The Use of Numerical Regression Analysis in Modeling Various Types of Experimental Friction

John Nydahl, Nancy Peck, and Scott Morton
Department of Mechanical Engineering, University of Wyoming

Abstract

Modeling is vital to engineering, yet students have difficulty understanding and appreciating the concept. This paper describes a series of dynamic experiments that utilizes a rotating disk to reinforce concepts involved in creating an empirical model. These experiments have many positive pedagogical attributes. The apparatus is simple, plus the required mass, length and time measurements are not difficult to make with simple instruments. The equipment is inexpensive and portable for both classroom and laboratory use. The experiments range from simple moment of inertia concepts to the testing of more complex friction models and may be easily modified to vary the results. The disk is an appropriate system for sophomore level students to analyze, since the solution of its angular momentum differential equation results in a simple angular displacement versus time relationship, \( \theta(t) \), even though the frictional model is non-linear, varying with the angular velocity \( \omega \) raised to some unknown power. This permits superior results since the \( \theta \) vs. \( t \) data set can be accurately determined over the range of angular velocities using an ordinary stopwatch. The drag devices generate kinetic friction (\( F_\mu \propto \text{constant} \)), eddy current friction (\( F_\mu \propto \omega \)) and aerodynamic friction (\( F_\mu \propto \omega^2 \)). Trend lines that undergraduates typically use to correlate data are inappropriate here because none have the proper functional form. Excel is utilized because a) it is a natural platform to record and manipulate experimental data, b) its Solver package permits an iterative nonlinear regression analysis to determine the unknown empirical coefficients by minimizing the error between experimental and model predictions, and c) Visual Basic function modules may be utilized to calculate the empirical model values. The discovery-based experiments, run as coupled or independent experiments, may also be utilized as demonstrations since the empirical results are in excellent agreement with accepted physical models. Further, the exceptional agreement obtained between the general empirical model results and the various friction laws allows students to deduce specific relationships from more general relationships.

I. Introduction

Lower division students often blindly accept authority-given knowledge and then try to regurgitate and/or apply this information without regard to the implicit assumptions and limitations. An implied goal of any higher learning institution is to move learners from naiveté to a questioning and critical thinking state\(^1\). This situation is particularly acute when students are asked to integrate material from different courses\(^1\) in discovery-based laboratory exercises, since the bulk of entering engineering students lack laboratory experiences. Consequently, many educators are stressing the need for hands-on teaching of fundamentals\(^3\)-\(^9\). Over the past ten
years, the Mechanical Engineering Department of the University of Wyoming (UW) has used alumni recommendations\textsuperscript{10}, Accreditation Board of Engineering and Technology (ABET) 2000 criteria\textsuperscript{11}, as well as UW’s Engineering Task Force on Undergraduate Education recommendations\textsuperscript{12} to help transform our laboratory courses into an integrated package. The principal goals of the introductory, discovery-based laboratory are to stimulate the development of sophisticated thinking, incorporate teamwork approaches and improve communication skills that are required in engineering practice. Specifically, students are required to integrate their fragmented knowledge base in mathematics, dynamics, statistics and programming to accurately model and test various simple dynamic systems, and clearly and concisely report the results.

This paper describes a friction module, consisting of two “linked” experiments utilizing a rotating disk that successfully incorporates the course goals and the prerequisite course materials. The results obtained from the respective empirical models dramatically demonstrate the validity of the classical friction models for kinetic, eddy current, and aerodynamic friction. Only basic experimental instrumentation in the form of a scale, stopwatch, ruler and protractor are utilized so that experimental errors and experimental design concepts may be emphasized. This approach focuses on the studied phenomena and the development of engineering judgment skills without the distraction of sophisticated equipment that may convey the illusion of accuracy\textsuperscript{3-9}.

