The Use of GeoGebra Virtual Interactives in Statics to Increase Conceptual Understanding

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Abstract

One of the distinct advantages of interactive digital media versus print media is the ability of the learner to manipulate the digital examples to test their conceptual understanding. A non-moving example diagram demonstrates one version of a case study, which is useful in the initial stages of learning. However, allowing a learner to manipulate and 'play' with a diagram feeds into a learner's curiosity and lets them test the boundaries of their conceptual understanding. This paper will discuss virtual interactives developed in the Geogebra platform as learning tools for Engineering Mechanics: Statics at Colorado State University. Topics will include: the learning curve and use of Geogebra, how assignments built around interactives encourage student use, and future plans for the use of Geogebra in Statics and Dynamics.

Educational Basis of Study

The use of active learning (across its various forms) has been broadly found to enhance student learning [1], however care must be taken to create active learning tools which are well-structured and delivered. As one example of active learning, the use of physical and virtual interactives (or the combination thereof) for both in-class teaching and outside of class learning has often been shown to enhance student learning [2]. Similar to all other learning materials, the way in which interactives are used can be as influential to student learning gain as the interactive itself. In additional to conceptual and computational analysis, writing has known benefits of enhanced learning and development of effective communication [3].

There are strong parallels between learning engineering and scientific discovery. Both are likely the most productive when those involved do not see their task as hard labor, but instead as an 'pure play' [4]. In the words of Albert Einstein, *"Teaching should be such that what is offered is perceived as a valuable gift and not as hard duty."* Students have a broad range of learning styles and as such instructors should develop a broad range of tools to match [5]. Using the Index of Learning Styles [6] a study at Iowa State University found that the majority of students fall into the categories of active, sensing, and visual learners. All these categories suggest that the majority of students who are allowed to actively play with virtual interactives may increase their learning over other methods.

A solid conceptual understanding in basic engineering topics like Statics and Dynamics has the potential to make more advanced topics in engineering education easier to learn and retain, and is the basis of developing expertise [7]. Conceptual inventories have been developed for both Statics [8] and Dynamics [9] in recent years. This paper works to bring an active learning tool for students to play with to bolster their conceptual understanding.

GeoGebra Background

GeoGebra[1] is an open source dynamic mathematical tool designed for educational purposes. GeoGebra was developed by Markus Hohenwarter in beginning in 2001 at the University of Salzburg in Austria. Since that time GeoGebra has been translated into over 25 languages and is available both as a web-based version and as a downloadable app (across all major platforms) [10]. GeoGebra is written as Java source code which is then translated to JavaScript code to run in a standard HTML5 browser without the need for a Java plugin[2]. The name GeoGebra is a mash-up of the words Geometry and Algebra, highlighting its two-pronged mathematical origins. Current versions also include statistics and calculus tools. As a graphical mathematics tool, GeoGebra has a broad range of built in functions (see Appendix A). One of the inherent strengths of GeoGebra is that it's not simply a graphics editor, but the graphics (whether 2D or 3D) are all computed in an explicit spatial environment. Thus, every point, line, circle, vector, ray, conic section, or polygon is defined with a mathematical function. Furthermore, all the elements can be dynamically altered via (1) direct touch/mouse manipulation, (2) algebraic computations, or (3) even animation. The end effect can be an interactive learning environment where the learner is able to manipulate the Geogebra learning tool.

While my observation suggests that GeoGebra is most often used in K-12 math and physics education, it is being used in higher education at a moderate rate. A search for 'GeoGebra' in the ASEE Annual Conference proceedings found 9 other papers mentioning its use (all since 2012).

Study design

This paper focuses on the use of virtual interactives to increase conceptual understating in a 2nd year Engineering Mechanics: Statics (hereafter 'Statics') course at Colorado State University. This paper is designed as a demonstration of developing virtual interactives in the GeoGebra platform and using these interactives as the basis of learning exercises completed out of class within the Canvas Learning Management System (LMS) platform.

