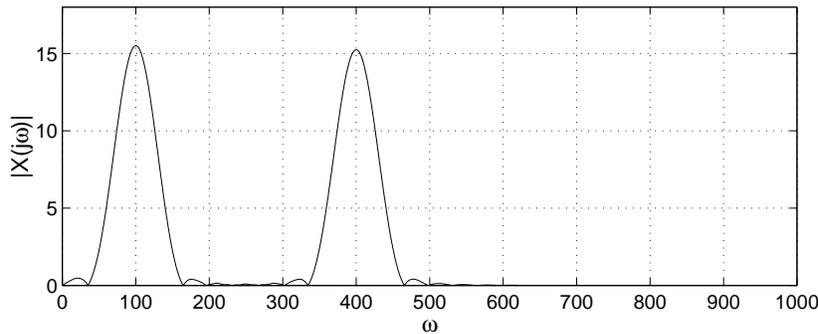
**Question 25**

Consider a real, continuous-time signal  $x(t)$ , which contains two narrowband pulses (windowed sinusoids). Figures 1(a) and 1(b) below depict  $x(t)$  and its Fourier transform magnitude  $|X(j\omega)|$ . The signal  $x(t)$  is the input to a real LTI filter with the frequency response magnitude  $|H(j\omega)|$ , shown in Figure 1(c). Figure 1(d) on the next page shows four possible output signals  $y_a(t)$  through  $y_d(t)$ . Which of these four signals could be the output of the filter in Figure 1(c) when  $x(t)$  is the input?

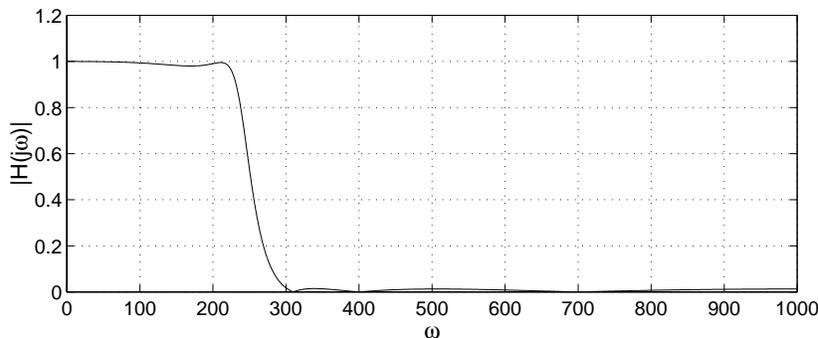
- (a)  $y_a(t)$                       (b)  $y_b(t)$                       (c)  $y_c(t)$                       (d)  $y_d(t)$



(a) Input signal  $x(t)$  for Question 25.

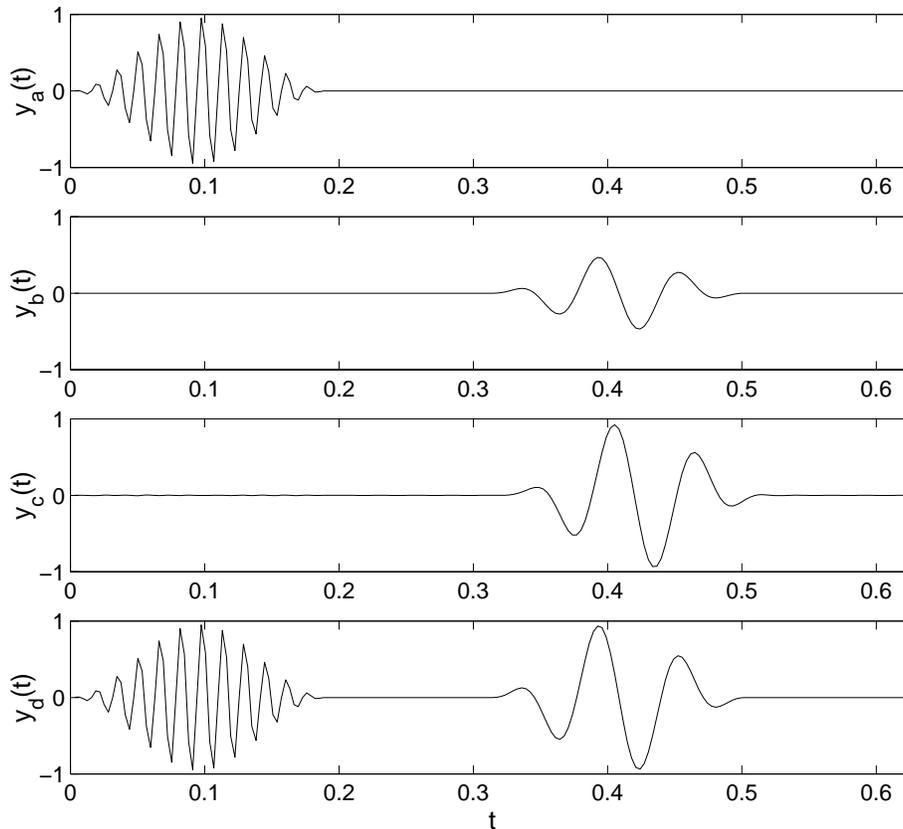


(b) Input signal spectrum magnitude  $|X(j\omega)|$  for Question 25.



(c) Filter magnitude response  $|H(j\omega)|$  for Question 25.

Figure 1: CT-SSCI Question 25.



(d) Output signals  $y_a(t)$  through  $y_d(t)$  for Question 25.

Figure 1: CT-SSCI Question 25.

answered the question correctly. Many (39%) of the students confused the low- and high-frequency pulses and chose  $y_a(t)$ , presumably due to the fact that they associated the high-frequency pulse, which is on the left side of the time plot, with the low-frequency spectral components on the left of the frequency plot.

To analyze student misconceptions about filtering, we drafted three simpler questions for version 2.0 of the CT SSCI. These questions test the three component concepts (enumerated above) of the filtering synthesis question. The first of the new questions is CT SSCI Question 1 shown in Figure 2. This question shows plots of four sinusoids and asks which has the highest frequency. Distractors probe whether the students confuse high frequency with high amplitude or large period. The second new question (CT SSCI Question 6 shown in Figure 3) considers the filtering of infinite-extent sinusoids with an LTI system. Students are given plots of the magnitude and phase of a lowpass filter and asked to determine the output when the input is a cosine. The third new question explores time-frequency relationships. This question (CT SSCI Question 7) is shown in Figure 4. Students are given a plot of the time signal and the corresponding Fourier transform magnitude for one narrowband pulse. They are shown the time signal plot for a second narrowband pulse of higher frequency and asked to determine which of the choices in Figure 4(b) is a plot of the Fourier transform

magnitude for the higher frequency pulse. In addition to these three new questions, we retained the synthesis question as the last question (Question 25) on version 2.0.

