Torsional Strength of Steel Machine Screws

Harvey Abramowitz and Laura Elliott

Department of Mechanical Engineering
Purdue University Calumet
Hammond, IN 46323

Abstract

An experiment that measures the torsional strength of metallic screws is described. The major objective is to use this experiment as the vehicle for students to gain an understanding of some basic statistical methods for analyzing data.

Introduction

Students majoring in Mechanical Engineering take a course in experimental methods during their junior year. This course includes some statistical methods for analyzing data, utilizing a well known text.[1] However, it has been generally found that a senior student has forgotten much of those techniques. In an effort to reinforce and solidify the learning of these methods, an experiment was devised that uses torsion testing of metallic screws as the vehicle for understanding and, hopefully, remembering some basic techniques of data analysis. In addition, the students learn something about product testing and the variability of real commercial products.

The specific objectives are:

1. To introduce statistical techniques for evaluating and comparing the means and variances of different samples.
   a) To determine the mean of a sample.
   b) To determine the standard deviation of a sample.
   c) To determine if a sample property is normally distributed.
      1) To construct histograms.
      2) To construct quantile plots.
      3) To construct normal probability plots.
   d) To determine confidence intervals.
   e) To determine if the variances are different for two samples, using the *F* test.
   f) To determine if the means are the same for two different samples, using the Student *t* test.

2. To teach students about types of fasteners, and parts of a screw.
3. To teach students about, at minimum, one important property of machine screws.

It is expected that the student will be familiar with PC's, spreadsheets and some very elementary statistical tools.

Fasteners

Types
There are many types of common threaded fasteners. Figure 1 shows a number of them in schematic form. Of particular interest for this experiment are the different types of recessed drives found on the heads of the fasteners. Most common in ordinary use are the slot, hex, phillips and torx heads. The focus here will be on the slot, phillips, and combination slot/phillips head screws (Figure 2).

Machine Screws
The elements of a machine screw are illustrated in Figure 3. The following features are clearly identified: crest, root, pitch, depth of thread, base of thread, thread angle, helix angle, major diameter, minor diameter, and pitch diameter.

The sizing of these screws is given in Table I. Numbers 8 and 10 size steel machine screws will be used in this experiment.

Torsional Strength Test

The torsional strength test apparatus for this experiment is shown schematically in Figure 4. The actual machine can be seen in Figure 5. The manufacturer is Greenslade and Company, Inc., Fort Worth , TX. Basically, the screw specimen is put into a fixture which is held securely in place in the base of the unit. An appropriate screw bit is attached to a torque wrench. A torque is then applied to the test specimen until failure.

Note to Instructors: While such equipment when new can cost well over $1000, all the equipment described here was purchased at auction for $22.

Statistics for Analyzing Data

Mean
The mean, $\overline{x}$, is simply the average of a sample from a population. It is defined as

$$\overline{x} = \frac{\sum_{i=1}^{i=n} x_i}{n} \qquad \text{Eq. 1}$$

When n goes to infinity, the symbol μ is commonly used. Thus the total population is considered and s is also called the 'population mean.'

$$\mu = \lim_{n\to\infty} \frac{\sum_{i=1}^{i=n} x_i}{n} \qquad \text{Eq. 2}$$

An EXCEL spreadsheet is easy to use to find mean values. As an example, put the values of the sample in a column of a spreadsheet. Go to the cell where the mean should appear, click *fx*, and choose *Statistical* as the Category, followed by *Average* . Left click OK for *Average* and then highlight the numbers that will be used in the calculation. Then click OK and the mean or average ($\overline{x}$) will be determined. When *n* becomes large $\overline{x} \to \mu$.

Variance

Given a sample of a population, it is important to know how much deviation the sample has from the mean. The deviation for each value in the sample from the mean can be written as:

$$d_i = x_i - \overline{x} \qquad \text{Eq. 3}$$

The mean deviation, or residual, would be:

$$\overline{d} = \frac{\sum_{i=1}^{i=n} (x_i - \overline{x})}{n} \qquad \text{Eq. 4}$$

and can be shown to always equal to zero.

$$= \frac{\sum_{i=1}^{i=n} x_i}{n} - \frac{\sum_{i=1}^{i=n} \overline{x}}{n} = \overline{x} - \frac{n\overline{x}}{n} = \overline{x} - \overline{x} = 0 \qquad \text{Eq. 5}$$

Therefore, a different approach is needed to express deviation from the mean. A zero sum can be avoided if the residuals are squared and then summed. The summation will thus be a positive number.

$$\sigma^2 = \frac{\sum_{i=1}^{i=n} (x_i - \overline{x})^2}{n} \qquad \text{Eq. 6}$$

Standard Deviation

When the population is considered infinite, the standard deviation, σ, is:

$$\sigma = \lim_{n\to\infty} \left[\frac{\sum_{i=1}^{i=n} (x_i - \mu)^2}{n} \right]^{1/2} \qquad \text{Eq. 7}$$

Most often, the sample size, *n,* does not become infinite, and is only a representative sample of the total population. For these cases the standard deviation commonly used is:[2]

$$\sigma = \left[\frac{\sum_{i=1}^{i=n} (x_i - \bar{x})^2}{n} \right]^{1/2} \qquad \text{Eq. 8}$$

and is the square root of the variance found in Eq. 6. Even more common and for when *n* is small, the 'Bessel correction term,' *n-1*, is used in the denominator.[2]

$$s = \left[\frac{\sum_{i=1}^{i=n} (x_i - \bar{x})^2}{n-1} \right]^{1/2} \qquad \text{Eq. 9}$$

The standard deviation has the same units as the measurement for x_i.
Like the mean, EXCEL can be used to find the standard deviation. The method is similar to that of the mean. Go to the cell where the standard deviation should appear, click *fx*, and choose *Statistical* as the Category, followed by *STDEV*. Left click OK for *STDEV*and then highlight the numbers that will be used in the calculation. Then click OK and the standard deviation will be determined. *STDEV* uses the n-1 correction term. There are other standard deviation functions in EXCEL that consider the total population.

As *n* becomes larger *s* becomes closer to σ. The percentage difference between σ and s is:[2]

$$d = \left(\frac{s - \sigma}{s} \right) \times 100\% \qquad \text{Eq. 10}$$

Substituting σ (Eq. 8)and *s* (Eq. 9) into Eq. 10, *d* becomes:

$$d = \left[1 - \left(\frac{n-1}{n} \right)^{1/2} \right] \times 100\% \qquad \text{Eq. 11}$$

For the purposes of this experiment *n* will range from 50 to 100. Therefore, *d* will range from 0.5 (100 samples) to 1.0 percent (50 samples). The differences will not be highly significant.

Normal Distribution
The 'bell shaped curve' is an expression that is familiar to most people, even if they are not aware of its statistical significance. A normal distribution can be found in Figure 6. It is symmetric about the mean. The area under the curve is indicated for standard deviations up to $\pm 6\ \sigma$. The curve is formed by using the 'normal probability density function':

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \qquad \text{Eq. 12}$$

When the density function is integrated between $-\infty \le x \le +\infty$, the area obtained is 1. The density function is a normalized one. The probability that x lies between $-\sigma$ and $+\sigma$ can be found by the following integration.

$$P(\mu - \sigma \le x \le \mu + \sigma) = \frac{1}{\sqrt{2\pi}} \int_{\mu-\sigma}^{\mu+\sigma} e^{-(x-\mu)^2/2\sigma^2} dx \qquad \text{Eq. 13}$$

To enable solution the random variable z is introduced.[2]

$$z = \frac{x-\mu}{\sigma} \qquad \text{Eq. 14}$$

with z having a mean of 0 and a standard deviation of 1. So Eq. 12 becomes:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-(z)^2/2} \qquad \text{Eq. 15}$$

Integrating now becomes

$$P(-1 \le z \le +1) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{+1} e^{-z^2/2} dz \qquad \text{Eq. 16}$$

and can only be solved numerically. Figure 7shows $f(z)$ as a function of z. The probability of z lying between $\pm\sigma$ is 0.68, or 68% of all sample values lie between $\pm 1\sigma$ (shaded area). Figure 6 shows the areas for z lying between $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$, $\pm 4\sigma$, $\pm 5\sigma$, and $\pm 6\sigma$.

In order to compare variances and means of different samples, it is statistically easier if the samples are normally distributed. Therefore, several methods of testing for normality follow.