Class description

To better understand the use of the virtual interactives, it may be valuable to understand the format of the course where they are being used. Statics is typically taken by students in their 3rd semester of their engineering undergraduate curriculum. They must take and pass both Physics for Scientists and Engineers I (covering forces, energy, momentum, and angular momentum) and Calculus for Physical Scientists I (covering limits, continuity, differentiation, and integration of elementary functions with applications). Statics is a required course for all Civil, Mechanical, Environmental, and Biomedical Engineering students and can be taken as an elective by Electrical and Chemical Engineering students. Section size varies between 100-150 students with three 50 minute lectures per week with no recitations or labs. During a 16 week semester, weekly homework sets and learning activities are completed by all students. The course currently does not use any commercial online courseware (Pearson Mastering Engineering or similar).

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[1] https://www.geogebra.org/
[2] https://dev.geogebra.org/trac/wiki/GeoGebraWeb

Using GeoGebra Interactives in Statics

My vision for using GeoGebra in Statics comes from the hypothesis: students given an opportunity to 'play' with a concept develop a deeper conceptual understanding than students viewing static material. Hence, students who raise conceptual questions and then test these questions are essentially creating micro-experiments. These micro-experiments are a cycle where questions are raised; hypotheses are developed; data is gathered and analyzed; conclusions are made; and new micro-experiments are developed. My vision is that interactives in online material could replace all figures in traditional textbooks. Imagine the power of students being able to 'play' with every figure they view.

I use interactives in Statics primarily for student learning. This takes place in three ways. I display and use some interactives during in-class lectures. Other interactives are simply posted to our LMS as supplemental learning tools. Finally, I assign other interactives along with designated scenarios force students to work through a series of the micro-experiments. A case study of one of these assignments can be found below.

Case Study: Resultant Force and Moment Assignment

Two common questions students have in the first month of Statics are:

- What exactly are the differences between a force, a moment from a force, and a couple moment?
- How do forces and couple moments interact to determine the resultant force and resultant moment about a designated point?

Assigning traditional homework problems do build a student's conceptual and computational skills. However, as they are typically completing each scenario once, they are not given the opportunity to create and test hypotheses in the micro-experiments I described previously. After creating a number virtual interactives to answer the need for micro-experiment testing tools, I found that many students were not using the interactives. I was not able to test for the underlying reason for their lack of use, but I theorize it is due to either (1) students having difficulty in knowing which questions to ask of the interactive or (2) lack of motivation to complete ungraded material. Hence, by giving students a series of assigned scenarios, it forces them to work through the interactives and hopefully piques their curiosity to create their own micro-experiments in the process.

The Resultant Force and Moment Assignment is coded into our LMS Canvas as two items. First, the GeoGebra interactive is embedded in a stand-alone Page (Figure 1) which can also be accessed outside of the assignment. Second, assigned scenarios are input as a Quiz wherein Canvas selects a random order (and set) including 8 of the 11 current scenarios (Appendix B). The final part of the Canvas Quiz was a single feedback question "Was this a worthwhile exercise which helped you to better understand Resultant Forces, Resultant Moments, and Couples?" (scored between 0 = 'not worth your time' to 10 = 'very useful')

Responses to the assigned scenarios are open-ended as one of the secondary objectives of this assignment is to have students write short technical responses. Given the volume of students

completing the assignment (100-250 depending on the semester) graders scan the answers from each student to make sure they are complete and on topic, but do not grade for correctness. Logistically, we use the Student Analysis report within the Quiz Summary tool to provide a spreadsheet summary of all student responses which can be visually scanned. Additionally, example answers to each scenario are provided upon completion of the assignment for students to self-reflect. Students are invited to complete the assignment more than one time if they want to demonstrate knowledge gained in previous iterations or see a broader range of the possible scenarios.

Course: Engineering Mechanics: Statics
Topic: Resultant Force & Moment
This interactive allows you to find the resultant force (Pink Vector) and moment of a system which includes 2 external forces and 2 external couple moments. Moment magnitude is represented by the size of the circle with green representing positive and red representing negative values (according to the right hand rule). You are able to change the:
= Angle of body ('Angle' point)
= Size of body ('Size' point)
= Magnitudes of forces (red sliders)
= Location of forces (pink tip points)
= Angle of forces (orange tail points)
= Location of couple moments (purple dots)
= Magnitude of couple moment (purple sliders)
= Location of resultant point ('Point')