II. Mass Moment of Inertia Experiment

The experimental objective of the first experiment is to determine the disk’s mass moment of inertia $I_D$, while the pedagogical objective is to gain an understanding of basic error analysis and the value of experimental design. The following five established relationships from statics and dynamics are used to determine $I_D$ in terms of directly measured parameters:

1. Static measurements: $I_D = \frac{1}{2} m_d R^2$
2. Angular momentum imparted by a mass falling distance $h$ in time $t$:
   
   $I_D = m_p R^2 (1 - g t^2 / 2h)$
3. Mechanical Energy Balance: $I_D = 2m_p gh / \omega^2$
4. Non-centroidal oscillations with period $T$, for large amplitude $\theta_0$\textsuperscript{13}:

   \[ I_D = m_0 r_0 \left[ g \left[ 2 \pi \left[ 1 + \frac{1}{2} \sin^2(\theta_0 / 2) + \left( \frac{1}{2} \frac{3}{4} \right)^2 \sin^4(\theta_0 / 2) + \left( \frac{1}{2} \frac{3}{4} \frac{5}{6} \right)^2 \sin^6(\theta_0 / 2) + \Lambda \right] \right]^{-2} \right] \\ - r_0 \] 

5. Which reduces to the classical, small amplitude relationship: $I_D = m_0 r_0 [g(T/(2\pi))^2 - r_0]$

Diagrams of the experimental setups for methods 2 through 5 are indicated in Figure 1.
Prior to performing these experiments, students are asked to estimate the maximum probable errors, $\sigma_{I_D}$, for each of these methods. These assessments are based upon the measuring accuracies of the available instrumentation and the following simple statistical relationships:

$$
\sigma_{I_D} = \text{Maximum} \left( \frac{S}{\sqrt{n}} \cdot \sqrt{\frac{1}{\sqrt{12}}} \text{ (smallest division of measuring instrument)} \right) $$

where $\sigma_{I_D}$ is the error associated with direct experimental measurement $x_i$, $n$ is the sample size, and $S$ is the sample’s standard deviation

$$
S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}
$$

This simplified procedure allows students to easily calculate the gross statistical characteristics of their experimental data by analytical means if the experimental result is an uncomplicated explicit function of the measured variables, e.g. Method 1. The analytical evaluation of the partial derivatives in the maximum probable error relationship is usually quite tedious for several of the above methods. Consequently, the utilization of numerical partial differentiation is introduced, and the students are then asked to compare the results obtained from both forms of differentiation for a simple case.
The students gain practical experience and insight into the importance of experimental design by using estimated values to calculate the maximum probable error for each method prior to performing the experiments. Further, students are encouraged to use creativity to minimize measurement errors, e.g. measure the disk circumference instead of disk radius and use direct rise and run lengths to calculate an angular position instead of using a protractor. After performing the experiments, the maximum probable errors are again calculated with actual experimental data and instrumentation accuracies. Table 1 lists typical values for these experiments. Of the five methods presented, an experimental design analysis predicts the simplest (the analytical static determination) should, and does, produce the most reliable measurement. In summary, this is an excellent after the fact hands-on demonstration of the value of experimental design.

Table 1. Typical Experimental Mass Moment of Inertia Values with Associated Maximum Probable Errors for Particle Board Disks, $I_D + \sigma_I$

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Predicted Values of $I_D$ (kg-m$^2$)</th>
<th>Experimentally-Based Values of $I_D$ (kg-m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Determination</td>
<td>$0.180 \pm 3.47 \times 10^{-4}$</td>
<td>$0.185 \pm 2.78 \times 10^{-4}$</td>
</tr>
<tr>
<td>Angular Momentum</td>
<td>$0.180 \pm 4.57 \times 10^{-3}$</td>
<td>$0.180 \pm 4.58 \times 10^{-3}$</td>
</tr>
<tr>
<td>Angular Acceleration</td>
<td>$0.180 \pm 8.01 \times 10^{-3}$</td>
<td>$0.158 \pm 7.32 \times 10^{-3}$</td>
</tr>
<tr>
<td>Mechanical Energy Balance</td>
<td>$0.180 \pm 8.01 \times 10^{-3}$</td>
<td>$0.158 \pm 7.32 \times 10^{-3}$</td>
</tr>
<tr>
<td>Natural Frequency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large Angles ($\theta \approx 85^\circ$)</td>
<td>$0.180 \pm 2.14 \times 10^{-3}$</td>
<td>$0.176 \pm 2.14 \times 10^{-3}$</td>
</tr>
<tr>
<td>Small Angles ($\theta \approx 15^\circ$)</td>
<td>$0.180 \pm 2.38 \times 10^{-3}$</td>
<td>$0.154 \pm 3.86 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