Tests for Normality

Histograms

Histograms provide a visualization of data which can help in the determination of the type of distribution for a given sample. If the plot is "unimodal, is symmetrical, and tapers off at the tails, normality is a definite possibility and may be sufficient information in many practical situations. The larger the sample size, the better the judgement of normality. A minimum sample size of 50 is recommended."[3]

The following steps are suggested by Besterfield[3] to construct a histogram:

1. Collect data and construct tally sheet
2. Determine the range

   $R = X_h - X_l$

where R = Range; X_h = highest number; X_l = lowest number

3. Determine the cell interval
   The cell interval is the distance between adjacent cell midpoints. Whenever possible, an odd interval such as 0.001, 0.07, 0.5, or 3 is recommended so that the midpoint values will be to the same number of decimal places as the data values. The simplest technique is to use Sturgis' rule, which is

$$i = \frac{R}{1 + 3.322\log n} \quad \text{where n is the sample size} \qquad \text{Eq. 17}$$

4. Determine the cell midpoints
5. Determine the cell boundaries.
   Cell boundaries are the extreme or limit values of a cell, referred to as the upper boundary and the lower boundary.
6. Post the cell frequency.

Quantile Plots[4]

Suppose the fraction, *f*, of a sample is less than or equal to the value *q(f)*. *q(f)* is termed the quantile for that fraction. For a particular probability distribution, *q(f)* can be determined for any value of *f* and any sample size. It is expected that the experimental quantile (i.e. the quantile for the data obtained in the experiment) will be very similar to the theoretical quantile if the data follow the theoretical distribution closely. Put another way, if the experimental data is plotted against the theoretical quantile the points would be expected to lie along a straight line. If true, then the sample is considered to be normally distributed. A quantile plot may be constructed as follows:

1. Order the experimental data x_i from smallest to largest.
2. Calculate the fraction of values, f_i, less than or equal to the i^{th} value using the relationship:
   $f_i = (i\text{-}3/8)/(n+1/4)$ Eq. 18
   where n is the number of values in the sample.
3. Calculate the qunatile, $q(f_i)$, for all the fractions, f_i (i=1 to n), for the standard normal distribution using:

   $q(f_i) = 4.91[f_i^{0.14} - (1-f_i)^{0.14}]$ Eq. 19
4. Plot a graph of x_i versus $q(f_i)$

Normal Probability Plots[3]

Another test of normality is the plotting of the data on normal probability paper. Different probability papers are used for different distributions. The step by step procedure follows.

1. Order the data. Each observation is recorded from the smallest to the largest. Duplicate observations are recorded.
2. Rank the observations. Starting at 1 for the lowest observation, 2 for the next lowest observation, and so on, rank the observations.
3. Calculate the plotting position. This step is accomplished using the formula
   $PP = 100(i-0.5)/n$
   where i = rank
   PP = plotting position in percent
   n = sample size

The first plotting position is 100(1-0.5)/*n*

4. Label the data scale. The coded values are the y values. The horizontal scale represents the normal curve and preprinted on the paper.
5. Plot the points. The plotting position and the observation are plotted on the normal probability paper (Figure 8).
6. Attempt to fit by eye a "best" line. A clear plastic straightedge will be most helpful in making this judgement. When fitting this line, greater weight should be given to the center values than to the extreme ones.
7. Determine normality. This decision is one of judgement as to how close the points are to the straight line. If we disregard the extreme points at each end of the line, we can reasonably assume that the data are normally distributed.

If normality appears reasonable, additional information can be obtained from the graph. The mean is located at the 50th percentile. Standard deviation is two-fifths the difference between the 90th percentile and the 10th percentile. A minimum sample size of 30 is recommended.

Confidence Intervals[5]

It is desired to test the hypothesis that the true value of the population mean μ is $\mu_{,(}$ any real number including zero).

$$H_o:\mu = \mu_o \qquad \text{Eq. 20}$$

Further, it is desired to test this hypothesis on the basis of a random sample of size *n* drawn from the population. Implied in the statement of the null hypothesis of Eq. 20 is the alternative hypothesis

$$H_1 : \mu \neq \mu_o \qquad \text{Eq. 21}$$

To proceed with the test of null hypothesis H_o it is first necessary to select an appropriate level of significance α

$$\alpha = P\{\text{rejecting } H_o | H_o \text{ is true}\} \qquad \text{Eq. 22}$$

Equation 22 is to be read "α is equal to the *probability* of rejecting H_o given that H_o is true."

After having selected the level of significance α, it is necessay to define the critical (rejection) region for the sample statistic z, by finding a value z_o such that

$$P\{-z_o \leq z \leq z_o\} = 1 - \alpha \qquad \text{Eq. 23}$$

With the critical region established by Eq. 23 as depicted in Figure 9, the next step is to draw the random sample and calculate

$$(z)_{sample} = \frac{x - \mu_o}{\sigma} \qquad \text{Eq. 24}$$

either exactly or approximately, depending on whether σ is known or must be estimated. Then, on the basis of Eq. 24, if z_{sample} as calculated falls into the rejection region as defined by Eq. 23

and Figure 9, the hypothesis H_o is rejected at the α level of significance. If z_{sample} does not fall into the critical region, the null hypothesis H_o is not rejected. As an example, suppose that one wishes to test the hypothesis

$$H_o{:}\mu = 50{,}000 \text{ psi} \qquad \text{Eq. 25}$$

based on the sample mean of 12 specimens drawn from the population. Suppose that independent estimates of the population standard deviation indicate that $\sigma = 2000$ psi. It is desired to test the hypothesis of Eq. 25 at the 0.05 level of significance. Assume that z has a standard normal distribution. For a significance level of 0.05, with a symmetrical distribution as shown in Figure 9, the value of z_o is the z value corresponding to $(1-\alpha/2)$ accumulated area under the distribution curve. A table,[5] which lists the Cumulative Distribution Function for the Standard Normal Distribution shows that z_o is 1.96. The next step is to calculate the statistic z based on the sample, namely

$$z_{sample} = \frac{\overline{x} - \mu_o}{\sigma / \sqrt{n}} \qquad \text{Eq. 26}$$

$\overline{x}$ is the sample mean of a random sample drawn from the population. The sampling distribution for $\overline{x}$ can be shown to have a mean μ and standard deviation $\sigma / \sqrt{n}$. Now suppose that the 12-specimen sample yields a sample mean of 47,500 psi. The statistic of Eq. 26 would be

$$z_{sample} = \frac{47{,}500 - 50{,}000}{2000 / \sqrt{12}} = -4.2 \qquad \text{Eq. 27}$$

Since z = -4.2 lies in the rejection region, the hypothesis H_o that μ = 50,000 psi is rejected at the 0.05 level of significance.

Testing hypotheses such as the one given in Eq. 25 is intuitively unappealing since the probability of finding that the mean μ is exactly equal to 50,000 psi must be very small indeed. A better approach to the estimation of population parameters is the technique of establishing ***confidence limits*** on the parameter in question. To establish $100(1-\alpha)$ percent confidence limits, it is necessary only to solve the equations $z = \pm z_o$ from Eq. 23 for the parameter. These solutions yield the $100(1-\alpha)$ percent confidence limits for this parameter.

For example, suppose that it is desired to find the 95 percent confidence limits on the mean μ of a given population, utilizing the sample mean $\overline{x}$ of a random sample drawn from the population. Again, it is known that

$$z_{sample} = \frac{\overline{x} - \mu_o}{\sigma / \sqrt{n}} \qquad \text{Eq. 28}$$

since the sampling distribution for $\overline{x}$ can be shown to have a mean μ and standard deviation $\sigma / \sqrt{n}$, in view of the ramifications of the central limit theorem, which will not be further discussed here. Specifying a confidence of 95 percent implies a significance level α of 0.05. Consider, then, based on Eq. 23,

$$P\left\{-z_o \le \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} \le z_o\right\} = 1-\alpha = 0.95 \qquad \text{Eq. 29}$$

The critical value of z_o corresponding to a symmetrical two-tailed critical region with $\alpha = 0.05$ is

$$z_o = 1.96 \qquad \text{Eq. 30}$$

Therefore, Eq. 29 may be written as

$$P\left\{-1.96 \le \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} \le 1.96\right\} = 0.95 \qquad \text{Eq. 31}$$

The inequality to the left in Eq. 31 may be written alone as

$$-1.96 \le \frac{\overline{x}-\mu}{\sigma/\sqrt{n}} \qquad \text{Eq. 32}$$

which when inverted yields

$$\mu \le \overline{x} + \frac{1.96\sigma}{\sqrt{n}} \qquad \text{Eq. 33}$$

Similarly, the inequality to the right in (Eq. 31) may be inverted to give

$$\mu \ge \overline{x} - \frac{1.96\sigma}{\sqrt{n}} \qquad \text{Eq. 34}$$

The results of (Eq. 33) and (Eq. 34) may then be incorporated in (Eq. 31) to give

$$P\left\{\overline{x} - \frac{1.96\sigma}{\sqrt{n}} \le \mu \le \overline{x} + \frac{1.96\sigma}{\sqrt{n}}\right\} = 0.95 \qquad \text{Eq. 35}$$

which, when written in corresponding confidence limit notation, becomes

$$C\left\{\overline{x} - \frac{1.96\sigma}{\sqrt{n}} \le \mu \le \overline{x} + \frac{1.96\sigma}{\sqrt{n}}\right\} = 95 \text{ percent} \qquad \text{Eq. 36}$$