In terms of presentation style, Questions 1, 6, 7, and 25 are typical of the questions on both SSCI exams. As noted above, both SSCIs use a combination of words, figures, and equations in the question statements. CT Question 6 (Figure 3) is an example of using all three modalities. The question describes the system (input and output) both in words and with a block diagram. The system frequency response is given as a plot of magnitude and phase, and the choices are specified using mathematical expressions. We would characterize all four of these problems as forward reasoning problems.

### 3.2 DT SSCI Questions

As indicated in Section 2, many of the questions on the DT exam parallel those on the CT exam. The four linked questions described above are also included on the DT SSCI. While the DT questions are very similar to their CT counterparts, there are some subtle differences, particularly for the question about sinusoidal frequency (CT Question 1, Figure 2). Figure 5 shows the DT version of this question. In discrete time, radian frequency is periodic with period  $2\pi$ , and  $\pi$  is the highest frequency possible. DT Question 1 shows five sinusoids and asks which has the highest frequency. The question probes whether students can identify the sinusoid  $\cos(\pi n)$  as having the highest frequency. Distractors include three signals that have obvious sinusoidal shapes (*i.e.*, they look like finely-sampled analog waveforms) and one signal with a frequency lower than  $\pi$  that looks less like a “typical” sampled sinusoid.

To further assess students’ understanding of DT sinusoidal frequency, an additional math background question was written for the DT SSCI. Question 5 shown in Figure 6 probes whether students know that the DT frequency variable is periodic with period  $2\pi$ . The question shows a plot of a signal  $\cos(\omega_0 n)$  and asks students choose the signal  $\cos((\omega_0 + 2\pi)n)$  from among five alternatives. Distractors for this question include cosine signals with larger amplitude, lower frequency, higher frequency, and with a phase change of  $\pi/2$ .

There are two other questions on the DT SSCI that have no CT counterparts. These are the questions about the concept of sampling. Figure 7 is the first sampling problem (DT Question 7). This question probes whether students understand the basic mechanics of the sampling process. They are told that the signal  $x_c(t) = \sin(2\pi(3)t)$  is sampled every  $T$  seconds to produce the DT signal shown in Figure 7(a) and are asked to determine the value of  $T$ . This is an example of a question that requires reverse reasoning, *i.e.*, students must determine a quantity ( $T$ ) that is one of the given quantities in typical homework problems.

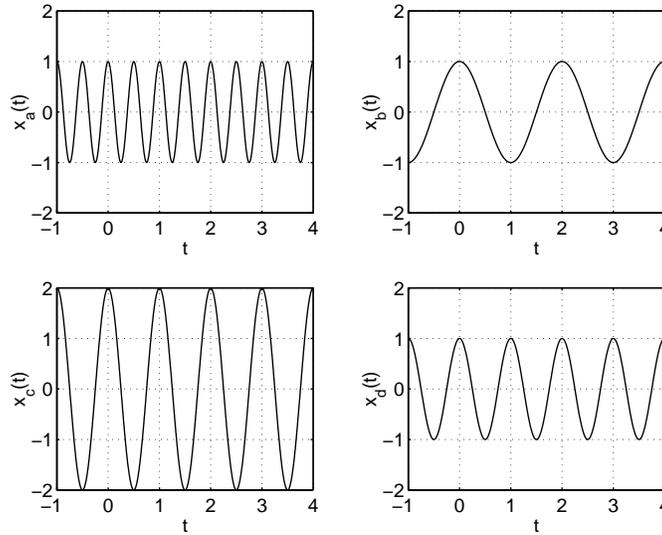
The second sampling question on the DT SSCI, shown in Figure 8 focuses on the Nyquist theorem. DT Question 8 asks students to determine which of the five sinusoids shown in the plot could be sampled at a rate of 5 Hz without aliasing.<sup>‡</sup> This question requires students to

<sup>‡</sup>There is a conflict in the use of subscripts in this question. Specifically,  $x_c(t)$  is used to denote both the

**Question 1**

Figure 2(a) shows four signals  $x_a(t)$  through  $x_d(t)$ , all on the same time and amplitude scale. Which signal has the highest frequency?

- (a)  $x_a(t)$                       (b)  $x_b(t)$                       (c)  $x_c(t)$                       (d)  $x_d(t)$

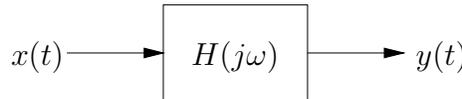


(a) Signals  $x_a(t)$  through  $x_d(t)$  for Question 1.

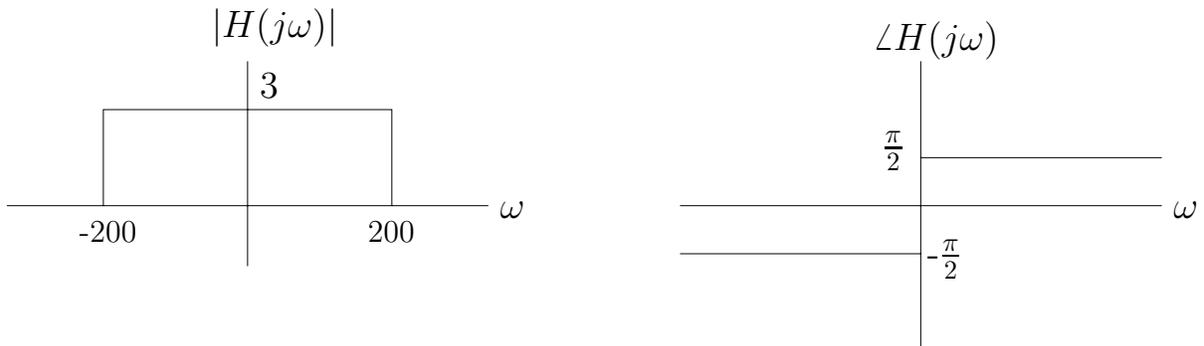
Figure 2: CT-SSCI Question 1.

**Question 6**

Consider the system with input  $x(t)$  and output  $y(t)$  shown in Figure 3(a). The magnitude and phase response (in radians) of the system are shown in Figure 3(b).



(a) System for Question 6.



(b) Magnitude and phase response of the system in Question 6.

Suppose that the input  $x(t) = \cos(50t)$  for all time. What is the output  $y(t)$ ?

- (a)  $3 \cos(50t + \frac{\pi}{2})$               (b)  $\cos(50t + \frac{\pi}{2})$               (c)  $3 \cos(50t)$               (d)  $3 \cos(200t)$

Figure 3: CT-SSCI Question 6.