III. The Friction Experiments

The second experiment in this module investigates the effects of friction on the dynamic characteristics of freely spinning disks. Three different friction devices - kinetic friction, eddy current friction, and aerodynamic friction - are set, and each team of students must model one of the prescribed friction effects. Figure 2 shows the disks with the three different friction devices. The disks run on ball bearings and are exposed to the viscous effect of the atmosphere, but the experimental friction devices dominate these natural damping forces. The kinetic friction mechanism is a spring-loaded wooden dowel sliding on a steel flange. The eddy current friction device consists of a magnet situated near an aluminum ring fastened to the side of the disk. The aerodynamic brake consists of multiple cups clipped to the outer rim of the disk.

The experimental procedure is quite simple: start the disk spinning and record revolutions as a function of time until the disk stops or slows down significantly. The experiments are performed at least twice to obtain a comparative data set. Coordination and communication between the team members is vital to minimizing the timing errors.
IV. Empirical Models

A reasonable and intuitive proposition is posed, that the frictional force occurring in a particular system, $F_{\mu}$, is proportional to velocity, $V$, raised to some power:

$$F_{\mu} = \mu_f V^\gamma$$  \hspace{1cm} (1)

where $\mu_f$, $\gamma$, and $V$ are the friction coefficient, velocity exponent, and linear velocity respectively. This empirical relationship results in the acceleration relationship

$$\frac{d\omega}{dt} = -\frac{\mu_f r (r\omega)^\gamma}{I_D} = \frac{d^2\theta}{dt^2}$$  \hspace{1cm} (2)

with the initial conditions $\theta(0) = 0$ and $\left(\frac{d\theta}{dt}\right)_{t=0} = \omega_0$

where $I_D$, $r$, $t$, $\theta$, and $\omega$ are the mass moment of inertia of the disk about its center of gravity ($kg\cdot m^2$), the characteristic radius of the friction source ($m$), time ($s$), angular rotation of disk
(rad), and angular velocity (rad/s), respectively. Equation 2 indicates that the dimensions of $\mu_f$ depend upon $\gamma$.

Separating variables and integrating
\[
\int_{\omega_0}^{\omega} d\omega = -\frac{\mu_f r^{\gamma+1}}{I_D} \int_0^t dt
\]
yields the result
\[
\omega = \left[ \frac{1}{1-\gamma} \frac{(1-\gamma)\mu_f r^{(1+\gamma)} t}{I_D} \right]^{1/(1-\gamma)} = \frac{d\theta}{dt}
\]
(3)

The angular displacement is obtained by a straightforward integration of the above differential equation even though the expanded form is somewhat intimidating
\[
\theta = \frac{I_D \left[ \omega_0^{2-\gamma} - \left( \frac{1-\gamma}{1-\gamma} \frac{(1-\gamma)\mu_f r^{(1+\gamma)} t}{I_D} \right)^{2-\gamma} \right]}{(2-\gamma)\mu_f r^{\gamma+1}}
\]
(4)

Students must note that this general solution for the angular displacement obviously does not hold for $\gamma = 1$ or 2. Also, the initial condition $\omega_0$ is not measured directly and must be determined from the experimental data. Further, the structure of this general solution is not compatible with any of Excel’s standard Trendline forms.

The intent of presenting only the above general empirical solution to the students is to have them discover the basic physical relationships without any preconceived theoretical biases. After completing the experiment and determining the best empirical correlation for $\gamma$ and $\mu_f$, the teams are asked to compare their results with the appropriate theoretical model as presented below.