To be more specific, suppose that a sample of 25 specimens has been drawn at random from a given population of aluminum bars and it is found that the mean strength calculated for the sample is $\overline{x}$ = 13,000 psi. Suppose further that the population standard deviation has somehow been estimated (perhaps from the same sample) to be σ = 2000 psi. Then the 95 percent confidence limits on the population mean, based on the statistic

$\overline{x}$, would be

$$C\left\{13,000-\frac{1.96(2000)}{\sqrt{25}} \leq \mu \leq 13,000+\frac{1.96(2000)}{\sqrt{25}}\right\}=95 \text{ percent}$$

Eq. 37

or $$C\{12,200 \leq \mu \leq 13,780\}=95 \text{ percent}$$ Eq. 38

This is to say that with 95 percent confidence it may be predicted that the population mean is in the range from 12,200 to 13,780 psi.

It may be noted that the length of the confidence internal $2z_o/\sqrt{n}$ is a function of the significance level α and also of the number of specimens in the sample. To decrease the length of the confidence interval, and thereby improve the quality of the estimate, one may either reduce the confidence or increase the sample size.

Determination of Number of Measurements to Assure a Significance Level

Based on the last section, the sample size can be determined given a significance level. In this experiment the torque wrench (0-100 in-lb) has a precision of ± 2.5 in-lb. It is desired to find the number of measurements necessary to establish the mean with a 5% level of significance, such that $\mu = \mu \pm 1$ in-lb. Using the notation of Holman,[6]

$$\Delta = z\sigma / \sqrt{n}$$ Eq. 39

with $\Delta = 1$

σ=2.5

n = ?

For a 5% level of significance z = 1.96. Therefore,

$$1 = (1.96)(2.5)/\sqrt{n}$$

and n = 24

It was suggested in previous sections that discussed testing a sample for normal distribution to use a higher n, such as 30. 50 samples from each population were chosen for testing.

Chauvenet's Criterion

Often times a set of experimental values contain some that seem not to belong to the set. This could be due to experimental errors. In order to determine if such points are indeed outliers, and be eliminated from the set, Chauvenet's Criterion can be utilized. The data is assumed to follow a Gaussian or normal distribution. According to the Criterion, "a reading may be rejected if the probability of obtaining the particular deviation from the mean is less than 1/2n. Table II lists values for the ratio of deviation to standard deviation for various values of n according to this criterion."[6] The application of the Criterion is as follows:

1. Calculate the sample mean and standard deviation, using all data points.
2. Compare the deviation of each point with the standard deviation of the sample set.
3. Eliminate all points not within the criterion, using Table III

4. Recalculate the mean and standard deviation for the new data set.

Comparison of Variances – *F* test

In order to determine if two sample means are the same or different, one must know if their variances can be considered the same or not. An *F* test is run determine this. Both samples are considered to be normally distributed. The following is a synopsis from Kennedy and Neville.[7]

The null hypothesis is that the populations from which the samples were taken have equal variances.

$$H_o : \sigma_1^2 = \sigma_2^2 \qquad \text{Eq. 40}$$

To find if this is true, the distribution of

$$\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \qquad \text{Eq. 41}$$

is studied. s_1^2 and s_2^2 are the variances for samples 1 and 2. Both of these samples were independently and randomly taken from normally distributed populations 1 and 2, having variances σ_1^2 and σ_2^2. The sample sizes are n_1 and n_2, respectively.

The random variable *F* (Eq. 42) follows a distribution known as the *F* distribution.

$$F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \qquad \text{Eq. 42}$$

The numerator and denominator have $\nu_1 = (n_1 - 1)$ and $\nu_2 = (n_2 - 1)$ degrees of freedom. If the null hypotheses Eq. 40 is true, then

$$F = \frac{s_1^2}{s_2^2} \quad \text{with} \quad \sigma_1^2 = \sigma_2^2 \qquad \text{Eq. 43}$$

The *F* distribution is also called the Snedcor *F* Distribution and has an associated probability density function *f(F)*. The total area under the curve for the *F* distribution is 1, and the shape of the curve can be found in standard texts on statistics.[5] A two sided *F* test is typically made.[5] To keep the ratio greater or equal to one, the numerator is made the larger number, $s_1^2 > s_2^2$
If the calculated value for *F* (Eq. 43) is greater than the tabulated value,[5] then the probability that the difference between the two variances is caused by chance alone is smaller than the specified probability (0.05, 0.01 or 0.001) and the null hypotheses can be rejected. Tables for the Cumulative Distribution Function for the F Distribution are used by choosing a value for $\alpha/2$ and knowing the degrees of freedom for each part of the fraction.[5]

Comparison of Means – Student *t* Test[8]

If the F test proves that for a given level of significance the variances of the two populations are equal, the Student t Test can be used to see if the populations have similar means.
The t statistic is defined as:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s_c\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad \text{Eq. 44}$$

where $\bar{x}_1$ and $\bar{x}_2$ are the two sample means, n_1 and n_2 are the two sample sizes, and s_c^2 is the combined population variance estimated from the two samples.

$$s_c = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \qquad \text{Eq. 45}$$

Each estimated variance is weighted by its number of degrees of freedom.

or

$$s_c^2 = \frac{\sum_{i=1}^{i=n_1}(x_1-\bar{x}_1)^2 + \sum_{i=1}^{i=n_2}(x_2-\bar{x}_2)^2}{n_1+n_2-2} \qquad \text{Eq. 46}$$

Another way of writing t is:

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s_d} = \frac{|d|}{s_d} \qquad \text{Eq. 47}$$

where

$$s_d = s_c\sqrt{\frac{n_1+n_2}{n_1 n_2}} \qquad \text{Eq. 48}$$

The t statistic has a t distribution with ($n_1 + n_2$ -2) degrees of freedom. The t distribution is unimodal and symmetrical about t=0.[5] Values for t for various levels of significance and degrees of freedom are found in published tables.[5]

The Student t test is started by choosing a level of significance at which the null hypotheses

$$H_o : \mu_1 - \mu_2 = 0 \qquad \text{Eq. 49}$$

would be rejected, such as 1 or 5 percent. Then t is calculated from Eq.44 or 47. If the calculated t is greater than the tabulated value,[5] the null hypotheses is rejected and the difference in means is considered significant.

Experimental Procedure

Equipment and Supplies

1. Torsional Strength Test Apparatus (Greenslade and Company)
2. Torque Wrenches
   Range 0-25 and 0-100 in-lb
3. Various Size Sockets to Fit Torque Wrenches
4. Assorted Steel Machine Screws
   10-24x1/2 Round Head (Slot and Phillips)
   10-32x1/2 Round Head (Slot and Phillips)
   8-32x1/2 Round Head (Slot and Phillips)
5. Software
   EXCEL
   MINITAB 14 (optional)
6. Graph Papers
   Rectangular Coordinate
   Normal Probability

Torsional Test Procedure[9]

1. Select a split collet that matches test screw's size and pitch (Figures 10-11).
2. Screw sample into collet. Leave at least 2 screw thread pitches above top surface of collet (Figures 10-11).
3. Place collet into split collet holder.
4. Place split collet holder into fixture's base, so screw is directly below rotating spindle of fixture(Figures 4 and 5).
5. Tighten clamp screw in fixture's base to prevent screw from turning in the collet (Figure 4).
6. Affix appropriate style and size driver to lower end of fixture's rotating shaft. Use adaptor or bit holder, as needed.
7. Lower upper arm until bit or socket fully engages screw's head or recess.
8. Clamp firmly upper arm to fixture's main shaft.
9. Adjust driver or socket's height relative to screw's head by rotating threaded adjustment wheel.
10. Engage torque wrench in upper end of rotating shaft's female square recess (Figure 12).
11. Set wrench indicator to "zero."
12. Exert smooth torsional force on the torque wrench in rotary manner until screw twists into two separate pieces.
13. Determine highest torque value observed on the wrench at any point during the test.

Selection of Sample

Type

Collect a variety of steel machine screws from two or more major hardware chains. The sizes should be a mix of 8-32x1/2, 10-24x1/2, and 10-32x1/2. Round heads with slot, Phillips, or combination drives can be used. These were the most common types of screws found off the shelf in retail stores.

