Toward a T-Shaped Integration of Mathematics in Mechanical Engineering

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Abstract

This paper presents a progress report structured to implement instructional methods presented in 3 earlier papers published by the author. Details of the coordinated instructional and assessment approaches were utilized by a faculty team in an engineering sciences core curriculum (ESCC) and are now extended to some upper level technical electives. These instructional guidelines have been part of the ABET continuous improvement process at the author's institution.

After the release of ABET 2000, various teaching and assessment methods have been explored by different faculty teams for engineering science topics. Their recommendations were implemented to develop pedagogy and assessment in ESCC. However, these courses failed to uniformly reinforce important mathematical concepts for various reasons. This shortfall was cited and discussed in recent publications. It is necessary to determine which effective mathematical tools are most needed to teach formulation and solution skills to mechanical engineering students, and how to train engineering faculty to use such tools.

Recently the departmental focus also shifted to topics of applied mathematics necessary to lead students into advanced research in continuum mechanics. Our initial attempt was to choose the most important mechanical engineering topics and demonstrate conceptual breadth and depth necessary for connectivity with previous topics. Last year a status update was published. This paper further presents a flowchart of mathematical preliminaries and their connectivity to advanced fluid mechanics. The examples presented here demonstrate student performance improvement in topics of aerodynamics and ideal flows. The focus group of students demonstrated remarkable clarity in expressing logical arguments. The author believes such recommendations should further be explored and implemented for other solid mechanics and continuum mechanics electives. The current results support a T-shaped integration of applied mathematics in mechanical engineering. They can be shown to link mathematical training of engineering students and desirable ABET outcomes.

Introduction

A few years ago, the college of engineering at Rochester Institute of Technology (RIT) appointed a new dean. The new dean added a new vision: *Build a T-Shaped Curriculum*. Few knew what it meant exactly. So, faculty in the college defined and accepted the vision as an axiom. The shape of the letter T illustrates the structure of such a curriculum – providing breadth (by the horizontal arm) and depth (by the vertical leg) of student learning in all engineering disciplines. The ESCC team in mechanical engineering (ME) had already designed an effective core engineering curriculum almost a decade before this time. It had to make changes according to this new focus. The effort in the present paper is to discuss the role of mathematics for implementation of such a T-shaped curriculum.

ME students learn a significant amount of applied mathematics to succeed functionally. How can the presentation style of conventional mathematical topics be improved so that students become better learners, and also retain mathematical thoughts for life? This is the research focus now.

We present an archived multiple choice (MC) examination question to begin discussion.

13) The acceleration of a particle in rectilinear motion starting from rest is given by the function $a = 3s^2$, where, a is given in ft/s^2 and s in ft. The velocity when the object has moved 2 ft. from its initial position is approximately:

(A) 7 ft/s
(B) 5 ft/s
(C) 4 ft/s
(D) 2 ft/s
(E) None of the above

Fig.1 Student performance assessment example from a Dynamics final examination

The above question was published in an assessment study [1] a few years ago by the Dynamics faculty of ESCC. The examination area involves integration of the acceleration function over a space variable. Most students with wrong answers chose option (E) instead of the correct choice (C). Lack of student performance on this and other examples quoted on the study created a new focus area for faculty to rectify. The deficiency area on this question had already been identified before (and instructional focus had already been implemented by ESCC), and yet there was a further twist in assessing student performance due to a new type of deficiency [2] identified in that particular study.

In 2016, another paper was published by the ESCC group which suggested remedial actions in an introductory/remedial statics course [2]. A follow-on report was published last year [3] which focused further on the remedial actions for specific mathematical deficiencies. The ESCC focus earlier had been to determine connectivity of common dynamics topics which would allow better retention of mathematical concepts. At that time, interest was not focused on details of mathematical connectivity in the whole ME curriculum. This paper presents an expanded implementation of recommendations presented in [3]. We have now broadened connectivity of specific mathematical topics necessary in the whole ME curriculum beginning with high school. The examples illustrated here are suitable for getting students ready for aerodynamics or, computational fluid dynamics (CFD). We gathered test results for assessing our approach on a small segment of ideal flow topics presented in an upper level undergraduate course in fluid mechanics. Similar connectivity may be established for other technical electives.