**Kinetic Friction**: The special case $\gamma = 0$ represents the classical kinetic friction situation where Equation 4 simplifies to
\[
\theta = \omega_o t - \frac{\mu_f r^2}{2I_D} \quad \text{for} \quad t \leq \frac{\omega_0 I_D}{\mu_f r}
\]
(5)

The kinetic model predicts a linear decrease in the disk’s angular velocity with time and a quadratic variation of the disk’s total number of revolutions with time. Here, the coefficient $\mu_f$ has the units of force and is equal to the familiar expression for kinetic friction:
\[
\mu_f = \mu_k N
\]
(6)

where $\mu_k$ is the coefficient of sliding friction and $N$ is the normal force.
Eddy Current Friction: The case $\gamma = 1$ corresponds to the theoretical eddy current friction model. Equation 2 becomes

$$I_D \left( \frac{d\omega}{dt} \right) = -\mu_f r^2 \omega$$  \hspace{1cm} (7)

Separating variables and integrating

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -\frac{\mu_f r^2}{I_D} \int_0^t dt$$  \hspace{1cm} (8)

which yields the result

$$\omega = \omega_0 e^{-\frac{\mu_f r^2 t}{I_D}}$$  \hspace{1cm} (9)

which in turn can be integrated to produce the angular displacement

$$\theta = \frac{\omega_0 I_D}{\mu_f r^2} \left( 1 - e^{-\frac{\mu_f r^2 t}{I_D}} \right)$$  \hspace{1cm} (10)

Aerodynamic Friction: The case $\gamma = 2$ represents the theoretical aerodynamic friction model. Equation 2 becomes

$$I_D \left( \frac{d\omega}{dt} \right) = -\mu_f r^3 \omega^2$$  \hspace{1cm} (11)

Again, separating variables and integrating yields

$$\omega = \frac{\omega_0 I_D}{I_D + \omega_0 \mu_f r^3 t} = \frac{d\theta}{dt}$$  \hspace{1cm} (12)

and

$$\theta = \frac{I_D}{\mu_f r^3} \ln \left( 1 + \frac{\mu_f r^3 \omega_0 t}{I_D} \right)$$  \hspace{1cm} (13)

For the aerodynamic friction model the coefficient, $\mu_f$, can be expressed as a function of a constant drag coefficient, $C_d$:

$$\mu_f = \frac{C_d \rho A n}{2}$$  \hspace{1cm} (14)

where $\rho$, $A$, and $n$ are the density of air ($kg/m^3$), the projected cross sectional area of a single drag inducing object ($m^2$), and the number of drag objects, respectively.
V. Data Reduction

Appendix I presents an example data set for the aerodynamic friction experiment in an Excel spreadsheet. The measured data is contained in the table’s Revolutions and Time columns (Appendix I, spreadsheet Columns F and G respectively). The general empirical relationship and the theoretical relationship ($\gamma = 2$) for the angular displacement for aerodynamic friction are also displayed on this spreadsheet. The initial angular velocity $\omega_0$ must be obtained by numerically differentiating the initial data and the accuracy of this result is quite sensitive to the procedure that is used. This is a good example where some forethought should be given on how to differentiate the data. A quadratic trend line was applied to the first three data points to estimate the angular velocity at the second data point ($\omega_0 = 5.446$ rad/s, see $\theta$ vs. $t$ plot in Appendix I). The data set was then truncated to start at the second data point (spreadsheet Columns H & I). Using the calculated initial angular velocity, a nonlinear regression analysis was then used to determine the unknown empirical coefficients of the general friction model. For all the cases, it should be noted that $\mu_f$ and $\gamma$ are dependent upon the initial angular velocity, $\omega_0$.