Sample Size

The selection of sample size is discussed in the section "Determination of Number of Measurements to Assure a Significance Level." Based on statistical methods and practical experience, a sample size of least 30 should be used. 50 is suggested.

Results and Discussion

10-32x1/2 Round Phillips Head Store A

The data for the trial using 10-32x1/2 Round Phillips Head from Store A is found in Table III. This data will be used to demonstrate the various statistical analysis techniques described previously.

Chauvenet's Criteria

Chauvenet's Criteria was applied to the set of data to find any data that could possibly be eliminate. It was found that one data point could be removed (Table IV). Removal of such point did not have a significant impact. The original mean was 51 and the standard deviation 2.5. With removal of the lone value, the new mean still remained at 51 and new standard deviation became 2.4. It would be expected that the standard deviation would decrease by removing the most outlying data. For other trials as well, application of this criteria did not have a significant effect.

Test for Normality of Distribution

Histogram

The methodology for plotting by hand has been outlined. The histogram can be plotted by hand, or using software like EXCEL or MINITAB. The resulting histograms should all look alike. Perhaps the easiest way to make a histogram is with MINITAB 14. The histogram drawn with this software appears to have a normal distribution (Figure 13). This was not true for all trials, and it was found that a higher sample number, such as 100, would make a better histogram. Based on plots such as these, the populations appear to be normally distributed.

Normal Probability Plots

A plot of this type, should first be drawn manually (Figure 14). Afterwards, the plot can be determined by software (Figure 15), like MINITAB 14. These two plots are similar and the distribution for this sample, as well as all those tested, will be considered normally distributed. These plots corroborate the findings of the histograms

Normal Quantile Plot

The third method is to construct a normal quantile plot using EXCEL. The Plot is found in Figure 16. While not a perfect straight line, there is only a small curvature, and once again, the sample will be considered normally distributed.

Confidence Interval

Using Eq. 36 $$C\left\{\overline{x}-\frac{1.96\sigma}{\sqrt{n}}\leq\mu\leq\overline{x}+\frac{1.96\sigma}{\sqrt{n}}\right\}=95 \text{ percent}$$

the confidence interval with a significance level of 95 %, and the mean (51) and standard deviation (2.5), the confidence interval is:

$$C\{50 \leq \mu \leq 52\}$$

Even if n is decreased to 49, based on Chauvenet's Criteria, the confidence limits do not change because values used are to the nearest whole numbers.

Comparison of Same Size Screws

8-32x1/2 Round Head Slot Screws from Stores A and B will be compared. Means and standard deviations are listed in Table V.

Comparison of Variances

The F test will be used with a significance level of $\alpha = 0.05$ or $\alpha/2 = 0.025$.
The degrees of freedom for a sample size of 50 is 50-1, or 49. ν_1 and $\nu_2 = 49$
The value for F is from Eq. 43

$$F = \frac{s_1^2}{s_2^2} = \frac{1.1^2}{0.8^2} = \frac{1.21}{0.64} = 1.89$$

From tabulated values,5 $F = 2.07$ for 30 degrees of freedom and 1.67 for 60 degrees of freedom. By interpolation, it will be assumed that the variances are equal. EXCEL with the statistical add in Analysis ToolPak (found under the TOOLS heading) and MINITAB 14 software can compute F tests. However, it is better for the student to use the Tables, at least the first time the calculation is performed.

Comparison of Means

The two means can now be compared by the Student t test (Eqs.44 and 45).

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{s_c\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad s_c = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and $t = 22.70$ when $s_c = 0.925$

With a significance level of $\alpha = 0.05$, a published table[5] gives an interpolated value close to 2. $t = 2$ for 60 degrees of freedom and $t = 1.98$ for 120 degrees of freedom. Since the calculated t is greater than the tabulated t, the null hypotheses (Eq. 49) is rejected, and the two sample means can be considered significantly different. Screws from Store A have a higher torsional strength than those from Store B. Therefore, the buyer of machine screws should be aware that the properties of such screws can depend on the manufacturer that a store uses.

Like the F test, it is suggested that the student do this t test calculation by hand at least once, before employing statistical software.

Conclusions

The torsional strength test for steel machine screws can be used to introduce students to the world of product variability, with particular emphasis on a property crucial for fasteners. In order to understand how to measure such variability, different statistical quantities and analytical techniques are taught. The techniques employ both hand calculations, and calculations using statistical software. In order to properly understand the products that can be tested in this fashion, various types of fasteners are discussed.

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Biographical Information

Harvey Abramowitz received a B.S. in Materials Science from the Henry Krumb School of Mines, School of Engineering and Applied Science, Columbia University. He completed M.S. and D.Eng.Sc. degrees from the same school in extractive metallurgy/mineral engineering. Prior to coming to Purdue University Calumet, he was a Research Engineer at Inland Steel Research Laboratories and Visiting Professor at the University of Missouri, Rolla. He has also been a Visiting Scholar at Northwestern University. Dr. Abramowitz is a Professor of Mechanical Engineering and is responsible for the materials sciences courses. Currently, he is also the AIST Foundation Ferrous Metallurgy Grant Professor. Major areas of research are the cryogenic treatment of steels and the treatment of waste streams for metal recovery.

Laura Elliott is a graduate student in the Department of Mechanical Engineering, Purdue University Calumet. She holds a B.S. in Metallurgy from the Illinois Institute of Technology, and has several years of industrial experience.

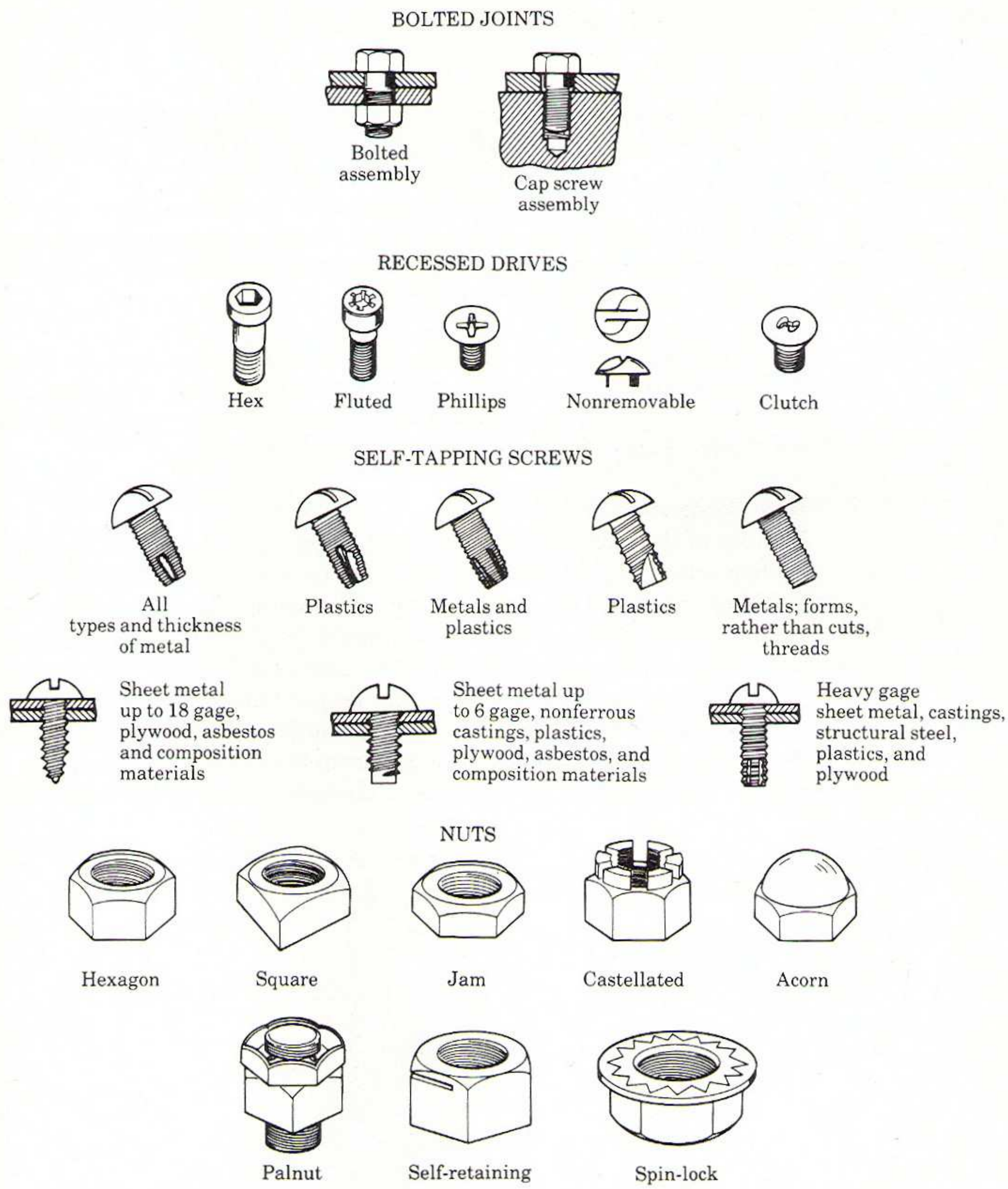

FIGURE 12.1 Selected examples of threaded fasteners. There are many other types and sizes of fasteners available. *Source:* Adapted from *Tool and Manufacturing Engineers Handbook*, 3d ed., McGraw-Hill, 1976.