Since 2006 ME faculty at RIT have systematically compiled examinations and performance data for ESCC use. Teams of faculty who participated in assessing performance on these examinations had developed a seamless teaching/learning environment for ME students which may be adapted to improve instructional strategies if necessary. When fully implemented, these help ME students learn better and meet ESCC direct assessment goals measured by their examination performance. A process for continuous improvement of the curricular design has been implemented in ESCC. The breadth of the RIT course offerings and experiential learning, together with the depth of upper level elective classes provide the RIT mechanical engineers a T-shaped integration of mathematical concepts. Since conceptual assessment is largely based on student responses on MC type examination questions, a brief discussion of MC performance analysis is necessary. Interested readers are requested to find more details in publications [4 – 7].

MC Assessment Process

ESCC faculty teams have studied causes of common failures on examinations using sets of carefully designed MC questions for several years. ESCC collects and preserves actual final examinations and student answers to both MC and analysis questions to develop a deeper understanding of the causes of failure. All ESCC courses have common home assignments and examinations. Some MC questions are always posed in addition to detailed analysis questions on examinations. *The hypothesis is: MC questions, when posed correctly, can accurately determine areas of conceptual difficulties and analysis questions may further provide supporting evidence for such claims.* Monitoring grading uniformity on analysis questions in large sections taught by several different instructors is very difficult. So, student performance on MC questions is always considered first.

Figure 2 shows analysis of 4 questions from ESCC faculty archives to clarify the process. In ESCC classification, a "well-posed" MC question has three requirements. The question must be 1) clearly written, 2) error-free, and 3) answerable within 3 minutes of testing time for average students. Faculty are asked to focus on one or two key concepts only to design the question. Otherwise the question is not posed as an MC question.