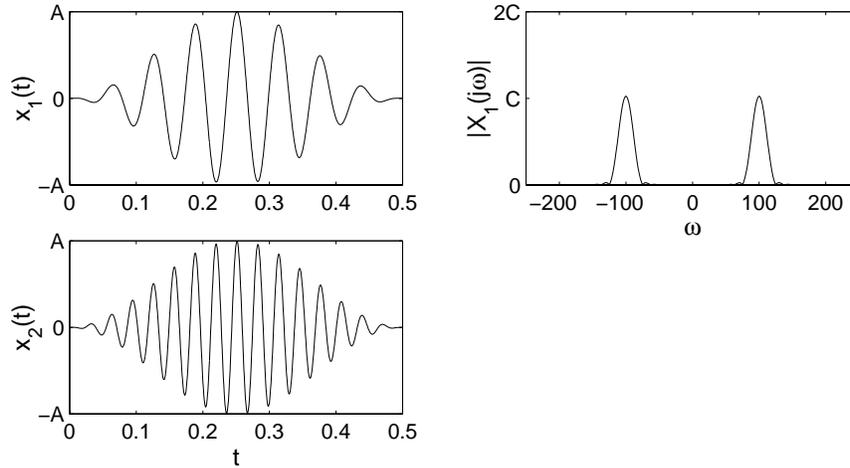
determine the frequency of each sinusoid by reading the plots and to understand the Nyquist sampling criteria. It is another example of a question that uses reverse reasoning because a typical Nyquist question would ask students to determine the minimum required sampling frequency for a single given signal.

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continuous-time signal and the third choice. We detected this conflict after version 1.0 was administered in fall 2001. Analysis of the fall exams indicates that the conflicting wording was not a major factor in determining students' answers to this question. Of the 53 students who answered DT Question 8, three chose (a), six chose (b), eight chose (c), 30 chose the correct answer (d), three chose (e), and three chose (f). As part of a survey, three of the students who answered (c) were later asked to justify their answers. The justifications they provided indicate that their choice of (c) was based on a conceptual misunderstanding rather than on the wording of the question. We will correct the wording of DT Question 8 prior to administering the exam in spring 2002.

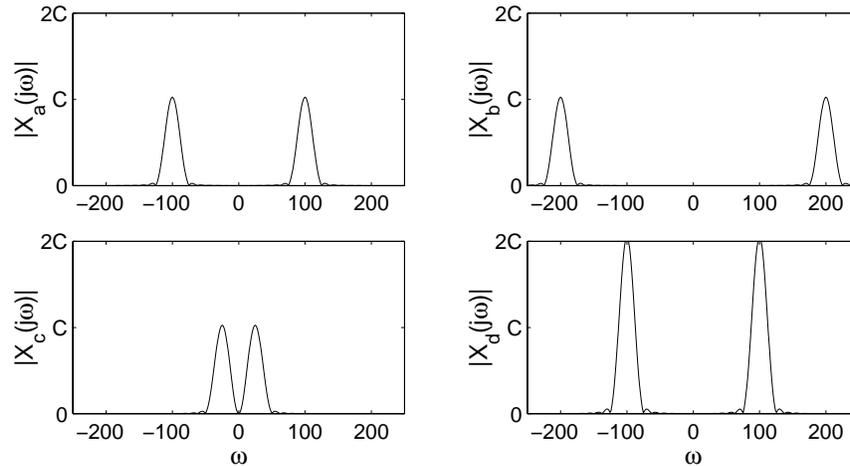
**Question 7**

Signals  $x_1(t)$  and  $x_2(t)$  are shown on the left hand side of Figure 4(a). The Fourier transform magnitude,  $|X_1(j\omega)|$ , for signal  $x_1(t)$  is shown on the right side of the figure.



(a) Signals  $x_1(t)$  and  $x_2(t)$  and the Fourier transform magnitude  $|X_1(j\omega)|$  for Question 7.

Which of the plots shown in Figure 4(b) could be  $|X_2(j\omega)|$ , the Fourier transform magnitude for signal  $x_2(t)$ ?



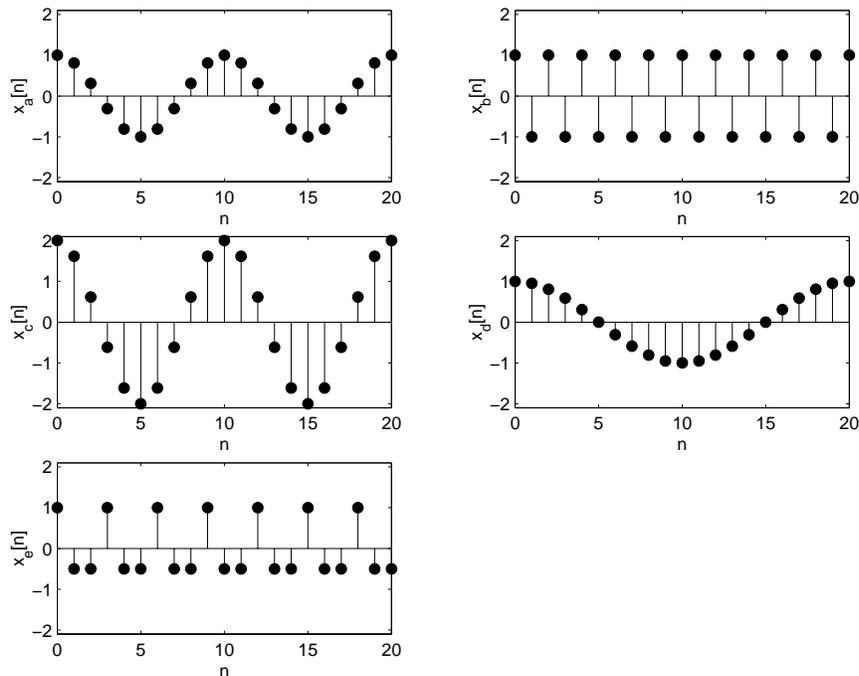
(b) Fourier transform magnitudes  $|X_a(j\omega)|$  through  $|X_d(j\omega)|$  for Question 7.

Figure 4: CT-SSCI Question 7.

**Question 1**

Figure 5(a) shows five discrete-time signals  $x_a[n]$  through  $x_e[n]$ , all on the same time and amplitude scale. Each of these signals has the form  $A \cos(\omega_0 n)$  with  $-\pi < \omega_0 \leq \pi$ . Which signal has the highest frequency  $\omega_0$ ?

- (a)  $x_a[n]$       (b)  $x_b[n]$       (c)  $x_c[n]$       (d)  $x_d[n]$       (e)  $x_e[n]$       (f) Other, please specify.

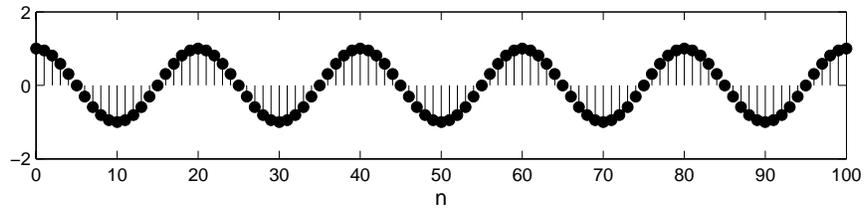


(a) Signals  $x_a[n]$  through  $x_e[n]$  for Question 1.

Figure 5: DT-SSCI Question 1.