Figure 1: Selected Examples of Threaded Fasteners
Ref. 10

Figure 2: Slot, Phillips, and Combination Screw Heads

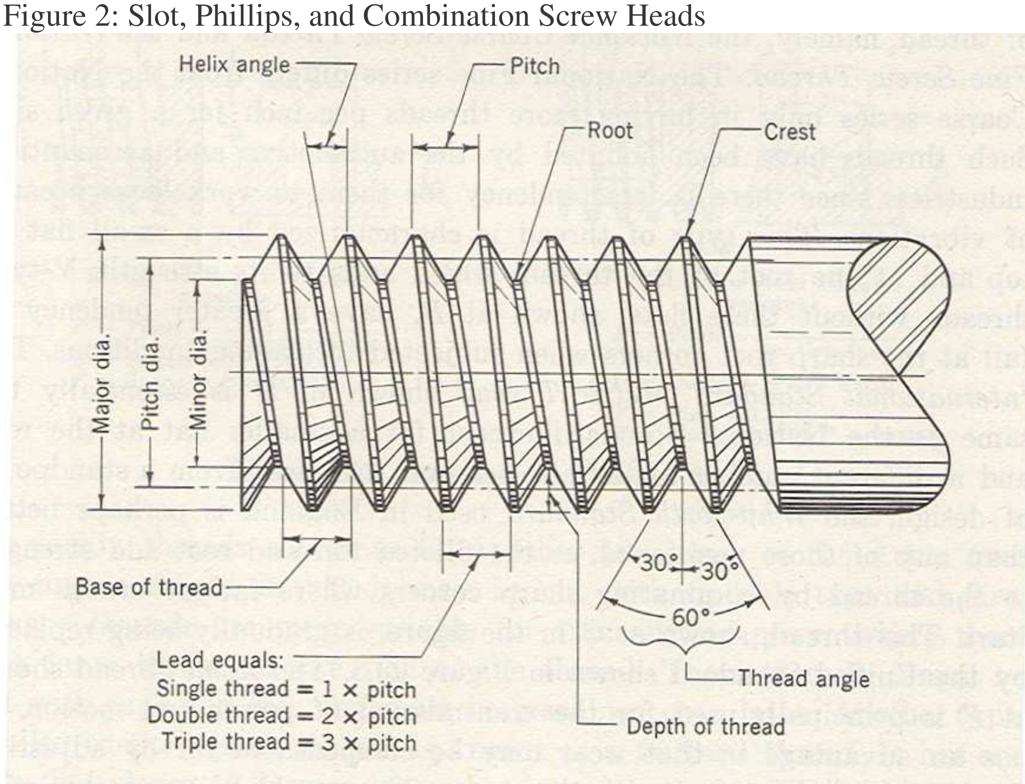

Figure 3: Schematic of a Machine Screw
Ref. 11

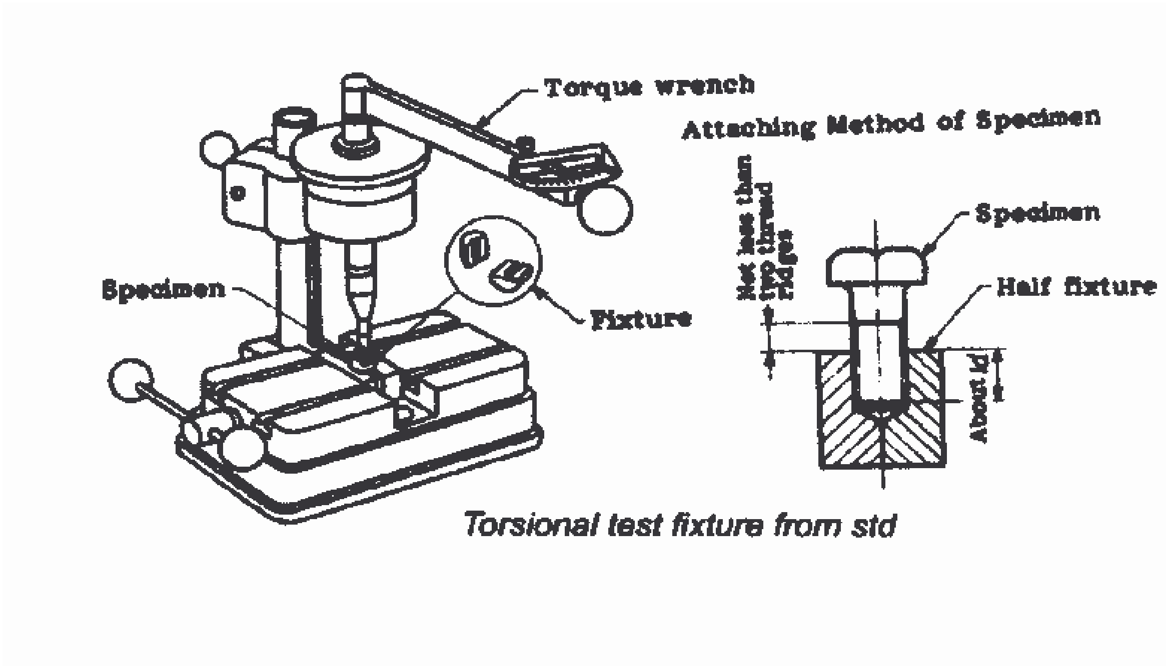

Figure 4: Schematic of Torsional Strength Test Apparatus Ref. 12

Figure 5: Torsional Strength Test Apparatus

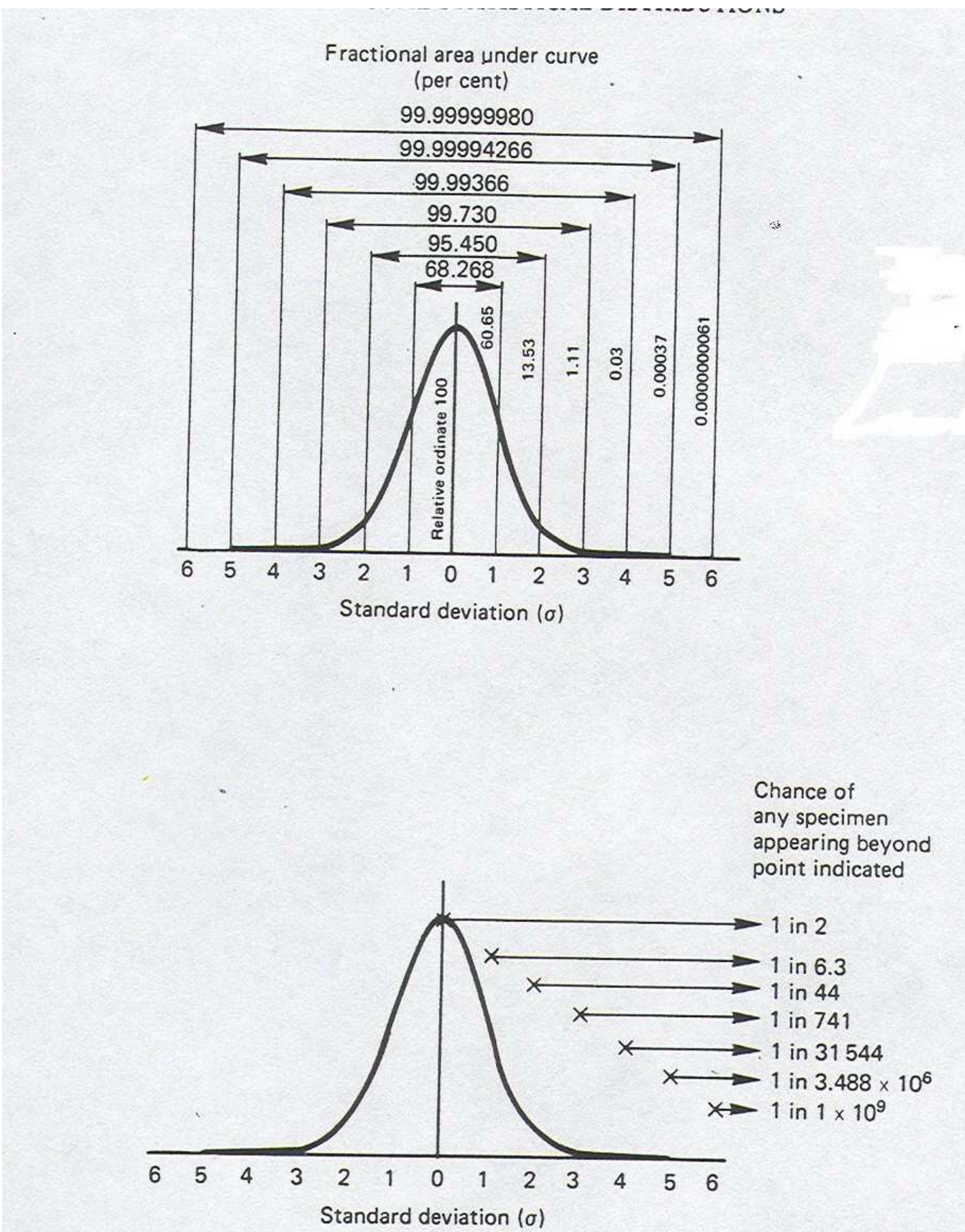

Figure 6: Normal Distribution – 'Bell Shaped Curve'
Some Properties of the Normal Distribution Carried to High Values
of the Standard Deviation
Ref. 13

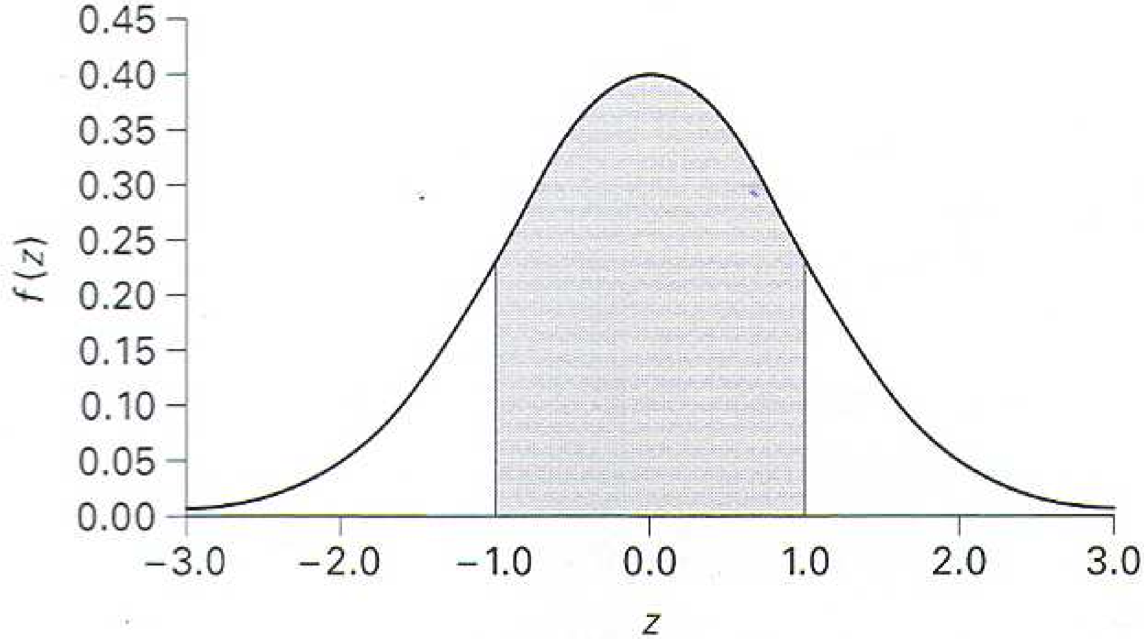

Figure 7: Variation of *f(z)* with *z*. The shaded area is equal to the probability that *z* lies between -1 and +1. Ref. 2

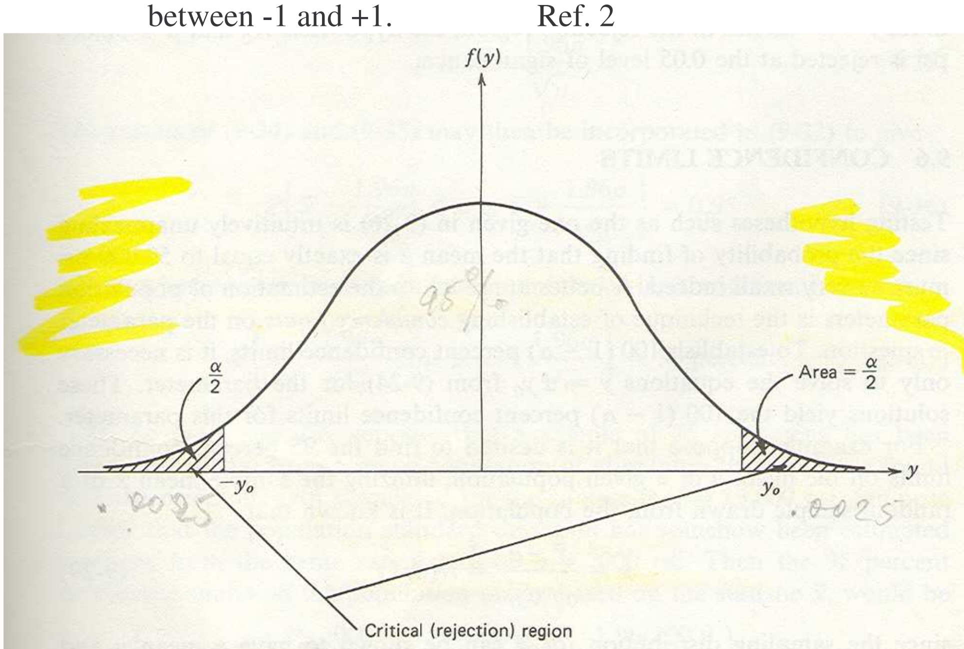

FIGURE 9.7. Graphical representation of the critical (rejection) region for testing $H_o: \mu = \mu_o$ for significance level of α using the sample statistic $y = \frac{x - \mu}{\sigma}$.

Figure 9: Critical or Rejection Region for a Given Significance Level, α.
y or *z*(used in text) = $(x-\mu)/\sigma$ Ref.: 5