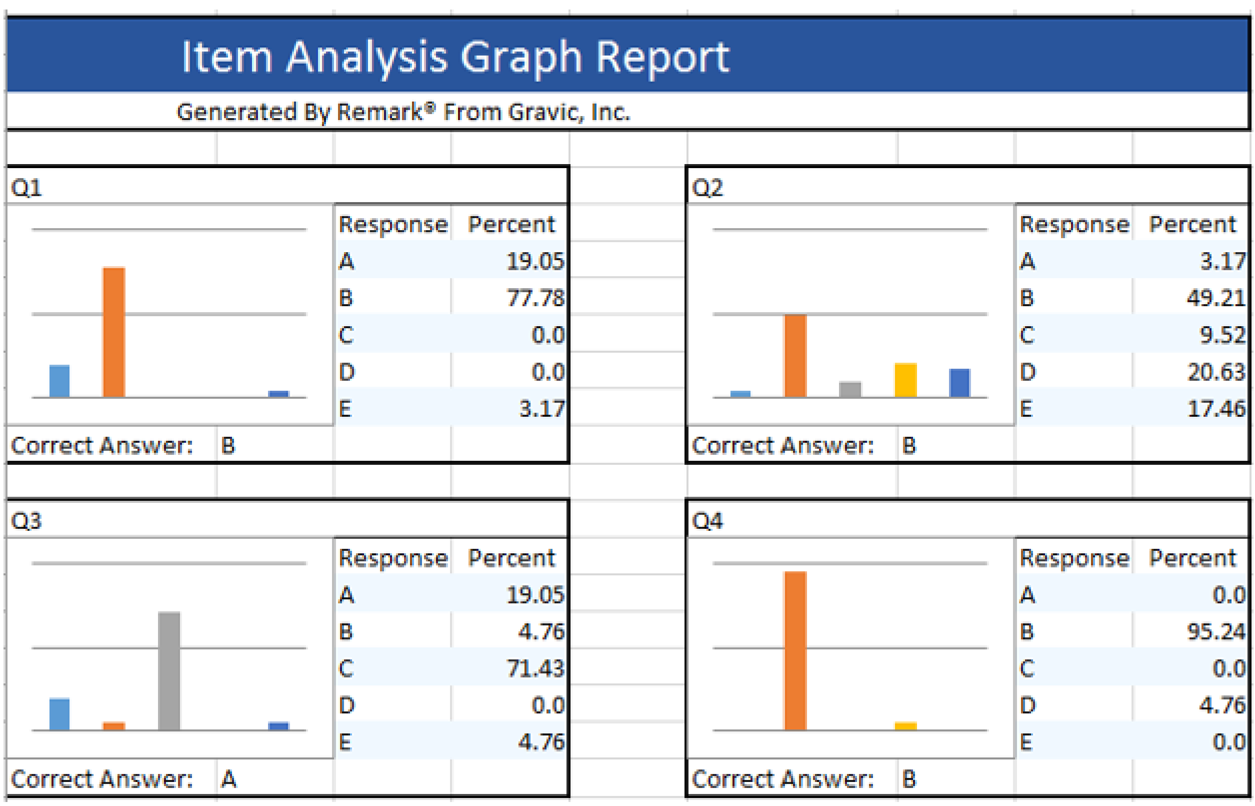


Figure 2. Performance analysis of 4 different MC questions in an examination

Category A questions are those in which questions are well-posed, and 60% or more of the class can answer them correctly. On figure 2, Q1 and Q4 fit this category. Category B questions are those where the questions are well-posed but less than 60% of the class can answer them correctly. Here Q2 fits that category. The response in Q3 on the other hand shows a completely different trend. Such responses may happen due to one of three reasons: 1) the question is incorrectly posed (viz. choices are ambiguous, figures are unclear, expressions/equations have errors, solutions are too long to obtain in the allotted time, etc.) or, 2) incorrect solutions are marked for processing (indicating a human error), or, 3) many students do not answer that question. The first two reasons are rectifiable. In figure 2 above, the reason 2) was applicable. After receiving the scantron report, the human error was quickly discovered and corrected before the examination grades were posted. Ill-posed questions are initially marked category C (needing corrections). After scrutiny and corrections, such questions may finally be considered in category A again.

Over the years ESCC faculty have learnt to design the answer choices on an MC question in such a way that reveals weak performance due to a specific reason. For example, choices D and E in Q2 have some such clues which would be explored further and corrective instructional strategies would be developed. For this purpose, the work space given for each MC question is collected after the test in addition to the answer sheet. Today, archived examination data are available for comparing test performance (on the same/similar question) on midterm examination with final examination, and performance on final examinations over the years. In the present paper we analyzed performance of both MC and analysis type questions on ideal flow topics. Additional description of instructional methods and delivery ideas may be found in references [2] and [3]. After this brief introduction of our assessment process, we now begin searching for mathematical concepts connecting ideal flows.

Mathematics as an Integrator

Mathematical physics has been the catalyst of all engineering advancements for mankind. Beginning with Newton all analytical thoughts found their basis in this subject until computer age began in the 20st century. Easy availability of powerful computational solutions detracts from mathematical thought development today. The purpose of this paper is to initiate a renewed focus on logical reasoning with emphasis leading to connectivity of relevant mathematical topics. We offer suggestions for optimized learning and retention of mathematical thoughts which, we believe, must be practiced by mechanical and aerospace engineers.