### Question 5

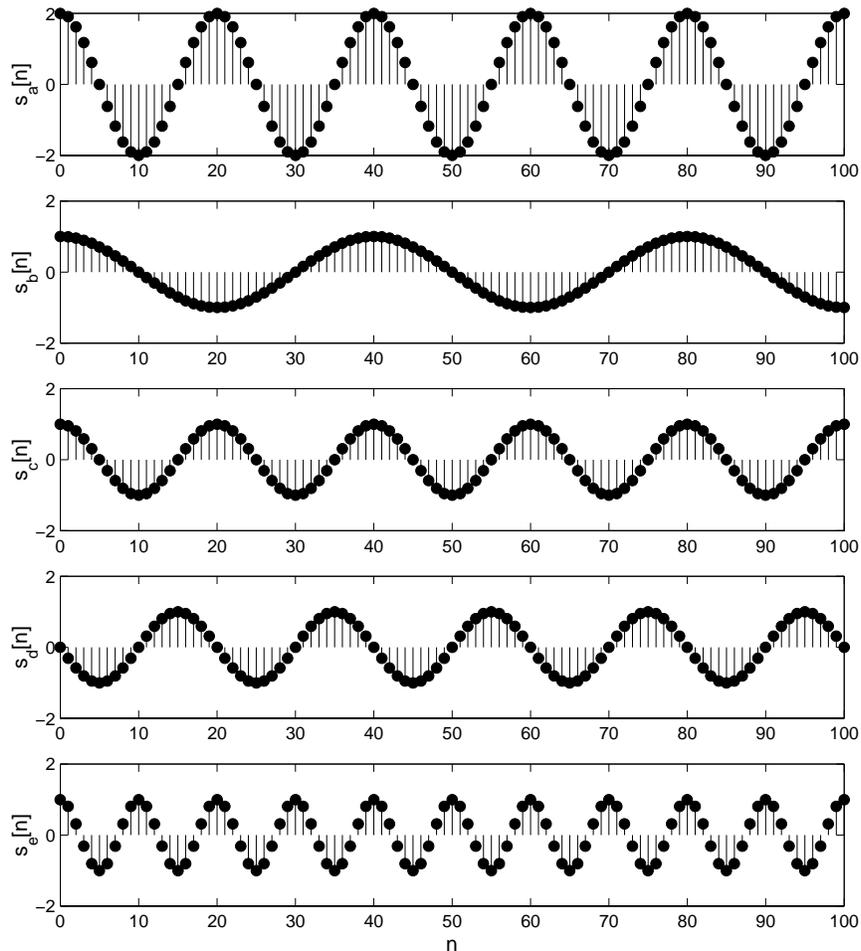
Figure 6(a) is a plot of the signal  $\cos(\omega_0 n)$ .



(a) Plot of  $\cos(\omega_0 n)$  for Question 5.

Which of the following signals is  $\cos((\omega_0 + 2\pi)n)$ ?

- (a)  $s_a[n]$     (b)  $s_b[n]$     (c)  $s_c[n]$     (d)  $s_d[n]$     (e)  $s_e[n]$     (f) Other, please specify.



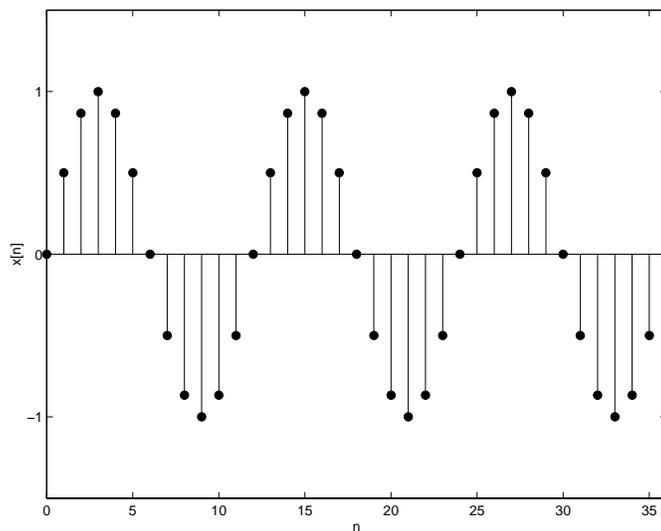
(b) Signals  $s_a[n]$  through  $s_e[n]$  for Question 5.

Figure 6: DT-SSCI Question 5.

### Question 7

A continuous-time signal  $x_c(t) = \sin(2\pi(3)t)$  is sampled every  $T$  seconds to produce the discrete-time signal  $x[n] = x_c(nT)$  shown in Figure 7(a). Which of the following choices of  $T$  would give the signal  $x[n]$  shown below?

- (a)  $1/72$       (b)  $1/6$       (c)  $1/12$       (d)  $1/18$       (e)  $1/36$       (f) Other, please specify.



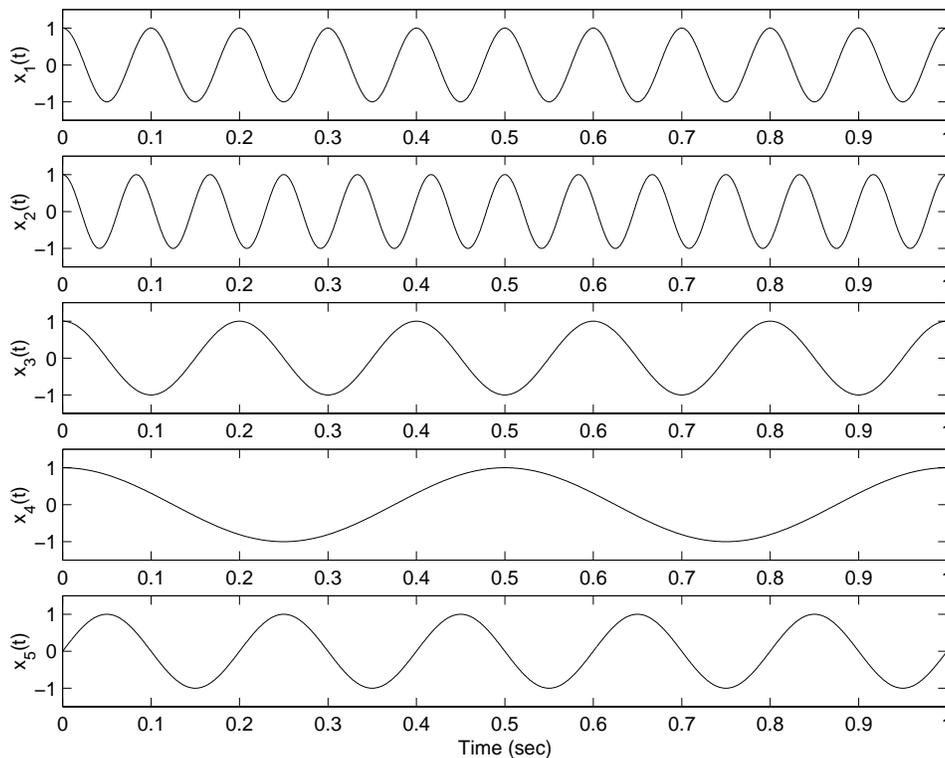
(a) Signal  $x[n]$  for Question 7.