Engineering students at UW are introduced to spreadsheets, specifically Excel, and the application of its Solver tool to nonlinear regression analysis in a first semester freshman course. The Solver package uses a generalized, reduced-gradient, nonlinear, multi-parameter optimization algorithm. For empirical modeling, Solver is a powerful tool to use as it permits an iterative nonlinear analysis to determine unknown empirical coefficients. The procedure is as follows. Using experimental data as a baseline and initial guesses for the unknown empirical coefficients (original guess values are displayed in Appendix I, cells D32 & 33), a particular model relationship is defined. The students are familiar with the least squares concept. The squared differences (Column K) between experimental data (Column I) and model prediction values (Column J) of angular displacements are calculated, then summed in the named cell $SS$ (cell E35). Cell $SS$ is then minimized to find the empirical coefficients (named cells $mu_{aero}$ and $gamma_{aero}$, (cells E32 & 33 respectively). The regression analysis is not limited to minimizing the sum of the square of the differences since any other appropriate relationship may be minimized, e.g., standard deviation, etc.

A Visual Basic function module calculates the model prediction values listed in column J of the spreadsheet. An example of the function call is displayed in Row 49 of Appendix I; the Visual Basic function code is listed in Appendix II. The use of Visual Basic introduces structured programming to the students, but its use is limited in this class to function calls that are easily taught by examples. The employment of a function module permits complicated functions, like Equation 4, to be readily implemented where they might otherwise be difficult to write directly in the worksheet.

This exercise reinforces the fact that realistic guesses for the parameters $\mu_f$ ($mu_{aero}$) and $\gamma$ ($gamma_{aero}$) must be made, and appropriate constraints must be set (see Figure 3) to obtain valid solutions. For this particular example, the only physical constraints are that both $\gamma$ and $\mu_f$
must be greater than zero. Running Solver results in the following optimized solution: \( \mu_f = 0.0235 \) and \( \gamma = 1.954 \).

The value of named cell SS drops from 51,855 (Appendix I, cell D 35) for the initial guess to 0.612 (cell E35) for this optimum configuration. This sum of squares is extremely low, indicating that the postulated model accurately captures this drag phenomenon. In addition, the empirical value of \( \gamma = 1.954 \) is within 2.5% of the aerodynamic drag theoretical value. When the same method is repeated for the \( \gamma = 2 \) model as expressed in Equation 13, \( \mu_f \) changed slightly from 0.0235 (cell E32) to 0.0234 (cell E40) and the sum of squares increased to 1.652 (cell E42). These results support the fluid dynamic concept that drag is often directly proportional to the dynamic pressure. This being true, the drag coefficient, \( C_d \) from Equation 14 is equal to 1.97, a value that is significantly higher than the 1.0 to 1.4 value of a hollow cup in linear motion. This discovery experiment indicates that some other phenomenon is having a significant influence on the aerodynamic drag, e.g. rotary motion and/or vortex shedding.

The use of Equation 4 for kinetic friction and eddy current friction also produced excellent correlation of the experimental data. The coefficient of sliding friction, \( \mu_k \), in the kinetic friction experiment was calculated to be 0.50 (Equation 6) for the \( \gamma = 1 \) model (Equation 5) with \( N \) being the normal force exerted by the linear spring. This compares very well with a published value of \( \mu_k = 0.49 \) for oak on cast iron\(^{15}\). Table 2 shows sample results for the three different friction cases. In all cases the exponents, \( \gamma \), were near to the accepted theoretical values.

<table>
<thead>
<tr>
<th>Dominant Friction Source</th>
<th>Calculated ( \omega_o ) (rad/s)</th>
<th>( \gamma )</th>
<th>( \mu_f )</th>
<th>( \gamma )</th>
<th>( \gamma )</th>
<th>( \mu_f )</th>
<th>Sum of Residuals Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinetic</td>
<td>12.729</td>
<td>0.10</td>
<td>3.00</td>
<td>13,078</td>
<td>0.170</td>
<td>5.916</td>
<td>0.657</td>
</tr>
<tr>
<td>Eddy Current</td>
<td>10.044</td>
<td>0.99</td>
<td>0.12</td>
<td>3,494</td>
<td>1.074</td>
<td>0.134</td>
<td>5.604</td>
</tr>
<tr>
<td>Aerodynamic</td>
<td>5.446</td>
<td>1.9</td>
<td>0.05</td>
<td>46,408</td>
<td>1.954</td>
<td>0.0235</td>
<td>0.612</td>
</tr>
</tbody>
</table>