Ideal Flows – Pedagogical Relevance

Traditionally ideal flows require classical methods of solution plus powerful complex analysis ideas such as complex potential, complex velocity, complex force, and the residue theorem. But the course on which ideal flows data was collected is an undergraduate elective course in fluid mechanics (Fluids II), and our ME students are not required to take a pre-requisite course introducing complex analysis. Since all students do not have the same background, only classical solution methods are taught. This makes the current course harder to teach. But employing three-course connectivity of topics before on a graduate level had proven to yield better student learning [4]. The same idea was replicated now at the undergraduate level with support from assessment data [3, 7]. As a result of these efforts, students in the present data pool benefited. Student teams demonstrated better reviewing methods, and individual students demonstrated better performance on class tests. The traditional lecture delivery was often flipped for enhanced learning in groups. Brainstorming was encouraged with modified assessment methods as explained later.

MC test questions (and occasionally analysis questions) were used to provide connectivity of topics (see figures 5 – 7 given later). Each of the questions used in this paper was actually posed on a recent examination. Archived data show where and which course earlier introduced the concept and where in the curriculum reinforcement was achieved. The next section presents three steps of conceptual connectivity required in the present approach, followed by some results and some new discoveries. Finally select recommendations are presented and concluded.

Establishing conceptual connectivity – Step 1

Since ideal flows is an essential topic to be understood by all mechanical engineers who wish to master the fluid mechanics area, this area was selected to implement the methodology. Continuity is tested tracing back to the high school and early college courses in mathematics. The reader may consider these recommendations and implement any remedial strategies contextual to his/her own university. This work is considered a developing field where focus has become necessary in recent times. Necessary data which have been the focus for corrective actions before would be recalled here and further connected with upper level conceptual understanding of mathematics. Although this is a study to enhance educational research, faculty involved with fundamental research in classical aerodynamics (or ideal flows) would find this work illuminating to understand mathematical bottlenecks experienced by average undergraduate students in mechanical engineering today.

Week	Text Sections	Course Content: Topics Covered	Home Work & Practice Problems	HW, Test/Quiz Coverage
08/27/18 through 08/31/18	Chapters 1 – 6 Reviews (Note: Chapters 3 and 5 expand topics of MECE210 class) 3.1 – 3.3	**Review of Fluid Mechanics and Calculus, Control Volume (CV) Approach, Flow Visualization Tools, Streamlines, Pathlines & Streaklines. Screening of the Movie: Vorticity (parts I & II)**	**HW1: 1-47, 2-50, 3-16, 4-14, 6-11** Practice : 1-43, 2-42, 3-10, 4-30, 6-11	**Sign and return the honor principle to earn the HW0 credit.** **Quiz 1 will test the pre-requisite concepts for this class**
09/03/18 through 09/07/18	3.4 – 3.5	**Sept. 3 (Labor Day, No Classes!)** **Flow Acceleration** **Read through Introduction.doc on the 550CD to get connectivity of all topics and detailed derivations.**	**HW2: 3-5, 3-36, 3-38, 3-41, 3-43** Practice: 3-14, 3-18, 3-27, 3-35	**HW1 due this week** **Quiz No. 2 will test Summary Reviews of Chapters 1 – 6 (see SummaryCh1-6.pdf on myCourses)**
09/10/18 through 09/14/18	5.1 – 5.5 7.1 – 7.8	**Euler's and Bernoulli equations Differential Fluid Flows, Euler, Bernoulli and the Navier-Stokes equations, Stream Function, Velocity Potential**	**HW3: 5-4, 5-9, 5-11, 5-21, 5-25** Practice: 5-19, 5-23, 5-26, 5-27	**HW2 due this week** **Quiz No. 3 will test on the Movie topics**

09/17/18 through 09/21/18	7.9 - 7.11	**Examples of Solutions of Laplace equation Elementary Plane Flows Superposition Principle**	**HW4: 7-4, 7-13, 7-15, 7-21, 7-24** Practice 7.6, 7-12, 7-17, 7-20, 7.26	**HW3 due this week** **Quiz No. 4 will test Ch3 and 5 Concepts**
09/24/18 through 09/28/18	8.1 – 8.5	**Superposition Principle (contd.) Introduction to Aerodynamics and Wind Tunnels; Dimensional Analysis and Similitude Review**	**HW5: 7-61, 7-67, 7-70, 7-77, 8-37** Practice: 7-47, 7-60, 7-73, 8-56, 8-60	**HW4 due this week** **Quiz No. 5 will test Ch. 7 Concepts**