Figure 7: DT-SSCI Question 7.

### Question 8

A continuous-time signal  $x_c(t)$  is sampled at sampling frequency  $f_s = 1/T = 5$  Hz to obtain a discrete-time signal  $x[n] = x_c(nT)$ . Which of the continuous-time sinusoids in Figure 8(a) could be sampled at this rate without aliasing?

- (a)  $x_a(t)$       (b)  $x_b(t)$       (c)  $x_c(t)$       (d)  $x_d(t)$       (e)  $x_e(t)$       (f) None of the above.



(a) Signals  $x_a(t)$  through  $x_e(t)$  for Question 8.

Figure 8: DT-SSCI Question 8.

## 4 Current Study

This section describes the studies currently underway to develop and refine both the CT SSCI and the DT SSCI. The CT SSCI exam is more mature and is roughly a year ahead of the DT SSCI in its development cycle. Consequently, the study for the CT SSCI is relatively sophisticated, looking to duplicate previous concept inventory exam evaluations published in Hake's survey of the FCI.<sup>1</sup> The DT SSCI study focuses on simpler goals of verifying that the question wordings are clear and determining which incorrect answers capture common student confusions.

### 4.1 CT SSCI Study

Version 2.0 of the CT SSCI is currently being tested at four schools: George Mason University (GMU), the U.S. Air Force Academy (USAFA), the U.S. Naval Academy (USNA) and the University of Massachusetts Dartmouth (UMD). Table 3 summarizes the curricular context of continuous-time signals and systems for each of the schools. Instructors are administering the CT SSCI exam as a pre-test and a post-test for the CT portion of their curriculum. Motivated by Hake's survey of the FCI,<sup>1</sup> we will compute the normalized gain  $g$  for each student, as well as normalized gains for each course (based on the average pre-test and post-test scores for each course). The normalized gain is defined as

$$g = \frac{\text{posttest-pretest}}{100\text{-pretest}},$$

*i.e.*, the fraction of the available improvement in score that was obtained during the course or courses. In analyzing the FCI, Hake has shown that normalized gain is a stable performance measure for courses that have similar pedagogical formats, regardless of variations in student background or instructor experience.<sup>1</sup>

Outside of the classroom and independent of the testing, the following data is being collected about each student participating in the study: class year, GPA, calculus grades, differential equations grade, circuits grade (where relevant), race, and gender. To protect student privacy, the academic and demographic data are linked to the exam scores through anonymous study IDs.

Course	Year	Circuits Course	DT/CT First	Text Used
GMU ECE 220	Soph 2	Co-Req	DT	<sup>9</sup>
USAFA EE 332, 333	Jr 1&2	Pre-req	CT	<sup>9</sup>
USNA EE 322	Jr 1	Pre-req	CT	<sup>6</sup>
UMD ECE 321	Jr 1	Pre-req	CT	<sup>7</sup>

Table 3: Curricular contexts of CT SSCI study courses.

The current study of the CT SSCI has four goals:

1. To gather information for ongoing refinement of the questions and the concept inventory list,
2. To begin to establish a baseline for the gain statistic on the exam, similar to Hake's results for the FCI,<sup>1</sup>
3. To investigate whether the exam has race or gender biases,
4. To examine the extent to which previous academic performance is a predictor of success on the CT SSCI exam.

This paper presents results from goals (1) and (2) in Section 5.1. Data for goals (3) and (4) are still being collected and analyzed, and will be reported in future publications.

## 4.2 DT SSCI Study

Version 1.0 of the DT SSCI is currently being tested at UMD and GMU. The goals of the DT exam study are more limited than for the CT exam study. Specifically, the DT SSCI study seeks to gather information for ongoing refinement of the questions and the DT concept inventory list, similar to what was done during the alpha-tests of the CT SSCI described in Section 2.1. In fall 2001, the DT exam was administered once during the semester to students in three different courses. Table 4 summarizes the curricular context of these courses, including the level (undergraduate or graduate), whether the DT SSCI was used as a pre-test or post-test, and the textbooks used.

In addition to the exam data, GMU ECE 410 also collected short answer justifications from students for Questions 7, 8, 12, 23 and 25 on the DT SSCI 1.0. This student input will be used to analyze common misconceptions for these questions. GMU ECE 320 will administer the exam to another set of students in spring 2002. We will revise the DT SSCI exam based on the statistical analyses of students' responses and the GMU justifications. This revised exam will be used starting in fall 2002 for a study with the same methodology and goals as the CT SSCI study in Section 4.1. Initial results of the DT SSCI are presented in Section 5.2.

Course	Year	Material	Pre/Post	Text
GMU ECE 320	Jr 1	DT Signals & Systems	Post	<sup>9</sup>
GMU ECE 410	Sr 1	DSP	Post	<sup>10</sup>
UMD ECE 574	Grad	DSP	Pre	<sup>11</sup>

Table 4: Curricular contexts of DT SSCI study courses.

## 5 Results

Sections 5.1 and 5.2 below report the results to date from the ongoing CT SSCI and DT SSCI studies described in the previous section

### 5.1 CT SSCI Results

This section reports the analysis of results for 97 students at GMU, UMD, and the USNA who took the CT SSCI as a pre- and post-test during the fall 2001 semester. We computed the normalized gain, denoted  $\langle g \rangle$  by Hake,<sup>1,12</sup> for each course based on the average pre-test and post-test scores of those students who took the exam both times. Table 5 reports the results for the pre-test, post-test, and gain for each campus. The values on this table are reported on a scale where 100 is a perfect score (25 correct answers) on the exam. The average gain over the three classes was  $0.24 \pm 0.08$  (std dev). This value is consistent with the FCI gain of  $0.23 \pm 0.04$  reported in Hake's survey<sup>1</sup> for "traditional" courses, in contrast with "interactive-engagement" (IE) courses, which have gains of  $0.48 \pm 0.14$  (std dev). All three of the signals and systems courses using the CT SSCI are considered traditional courses in Hake's characterization, since they make "little or no use of IE methods, relying primarily on passive-student lectures".<sup>1</sup>