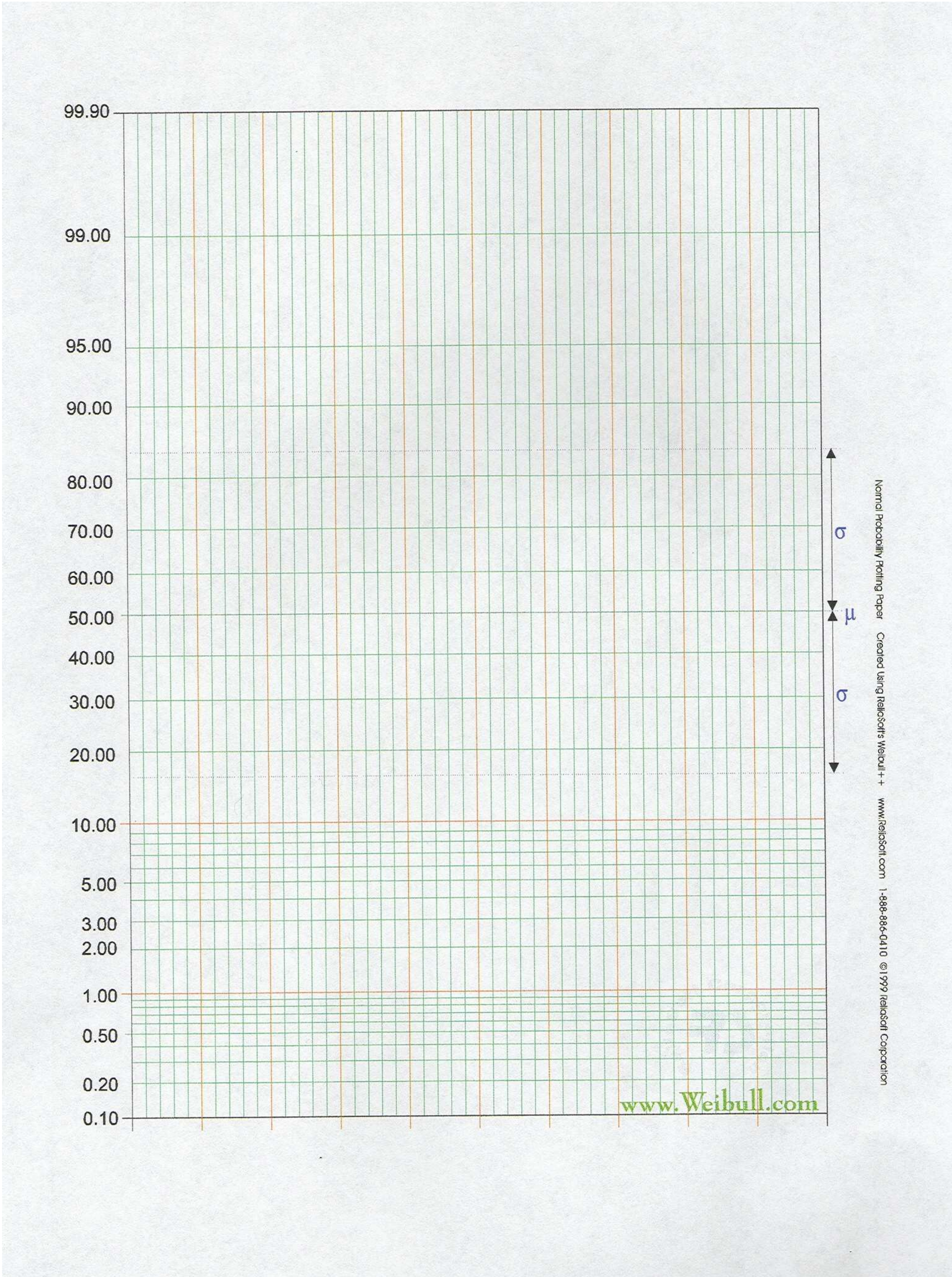

Figure 8: Normal Probability Paper [14]– for Plotting by Hand

Figure 10: Split Collet (Top)

Figure 11: Split Collet (Side)

Figure 12: Torque Wrench (0-100 in-lb)