Figure 3: Fluids II syllabus in part leading up to the ideal flow topics

Figure 3 shows a typical syllabus leading up to the topics of ideal flows presented in Fluids II or any upper level second fluid mechanics class. The course begins with discussion of continuum hypothesis, basic fluid properties and units. After the introductory discussion of fluid statics, selected gadgets such as manometers, pressure gages and pitot tubes are reviewed. These topics were first introduced in the first fluid mechanics course, but students must review them for problem solving [8]. All homework and practice problems are from this reference textbook. Kinematics discussion in fluid flows contrasts fluid behavior with rigid body dynamics and introduces two additional topics of shear deformation and dilatation. The substantial derivative representing fluid acceleration introduces both local and convective terms. These terms become difficult to conceptualize for average students unless stream tubes and other CV examples are used. Worked out examples, class notes, and complete derivations are provided on files archived at the class site in myCourses. The movie Vorticity developed by A. H. Shapiro [9] is an excellent learning aid for these topics. The movie is linked to open from the Internet on myCourses and also screened and discussed in class. Both Lagrangian and Eulerian flow descriptions are presented and contrasted. The class reviews the determination and sketching of streamlines. Then stream function and velocity potential are introduced, and connected conceptually to an orthogonal grid system seen later in CFD. The evaluation of partial derivatives and integration of functions of several variables are presented in full rigor. After reviewing dimensional analysis, Buckingham's pi theorem and streamline coordinates, Bernoulli equation is recalled. The added understanding due to Crocco's theorem recalled from the movie [9] relaxes the use of the application of Bernoulli equation between any two points in a flow field rather than two points on a particular streamline.

Later topics in the course present a big conceptual leap for students. Do they possess sufficient understanding to explain when solution of Laplace equation may replace the solution of Euler's equations? Have they understood ideal flow boundary conditions and pressure-velocity relations? Can they integrate functions of several variables? If they can answer these questions well, they should be mathematically prepared to learn ideal flow applications. We had earlier experienced difficulty in the implementation phase because reinforcement of mathematics was difficult in a single course to provide complete connectivity. Using the ideal flow concepts, we opened a backward tracing approach taking us back to college and high school mathematics which will be presented now in connectivity steps 2 and 3 below.

Mechanical engineers do not need to have the full mathematical rigor of aerodynamicists in ideal flows. But students must receive adequate background to appreciate its scope in meteorology, boundary layers and wind tunnel applications which emphasize the superposition principles. In four weeks, Fluids II class offers an abridged introduction leading to CFD. The inverse design problems and construction of Euler solvers in CFD require a complete understanding of governing differential equations and boundary conditions. Linking the undergraduate mathematical base requires reviewing some concepts first seen in high school. The next section presents specific topics that assist understanding formulation and problem solving in this course.

Connectivity – Step 2

After identifying the technical topics presented in figure 3 it is clear that the review focuses on both control volume analysis and differential equations. The relevant physical concepts link the following mathematical topics with our approach (Fig. 4). The analytical methods require mathematical concepts of Taylor series, line, surface and volume integrals, sign conventions of surfaces and stresses, review of directional lumping and differential element choices for CV and boundary conditions, Leibnitz rule, and integration by parts (depending upon problem selections). Also reviews of basic algebraic identities (applicable for polynomials), functions of several variables, partial differentiation, and various coordinate systems are necessary. Since tensor representations are not possible (our ME undergraduates are unfamiliar), vector calculus emphasized physical/geometrical interpretations, coordinate system fundamentals and some transformation ideas. In addition, dimensional analysis, Buckingham's pi theorem, similarity concepts and non-dimensional differential equations are required to extend concepts further. Note that only mathematical concepts useful for engineering problem solving and of relevance to the course on figure 3 are presented. The application areas are also limited due to time constraints in the course. Motivated students usually learn applications further from this level through project work and graduate studies.