* Note, the units of \( \mu_f \) depend on the value of \( \gamma \).
VI. Conclusions

A set of uncomplicated, inexpensive and portable friction experiments involving concepts relevant to sophomore level engineering students was successfully used to demonstrate the use of physical modeling. The particular model suggested for these experiments leads to a two-parameter, nonlinear relationship between angular displacement and time that does not permit students to use typical canned “trend line” tools to correlate data. Students are therefore compelled to implement their own nonlinear regression analysis. To present various students with individual data sets or to perform parametric studies, the physical parameters of the friction devices are easily changed, e.g., the spring load or the dowel material for the kinetic friction device, the size and spacing of the magnet for the eddy current friction device, and the number, size and/or shape of the cups on the aerodynamic friction device. In this paper a procedure is described that utilizes the Excel Solver tool to obtain a least squares fit to the data where a simple Visual Basic function is used to calculate the corresponding model values. This introduces the students to the use of spreadsheet macros, some basic programming and data reduction.

One of the most gratifying results of this set of experiments is the excellent agreement between the general empirical model results and the various established friction “laws”. This permits the students to deduce specific relationships from more general relationships. The experiments, intentionally designed with inexpensive and mechanically simple equipment and instrumentation that introduce significant errors, focus the students on problem solving and engineering judgement skills without their being distracted by gadgetry. This format has been shown to help engage students who might otherwise try to get through with minimal effort - “the survivalist” - and with minimum comprehension – “the ATM student”.

In summary, the developed set of experiments emphasizes dynamic analysis, experimental design, and numerical regression. The experiments have a variety of desirable attributes: the physical configurations are easily changed; the experiments may be performed singly or coupled; they may be utilized as demonstrations or discovery-based experiments; students can work the experiments individually or in teams; and the experimental results have excellent agreement with the various accepted empirical laws.

Bibliography


JOHN NYDAHL
John Nydahl is currently a Professor of Mechanical Engineering at the University of Wyoming and is actively involved in renewable energy research. He received his B.S. and M.S. degrees in Aeronautical Engineering from the University of Florida and a Ph.D. in Mechanical Engineering from Colorado State University in 1971.

NANCY PECK
Ann “Nancy” Peck joined the University of Wyoming’s Mechanical Engineering department in 1995. She is currently an Associate Lecturer. Her research interests include the analysis and design of composite structures and the use of structural optimization tools. She received her B.S. in Mechanical Engineering from Lehigh University and the M.S. and Ph.D. degrees from Rensselaer Polytechnic Institute (1992).

SCOTT A. MORTON
Scott Morton is a Research Scientist in the Department of Mechanical Engineering at the University of Wyoming. He received his BSAE and MSAE from the University of Wyoming. His professional interests are diverse and include instrumentation and electro-mechanical devices. Current research activities are in the areas of alternative energy and computer aided laboratory instruction.
Appendix I