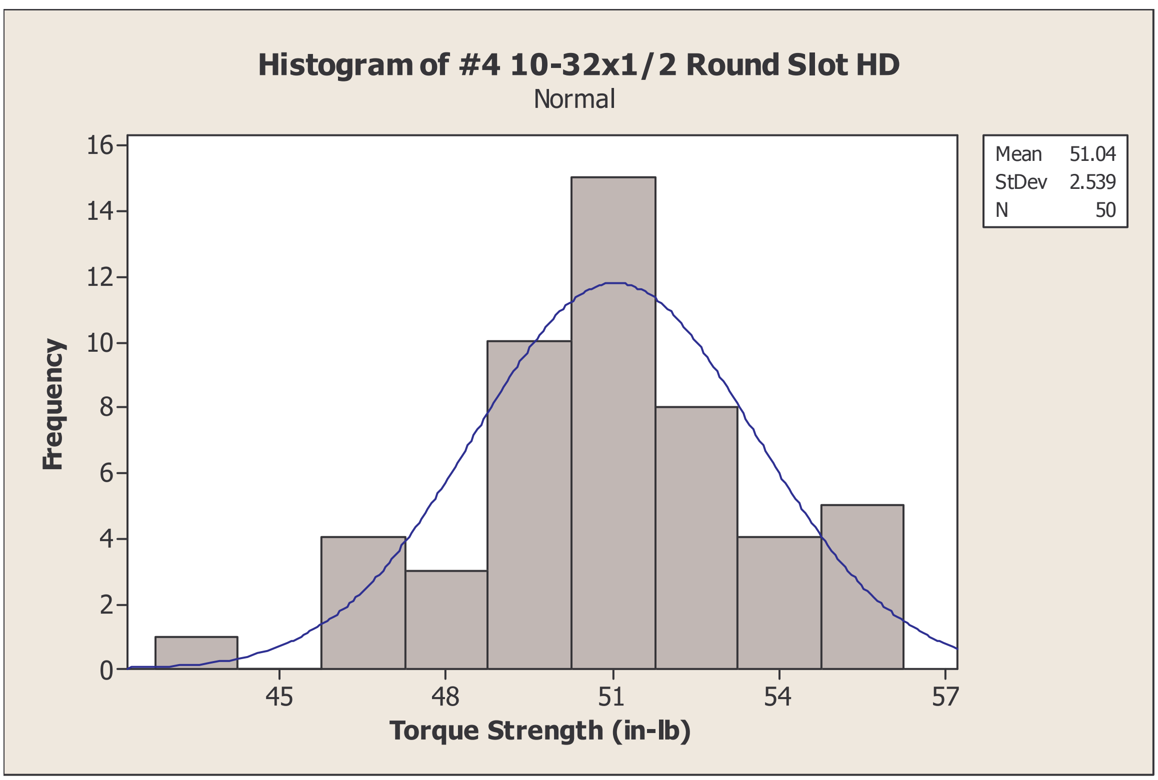

Figure 13: Histogram of 10-32x1/2 Round Head Slot Store A

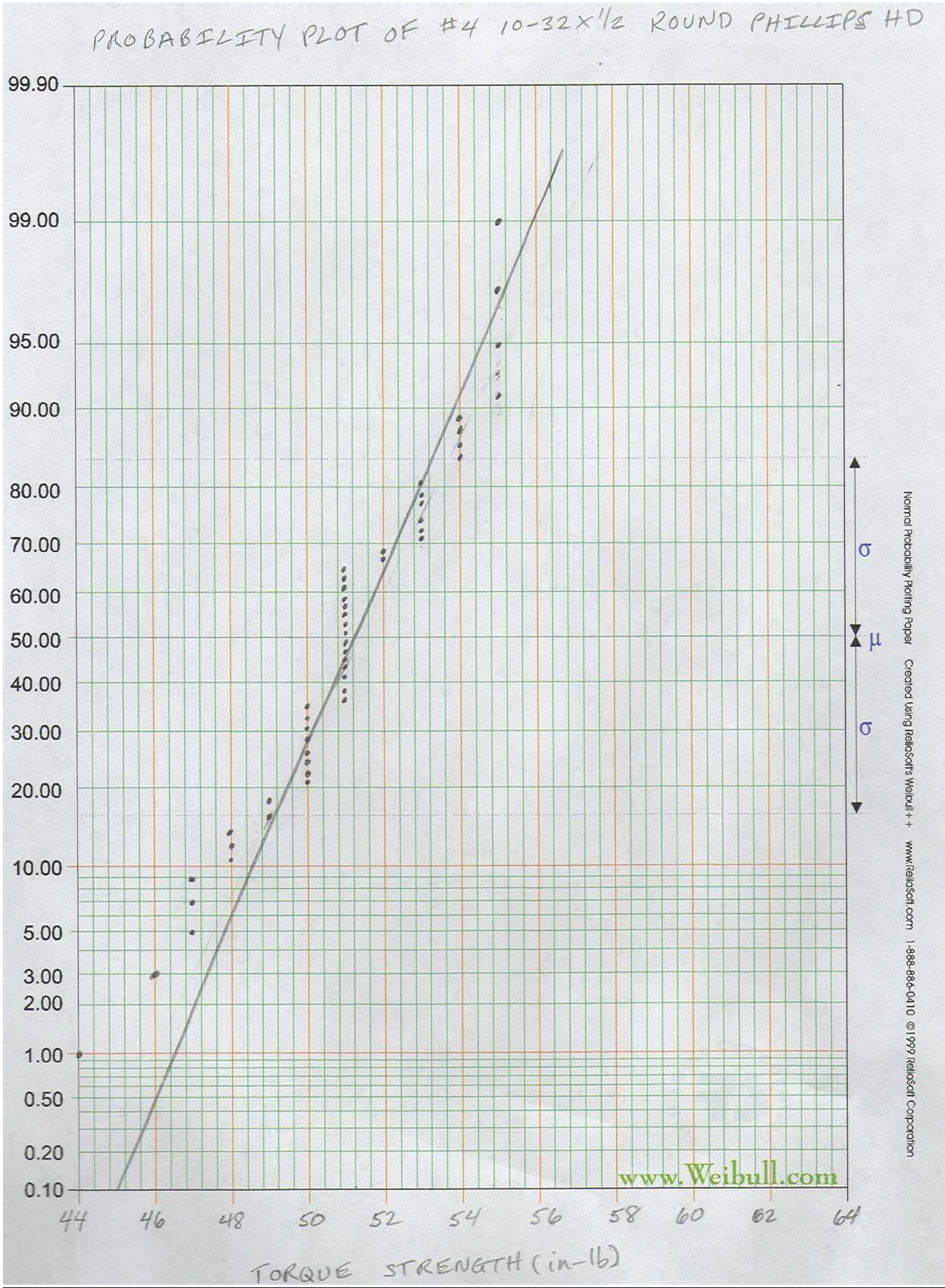

Figure 14: Hand Drawn Probability Plot for 10-32x1/2 Round Head Slot Store A

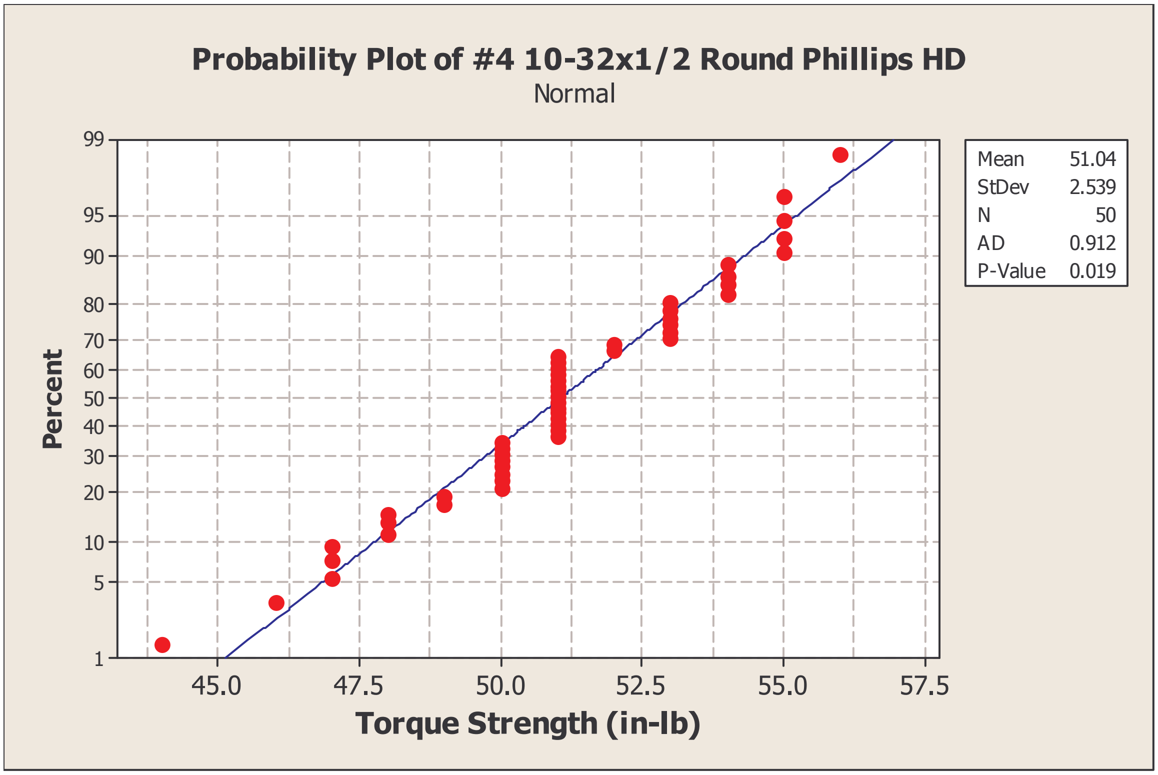

Figure 15: Normal Probability Plot for 10-32x1/2“ Round Phillips Head Store A

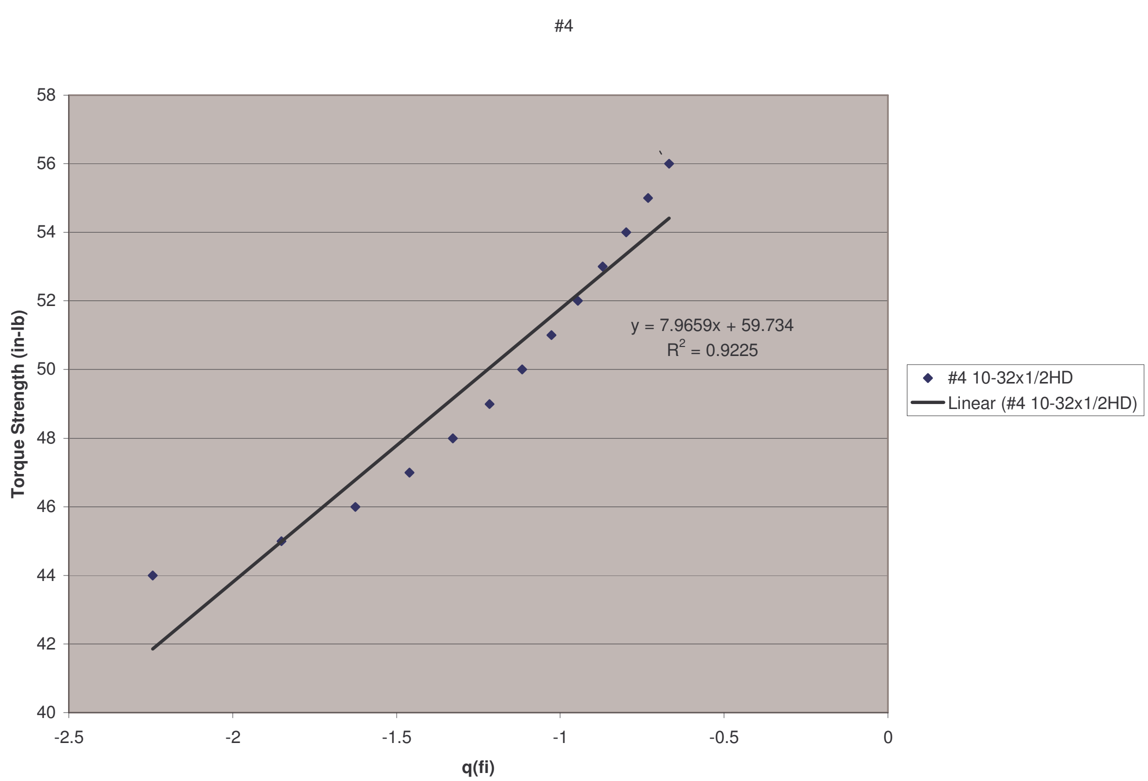

Figure 16: Normal Quantile Plot for 10-32x1/2” Round Phillips Head Store A

Table I: Recommended Tap-Drill Sizes for Standard Screw-Thread Pitches, American National Course-Thread Series

Ref.: 15

Number or Diameter	Threads per Inch	Outside Diameter of Screw	Tap Drill Sizes	Decimal Equivalent of Drill
6	32	0.138	36	0.1065
8	32	0.164	29	0.1360
10	24	0.190	25	0.1495
12	24	0.216	16	0.1770
1/4	20	0.250	7	0.2010
3/8	16	0.375	5/16	0.3125
1/2	13	0.500	27/64	0.4219
3/4	10	0.750	21/32	0.6562
1	8	1.000	7/8	0.875

Table II: Chauvrenet's Criterion for Rejecting a Reading

Ref.: 6

Number of Readings *n*	Ratio of Maximum Acceptable Deviation to Standard Deviation *dmax/σ*
3	1.38
4	1.54
5	1.65
6	1.73
7	1.80
10	1.96
15	2.13