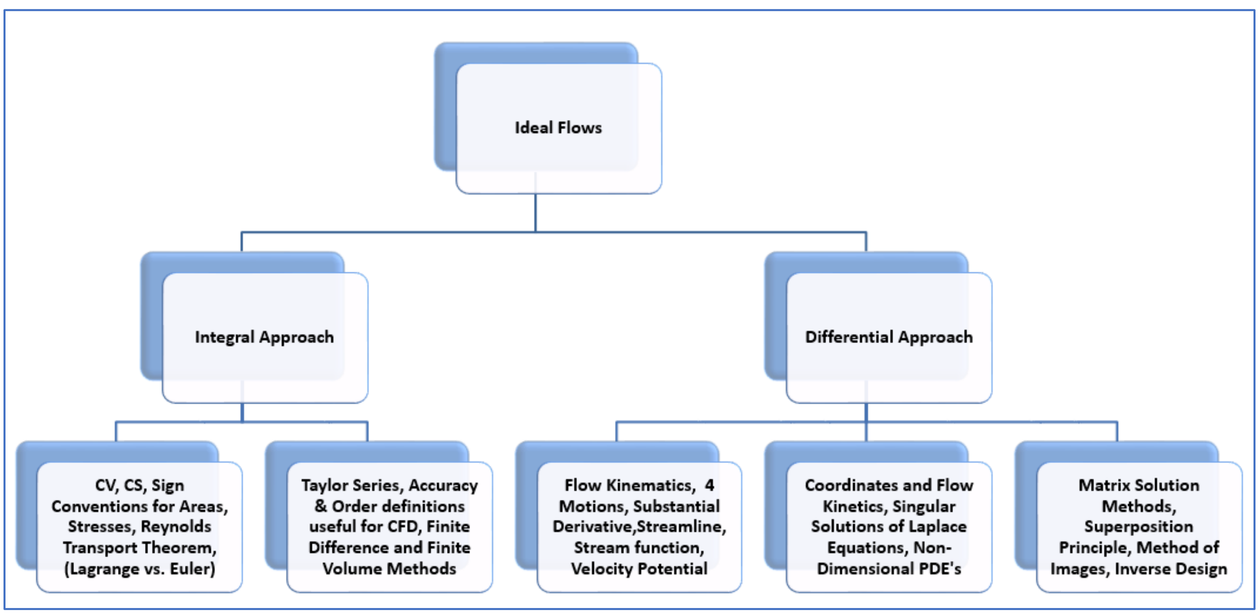


Figure 4. Organization of Ideal Flow Mathematical Topics

To learn the third level details in figure 4 students must have the necessary background of differential and integral calculus, differential equations, boundary value problems and concepts of numerical analysis. The next section presents the connectivity of these to fundamentals of algebra, geometry and trigonometry beginning from high school.

In this section we review such engineering conceptual strings in relation to mathematics beginning with high school education. Many mathematical concepts learnt in high school require earlier nurturing of logical thoughts beginning with arithmetic. Ask students to share their thoughts about unitary methods, fractions and percentages, and decimals in select groups if necessary. We shall focus only on definition based abstract thought development beginning with algebra, geometry and trigonometry to find connectivity later with college calculus. The recommended approach helps synthesis of concepts and techniques in a meaningful way instead of presenting mathematics as a subject full of rules and patterns only.

Focus – High School

Listed below are relevant subject areas followed by some discussions of connectivity.