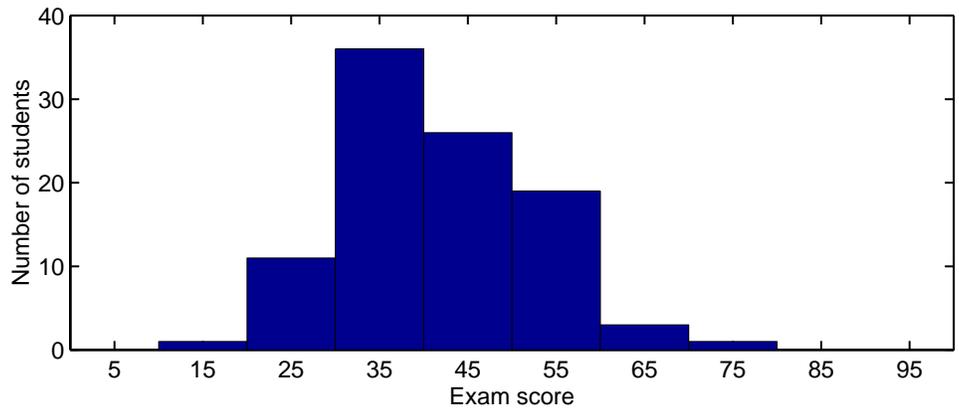
Campus	Students	Pre-Test		Post-Test		Gain $g$
		Mean	Std. Dev.	Mean	Std. Dev.	
GMU	43	40.4	9.6	49.6	9.9	0.15
UMD	13	44.3	13.1	60.0	15.5	0.28
USNA	41	44.7	11.0	60.1	11.3	0.28

Table 5: Results of fall 2001 CT SSCI study by campus.

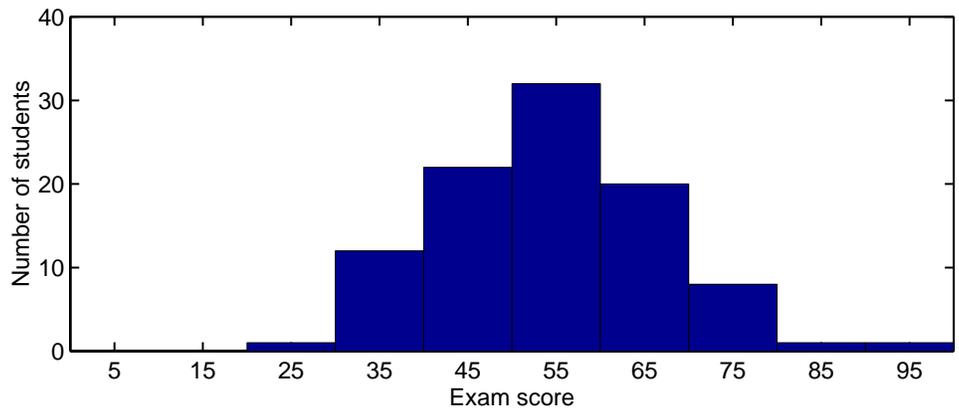
Additionally, 29 USAFA students took the pre-test during this semester, but have not yet taken the post-test. The USAFA curriculum divides the signals and systems material across semester boundaries, so these students will take the post-test after finishing the signals and systems material during the Spring 2002 semester. These results will be incorporated into this analysis and reported in future publications.

The CT SSCI study data were also analyzed by pooling all students across all campuses. Since all courses were traditional courses, and covered very similar syllabi, this appears well-founded. Figure 9 plots the histograms of the pre-test and post-test scores for all students taking the exam both times. The improvement in the students' conceptual understanding during the semester is clearly visible from this data.

Figure 10 plots a histogram of the normalized gain  $g$  for individual students. The gains are broadly spread, ranging from  $-0.43$  to  $+0.80$ . Fourteen students actually had negative gain, with post-test scores less than their pre-test scores. As suggested by Hake,<sup>12</sup> we also



(a) Pre-test score histogram.



(b) Post-test score histogram.

Figure 9: CT SSCI pre-test and post-test histograms.

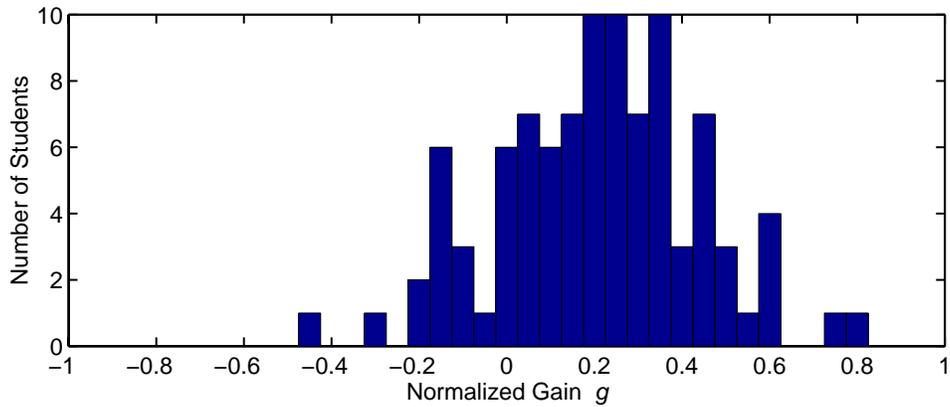


Figure 10: CT SSCI histogram of individual students normalized gains  $g$ .

analyzed the standard deviation of the gains within each class to obtain an estimate of the random error of each population, which is associated with the spread. Table 6 presents the standard deviation of individual student's  $g$  for each course, and indicates that the average standard deviation over all three classes is  $0.21 \pm 0.04$ . This random error of 0.21 exceeds the error of 0.08 calculated for the gain of  $0.24 \pm 0.08$  from Table 5, which suggests that random errors within the student population dominate any systematic errors between the courses.

Campus	Std. Dev $g$
GMU	0.19
UMD	0.19
USNA	0.26
Mean	$0.21 \pm 0.04$ (sd)

Table 6: CT SSCI random error analysis results.

Figure 11 shows scatter plots of individual student performance on the exams. The left panel, Figure 11(a), represents each student as a dot using the student's pre-test score as the abscissa and their post-test score as the ordinate. Note that there are fewer than 97 dots on the graph since some students obtained identical pre- and post-test scores. Almost all student scores fall above the diagonal solid line representing equal pre-test and post-test scores. Therefore, almost all students performed better on the post-test than the pre-test. The dashed line represents the mean normalized gain of 0.24 reported from Table 5. The right panel, Figure 11(b), represents each student's performance using the pre-test score as the abscissa and their normalized gain  $g$  as the ordinate. It is curious to note that although most students achieved a positive gain in score during the semester, eleven of the fourteen students with a negative gain scored better than the average pre-test score of 42.7.

As suggested in Hake's unpublished memo on diagnostic exams,<sup>12</sup> we also calculated the correlation coefficient between individual students'  $g$  and pre-test scores. This value,  $-0.33$ , is less than zero, which suggests that "the instruction favors students who have *less* prior knowledge of the subject as judged by the pre-test score".<sup>12</sup> It would be premature to draw strong conclusions about this without a larger sample size, but the preliminary indications are intriguing.