25	2.33
50	2.57
100	2.81
300	3.14
500	3.29
1000	3.48

Table III: #4 10-32x1/2 Round Phillips Head Store A

Observation Torque Strength (in-lb)	Rank i	Plotting Position
44	1	1
46	2	3
47	3	5
47	4	7
47	5	9
48	6	11
48	7	13
48	8	15
49	9	17
49	10	19
50	11	21
50	12	23
50	13	25
50	14	27
50	15	29
50	16	31
50	17	33
50	18	35
51	19	37
51	20	39
51	21	41
51	22	43
51	23	45
51	24	47
51	25	49
51	26	51
51	27	53
51	28	55
51	29	57
51	30	59
51	31	61
51	32	63
51	33	65
52	34	67
52	35	69
53	36	71
53	37	73
53	38	75
53	39	77
53	40	79
53	41	81
54	42	83
54	43	85
54	44	87
54	45	89
55	46	91
55	47	93
55	48	95
55	49	97
56	50	99

Table IV: Chauvenet's Criteria for 10-32x1/2 Round Head Phillips Store A

Chauvenet's Criteria	di/SD
44	-2.77
45	-2.38
46	-1.99
47	-1.59
48	-1.20
49	-0.80
50	-0.41
51	-0.02
52	0.38
53	0.77
54	1.17
55	1.56
56	1.95
Max for 50 samples	2.57
Eliminate 44 – 1 reading	
new Average	51
new Standard Deviation	2.4
old Average	51
old Standard Deviation	2.5

Table V: Comparsion of Same Size Screws

ID	Mean	Standard Deviation
Round Head Slot Store A (RSA)	31.8	1.1
Round Head Slot Store B (RSB)	27.6	0.8