Example Analysis Spreadsheet for Aerodynamic Friction

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fluid Friction (Aerodynamic Drag) on a Disk</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Calculation of Initial Angular Velocity ( \omega_0 ), using Angular Displacement vs. Time Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \omega = -0.0952t^2 + 6.453t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Calculation of ( \omega_0 ), using Angular Displacement vs. Time Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Initial Angular Velocity ( \omega_0 ) Calculation :</td>
<td>Constants</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>( \frac{d\theta}{dt} = \omega = 6.4318 \cdot 2^2 t^2 )</td>
<td>( I_D = 0.131 ) (kg m(^2))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>( \omega_{\omega \omega} = 5.446 ) (rad/sec) @ ( t = 5.29 ) (sec)</td>
<td>Radial Distance to Cups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>( r = 0.34 ) (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Moving to Excel Spreadsheet:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Revolutions</td>
<td>Time</td>
<td>Adjusted Time</td>
<td>Angular Displacement (Experimental)</td>
<td>Angular Displacement (Predicted)</td>
<td>Difference Squared</td>
<td>Angular Displacement (Particular Solution ( \omega=2 ))</td>
<td>Difference Squared</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(sec)</td>
<td>(sec)</td>
<td>(rad)</td>
<td>(rad)</td>
<td>(rad)^2</td>
<td>(rad)</td>
<td>(rad)^2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5.29</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10.58</td>
<td>4.29</td>
<td>4.29</td>
<td>18.36</td>
<td>18.36</td>
<td>18.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>15.87</td>
<td>9.29</td>
<td>9.29</td>
<td>87.01</td>
<td>87.01</td>
<td>87.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>21.16</td>
<td>14.29</td>
<td>14.29</td>
<td>206.05</td>
<td>206.05</td>
<td>206.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>25</td>
<td>26.45</td>
<td>19.29</td>
<td>19.29</td>
<td>415.10</td>
<td>415.10</td>
<td>415.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>31.74</td>
<td>24.29</td>
<td>24.29</td>
<td>624.15</td>
<td>624.15</td>
<td>624.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>35</td>
<td>37.03</td>
<td>29.29</td>
<td>29.29</td>
<td>833.20</td>
<td>833.20</td>
<td>833.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>42.32</td>
<td>34.29</td>
<td>34.29</td>
<td>1042.24</td>
<td>1042.24</td>
<td>1042.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For \( \gamma = 2 \),

\[
\theta = \frac{I_D}{\mu r} \ln \left[ 1 + \frac{\mu r}{I_D} \omega_0 \right] 
\]

Angular Displacement vs. Time

\( \gamma = 1.954 \)

\( \mu = 0.0235 \)

\( \omega_0, t \), in Cell J11


Proceedings of the 2002 American Society for Engineering Education Annual Conference & Exposition
Copyright © 2002, American Society for Engineering Education

Page 7.1134.12
Appendix II

Visual Basic Macro Function

Private Function Theta(ind, wo, Id, muf, rf, gamma, et)
    'This function calculates the predicted angular displacement

    ' SYMBOLS LIST
    'et = elapsed time (sec)
    'gamma = unknown linear velocity exponent (unitless)
    'Id = mass moment of inertia of disk (kg\cdot m^2)
    'ind = indicator of solution to use
    'ind' equals -1, the general friction model is used
    'ind' equals 0, the kinetic friction model is used
    'ind' equals 1, the eddy current friction model is used
    'ind' equals 2, the aerodynamic friction model is used
    'muf = unknown coefficient of friction (units unknown for general case)
    'rf = radius position to friction source (m)
    'Theta = predicted angular displacement (rad)
    'wo = initial angular velocity (rad/sec)

    If (ind = -1) Then
        ' General Friction Model is used
        a = Id / ((2 - gamma) * muf * rf ^ (1 + gamma))
        b = (wo ^ (2 - gamma))
        c = (wo ^ (1 - gamma))
        d = ((1 - gamma) * muf / Ig) * rf ^ (1 + gamma)
        e = (2 - gamma) / (1 - gamma)
        Theta = a * (b - (c - d * et) ^ e)
    ElseIf (ind = 0) Then
        ' Kinetic Friction Model is used
        a = wo
        b = (muf * rf) / (2 * Id)
        Theta = a * et - b * et ^ 2
    ElseIf (ind = 1) Then
        ' Eddy Current Friction Model is used
        a = (wo * Id) / (muf * rf ^ 2)
        b = (muf * wo * rf ^ 2) / Id
        Theta = a * (1 - Exp(-b * et))
    ElseIf (ind = 2) Then
        ' Aerodynamic Friction Model is used
        a = Id / (muf * rf ^ 3)
        b = (muf * wo * rf ^ 3) / Id
        Theta = a * Log(1 + b * et)
    End If
End Function