The CT SSCI exam results were also analyzed on a question by question basis. The difficulty index for each question on the exam is defined as the fraction of the students getting this question correct. Figure 12(a) plots the difficulty index vs. question number for the pooled student data. The pre-test data appears as a solid line, while the post-test data appears as a dashed line. Based on these scores, the normalized gain was computed for each question, and the results are also plotted in Figure 12(b). Questions 2, 3, 4, and 11 show high gains (over 0.5). The first three of these (2, 3 and 4) assess students' understanding of basic shifting and scaling operations. Question 11 requires the students to identify the correct spectrum for an amplitude modulated signal. Note that Figure 12(b) also shows that several questions actually have negative gains through the semester. Questions 9, 10 and 18 had fewer students get them correct on the post-test than did on the pre-test. Question 9 asks the students

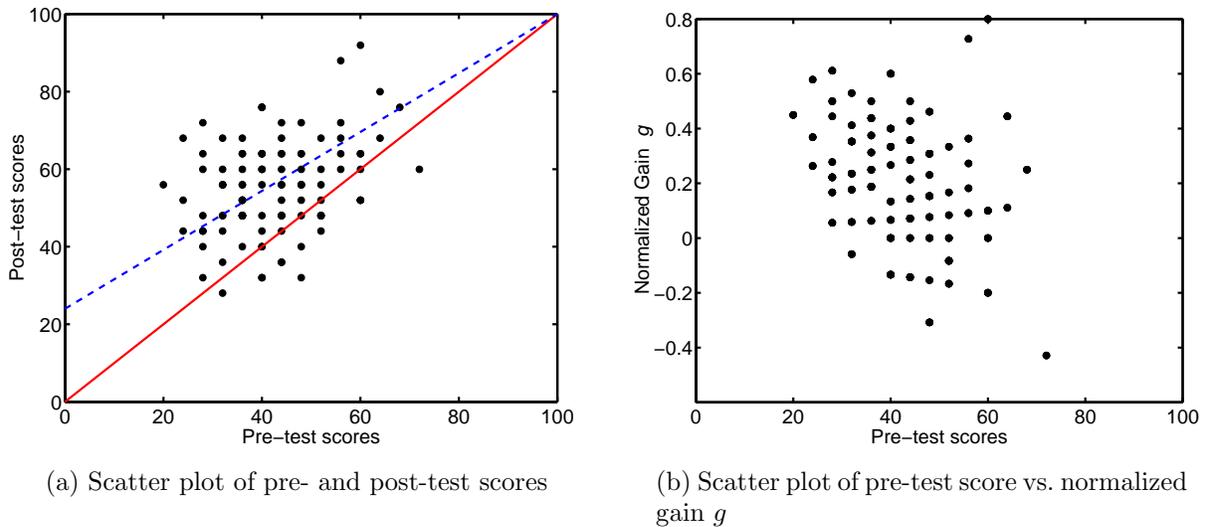
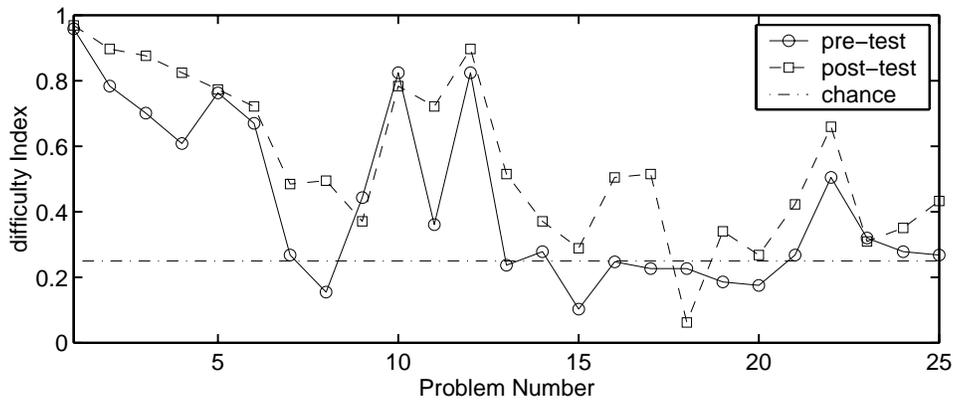


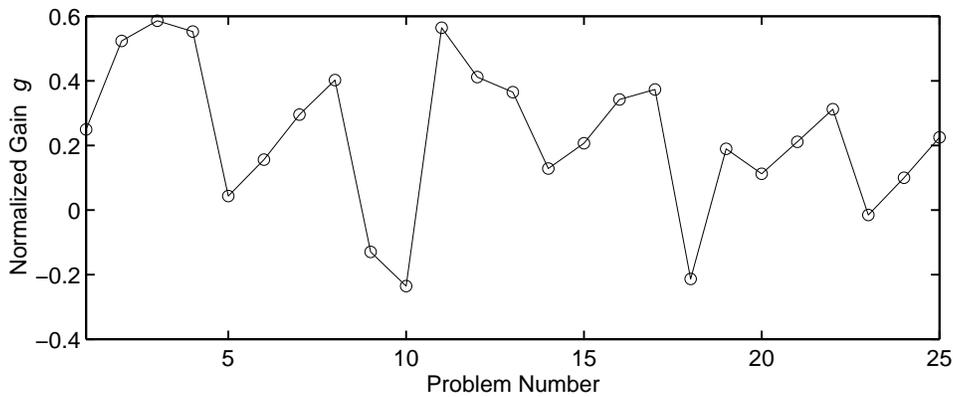
Figure 11: Scatter plots of individual student performance on the CT SSCI

to identify the spectrum of a signal which has been convolved with itself. Question 10 asks the students to identify the spectrum after the signal has been scaled by 2. Question 18 asks the students to identify which pole-zero diagrams correspond to real impulse responses. It should be noted that although the performance on Questions 9 and 10 was worse after the course than before, these were two of the highest scoring questions in both the pre-test and post-test, with about 80% of students getting them correct. Thus, the small loss in performance on Questions 9 and 10 may not be cause for tremendous alarm, especially for the modest population size. The results for Question 18, however, suggest that during the semester the students form strong incorrect ideas about the pole-zero locations for real signals. Twenty-three percent of the students got this question correct on the pre-test, which is roughly what would be expected by chance since there are four answers to choose from. For the post-test, only 6% of the students chose the correct answer, with a strong majority (77%) choosing a distractor that included only poles on the real  $s$  axis, but not complex conjugate poles.

Analysis of the four linked questions described in Section 3 sheds light on the problems students have in understanding frequency-selective filtering. Recall that CT SSCI Questions 1, 6 and 7 separately test the concepts that students must synthesize to get Question 25 correct. Both the pre-test and post-test data in Figure 12(a) suggest that the concept on Question 7 is the limiting factor in students' performance on Question 25. On the pre-test, the students scored 27% on both questions, while on the post-test 48% got Question 7 and 43% got Question 25. The scores for Questions 1 and 6 are much higher on both pre- and post-tests (over 95% for Question 1 and over 65% for Question 6). Consequently, it appears that confusion about the relationship between time and frequency domain signal representations limits students' performance more than their ability to synthesize the three distinct concepts required to solve the problem.



(a) Difficulty index by question number.