F1magnitude = 1.1
F2magnitude = 1.8
Couple'A'Magnitude = -2.5
Couple'B'Magnitude = -2

$\alpha = 19.89°$
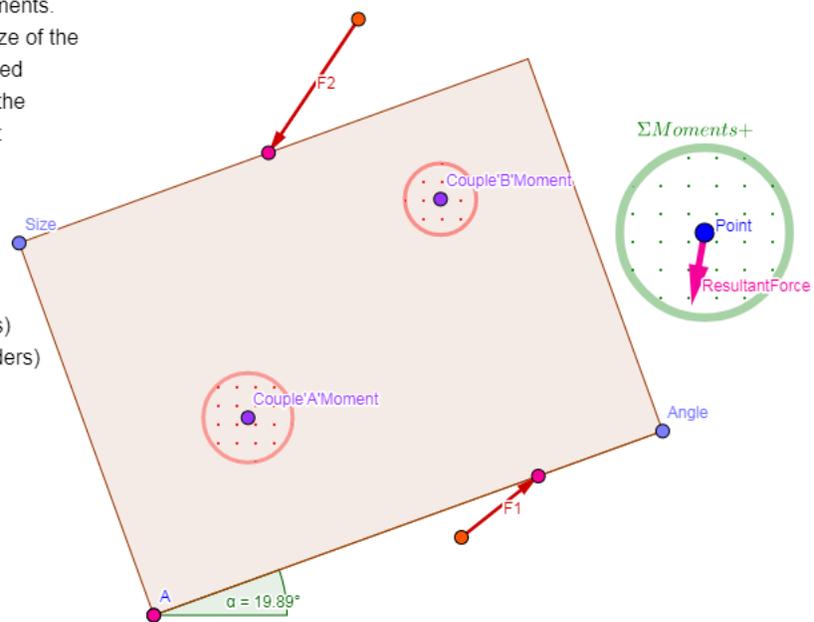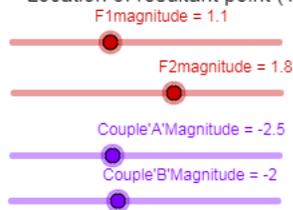
Figure 1: Screen shot of Resultant Force & Moment GeoGebra interactive available at https://ggbm.at/GqURw4N4. Students are able to manipulate all aspects of interactive and the interactive provides a graphical solution as to the resultant force and moment of the given applied forces and couple moments about the designated 'Point'

Fall 2017 data (231 respondents across two on-campus sections and 1 online section) to this assignment was analyzed. The feedback question yielded a median value of 7 and a mean of 7.3. Anecdotally, I find that anytime student's feedback to assignments hits above the mid-point of the scale (5 in this case) that assignments were an overall success from the student perspective. It is worth noting that very few students (2.1% of student completing the assignment in Fall 2017) took advantage of multiple attempts.

In our quick-grading of student responses we did not find any student responses which were off topic. This suggests that all the students which completed the assignment at least took the time to respond to the scenarios to the best of their knowledge.

Geogebra Learning Curve

Relative to my experience with various software packages, the learning curve for GeoGebra is moderate. You can get started right away, but as you progress you'll learn a number of efficiencies and enhancements. Here are a couple of my lessons learned:

- To speed your construction, think about geometry first, and algebraic equations 2nd. It is far easier to graphically add a tangent line at a point on a curve then it is to derive the equations algebraically. When the graphical tools no longer meet your goals, then use written equations to complete your construction.
- GeoGebra applies default names to each component. I have found it helpful to change these names to carry more intuitive meaning in your final project. For example, rename the Cartesian components of the force F from the defaults u and v to the more descriptive Fx and Fy.
- Construction sequence is a major factor in the design of your end product. For example, if you want one point (child) to move relative to another (parent), you will need to establish the parent point first and then create the geometry which will connect the two points (circle, parallel/perpendicular lines, etc) where the child often ends up as an intersection point between two lines/shapes.
- There are a variety of construction tools to aid in more complex interactives, including hidden objects and layers. Any construction item can be hidden, hence if lines parallel to a body are used in the construction, you can just see the intersection point or vector from this point in the final tool.

Conclusion

This paper outlined a process and case study which used the active learning inherent in virtual interactives to allow students to play with their conceptual understanding of Resultant Forces and Moments in a 2nd year Engineering Mechanics: Statics course. Students used a virtual manipulative developed in the GeoGebra platform to enforce their conceptual understanding by playing with an assigned set of scenarios assigned. Students reported that this assignment was a worthwhile learning experience.

The freely-available (and customizable) GeoGebra virtual manipulatives have been used for in-class demonstrations, outside of class studying, and also linked to assignments. Based upon usage rates of non-assigned interactives, I have found that linking them to assignments best guarantees that students interact with the learning tool. Future expansion of this study could include comparing student knowledge gained via the virtual interactives with student knowledge gained in a non-interactive assignment – possibly in a paired study across two course sections. This type of information would help to gauge future development efforts in this area. Additionally I am working to develop a network of instructors of Statics and Dynamics to collaborate on a robust set of virtual manipulatives for community use. The development and use of web-based virtual interactives for student learning will likely increase as tools for developing interactives (like GeoGebra) become more widely known in the engineering education community.

Acknowledgements

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Appendix A

Figure 2: Catalog of drop-down menus in the 2D Geometry view of Geogebra (image acquired from http://www.jensilvermath.com/2013/08/07/geogebra/)

Appendix B

Table 1: Scenarios used in Resultant Force & Moment assignment.

Topic	Instructions			Narrative Questions (top row) & Answers (below)	
	Set external forces to:	**Set applied couple moments to:**	**Now:**	**What do you observe about the Resultant Force? Why?**	**What do you observe about the Resultant Moment? Why?**
Just Forces	Both in same direction (parallel) but not collinear or equal	Set to Zero (middle of slider)	Change the magnitudes of the forces	Resultant force parallel to external forces	Equal to the sum of the (r x F)'s for each force
Just Forces	Same magnitude	Set to Zero (middle of slider)	Change direction of forces until the resultant force disappears	Opposite directions	Not zero, but equal to the couple of the two forces
Just Forces	Anything	Set to Zero (middle of slider)	Move the 'Point' over the tip (pink dot) of F1. Then change the direction and magnitude of F1	Resultant force adjusts as ∑ forces changes	Does not change as the line of action of the F1 goes through the 'Point' and thus the only moment comes from the (r x F2).
Just Forces	Your choice (non-zero) and leave constant	Set to Zero (middle of slider)	Move the resultant 'Point' around	No change in direction or magnitude of Resultant Force	changes in magnitude as the distance to the force vectors change
Just Forces	Your choice (non-zero) and non-parallel	Set to Zero (middle of slider)	Change direction of forces until the resultant force moment disappears but the resultant force is non-zero	∑ of the two forces	Zero because the moments of the two forces are equal and opposite

Topic	Instructions			Narrative Questions (top row) & Answers (below)	
	Set external forces to:	Set applied couple moments to:	Now:	What do you observe about the Resultant Force? Why?	What do you observe about the Resultant Moment? Why?
Just Forces	Your choice (non-zero)	Set to Zero (middle of slider)	Adjusting the force locations, magnitudes, and directions until both the resultant force and moment are both equal to zero	Equal to zero because the external forces are equal, opposite, and collinear	Equal to zero because the external forces are equal, opposite, and collinear
Just Couple Moments	Set all to zero	Set one to zero and the other to a non-zero value	Changing the 'Angle' of the body	Equal to zero	Resultant Moment equals Couple moment and does not change as couples are a free vector
Just Couple Moments	Set all to zero	Your choice of magnitude, but leave constant	Move Couple Moment points around	Equal to zero	Couple moment is equal to Resultant Moment and does not change as Couple is a free vector
Just Couple Moments	Set all to zero	Set to equal and opposite magnitude	Move resultant 'Point' around	Equal to zero	Couple moment is equal zero as the sum of moments is zero AND this does not vary with location
Forces & Couple Moments	Your choice, but leave constant	Your choice, but leave constant	Changing the 'Angle' of the body (note forces don't change direction)	Same magnitude as forces don't change	changes as position vectors change (r x F)
Location of Resultant	Your choice, but leave constant	Your choice, but leave constant	Moving the resultant 'Point' around	Same magnitude everywhere	Magnitude varies with location of point, zero value along a straight line