(b) Normalized gain  $g$  by question number.

Figure 12: CT SSCI difficulty index and normalized gain  $g$  for each question for pooled students.

## 5.2 DT SSCI Results

This section presents results from the preliminary testing of version 1.0 of the DT SSCI during fall 2001. As noted in Section 4.2, this study sought to verify that the questions were clearly written, and to confirm that the set of distractors covered the most common student confusions. A total of 55 students took this exam during the fall 2001 semester. Table 7 reports the statistics from each campus individually. Recall from Table 4 that these courses are for different class years. The only course in which the exam was administered as a pre-test was in a graduate DSP course, for which the material on the DT SSCI is taught in the prerequisite of the prerequisite, so the students should have been reasonably familiar with it. As one might expect, the post-test scores of the seniors in GMU ECE 410 were statistically similar to the pre-test scores of the new graduate students in UMD ECE 574. The overall mean score was 49.2 with a standard deviation of 16.7. Figure 13 shows a histogram of the pooled scores from all three classes.

Course	Students	Mean Score	Std. Dev.
GMU ECE 320	15	39	14.5
GMU ECE 410	23	55	15.2
UMD ECE 574	17	50	17.0

Table 7: Fall 2001 DT SSCI version 1.0 results.

Figure 14 plots the difficulty index of each question on the DT SSCI. The dashed line indicates the 20% performance expected from random guessing, since there are 5 answers for each question. Comparing these results with the CT SSCI scores reveals several interesting points. Similar to the CT exam, students did better on the basic background concept questions, tested by 1, 2, 3, and 5, than on most of the other questions. The DT SSCI Question 1 (Figure 5) was more difficult than the comparable CT SSCI question on frequency (Figure 2). This is consistent with our experience that many students find DT frequency a more subtle concept to grasp than CT frequency, particularly the notion that there is a highest effective frequency in DT. Intriguingly, the strong performance on DT SSCI Question 5 (Figure 6) demonstrates that the students do understand the periodic nature of DT frequency.

Examining the linked questions on filtering reveals a difference between the CT and DT results. Recall that Question 25 of the DT SSCI synthesizes three concepts, similar to Question 25 of the CT SSCI. DT SSCI Questions 1, 9 and 10 separately test the constituent concepts that appear together in Question 25. Figure 14 shows that, unlike the CT SSCI, there is no clear bottleneck question limiting performance on Question 25. All three of Questions 1, 9 and 10 have more than 50% of the students getting them correct, while only 36% of the students get Question 25 correct. One interpretation is that since the DT students we tested have had at least one more signals and systems class than those in the CT study, they are more comfortable with the time and frequency domain representations tested in DT Question 10 (the analog to CT SSCI Question 7 shown in Figure 4). The lower performance on DT SSCI Question 25 would be consistent with an explanation that the

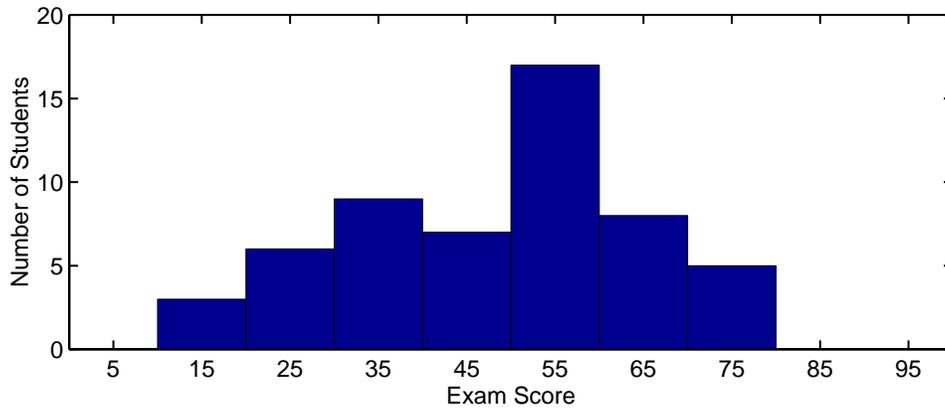


Figure 13: DT SSCI histogram of scores.

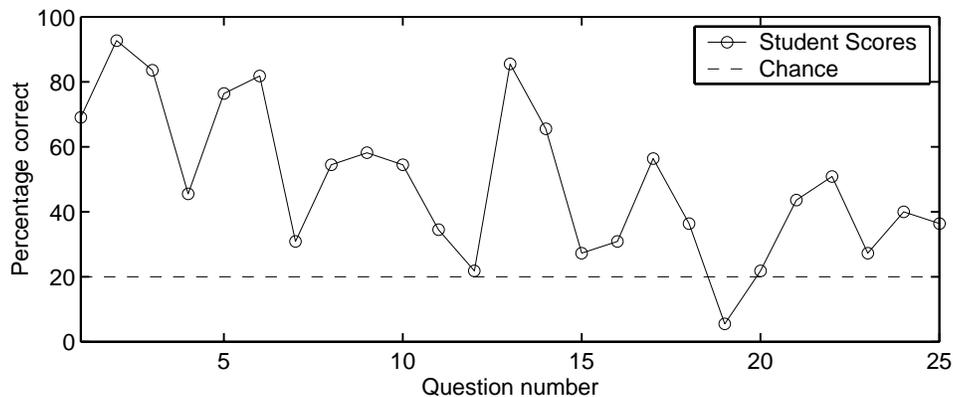


Figure 14: DT SSCI difficulty index by question number.

students understand the individual concepts in Questions 1, 6 and 7, but cannot synthesize their knowledge to get Question 25 correct. We would expect somewhat different results from schools which teach DT topics before CT topics in their signals and systems courses.

## 6 Conclusion

We developed concept inventory exams for continuous-time and discrete-time signals and systems. This paper described the content of these exams by presenting eight sample questions and reported the results of the current study being conducted on four campuses. In compiling statistics for the CT exam, we duplicated the analyses done by Hake for the Force Concept Inventory exam.<sup>1</sup> Analysis of the data for three of the participating schools shows a normalized gain of  $0.24 \pm 0.08$ , which is consistent with the reported results for other concept inventory studies of traditional lecture courses.

Study and development of the CT and DT SSCI exams is an ongoing project. In addition to incorporating the USAFA post-test scores into the analysis as they become available, we

plan to examine the demographic and academic data collected as a part of the current study. We will use this data to examine whether the exam has race or gender biases and investigate the correlation between previous academic performance and performance on the SSCI.

We are actively seeking additional instructors to participate in the SSCI study. In particular, we would like to recruit professors who are using interactive engagement techniques as a part of their curriculum. All who are interested in participating should contact John Buck or Kathleen Wage. Additional information about the study, including an information packet for instructors, may be found on the SSCI website, located at <http://ece.gmu.edu/~kwage/research/ssci>. The website also contains copies of current versions of the exams, which are available to signals and systems instructors or other interested non-students who request a password.

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