1. Straight Lines – Discuss 3 forms of equations in relation to the origin of coordinates.
2. Connectivity of two points on a straight line to linear interpolation & extrapolation
3. Limiting connections of straight lines to other curves (e.g., secants, tangents, etc.) Circles, Ellipses, Parabolas & Hyperbolas – specific forms and coordinate origins to relate focus and directrix in conic sections with eccentricity. – Use suitable wooden or inexpensive models to demonstrate these.
4. Algebraic ratios, operations with ratios (e.g., simplifying ratios, componendo-dividendo, etc.) may be introduced to gifted students.
5. Factorization techniques (must be reinforced later in college algebra for root finding of second and third order curves).
6. Algebraic identities, their connections to factorization and solution of problems with coordinate geometry.
7. Geometric focus on parallel lines with bisectors and equality of angles, external and internal angles of a triangle. Focus on right, isosceles, equilateral and similar triangles.
8. Trigonometric focus on definitions in relation to right triangles, sine and cosine laws.

Focus – College

1. Discuss slopes of straight lines & relations that yield connectivity with secants and tangents to curves. Parametric forms of curves.
2. Discuss matrix Methods of solving algebraic and differential equations.
3. Recall differences of shapes and properties of second order curves.
4. Contrast differential calculus of one independent variable vs. multivariable calculus – Review ordinary versus substantial derivatives, local and convective accelerations.
5. Emphasize conceptual connectivity of curvilinear coordinates to rectilinear coordinates. Similarities and differences in unit vectors between rectilinear and curvilinear systems.

6. Review and contrast determination of a streamline by definition with the alternate stream function approach. Focus on stagnation streamline.
7. Review Taylor series and emphasize on the accuracy and order of terms. Review engineering approximations and error analysis with examples from later courses.
8. Revive a focus on engineering formulations. Many engineering professors introduce engineering formulations without any conceptual relevance to mathematics. Instead formulations must be understood *a priori* by solvability and uniqueness of solution to mathematical equations. Disallow all assumptions and let students make them only to resolve formulation and solvability. Explain earlier used examples from Dynamics and Fluid Mechanics again to reinforce the ideas.

The above focus in college must be for earlier conceptual recalls and extensions to later engineering usage. *In our view, strengthening dynamics is extremely important for ME students because all formulations of Fluid Mechanics and ideal flows depend on the extension of rigid body dynamics concepts* [10]. Once basic concepts of moments and force-couple equivalency are understood, all new learning must be reinforced following these ideas. For example, topics such as centroids and center of gravity in Statics are excellent to reinforce line, area, and volume integration concepts. Center of gravity, center of mass, centroids, moments of inertia, parallel axis theorem, and radius of gyration should not be presented as formulas to be applied, but conceptually connected as a string of thoughts beginning with their definitions and stages of simplification. Furthermore, when submerged surfaces are discussed in fluid statics, students should refrain from using cookbook "I" formulas for location of centroid, force, and moment calculations. Not only are the integral concepts better, average students would develop advanced understanding without memorizing use of formulas. Topics such as parallel axis theorem are required to conceptually connect the Kinetic Diagrams [KD] in Dynamics, and they offer better understanding of dynamical constraints in motion. Thus, mathematics can be revived, reinforced, and retained as a powerful synthesizer.

Horizontal and vertical connectivity of each mathematical concept are required with examples from engineering mechanics concepts. For example, the normal-tangential coordinates are first introduced in Dynamics but are again used in Fluid Mechanics in defining streamlines. But the Dynamics course can never have full scope to discuss details of its connectivity. If this topic is recalled in Fluid Mechanics, an instructor can teach inquisitive students all the complexities in the governing differential equations due to rotating coordinates. For this purpose, it is important to discuss in Dynamics how the time derivative of unit vectors is obtained. Many instructors choose to skip the derivation. This important derivation recalls some geometrical properties of curves and reviews secants and tangents also. Furthermore, a unique association of magnitude and directional changes of any vector (e.g., velocity) may be followed later. Once n-t derivation is completed, ask student teams how they would obtain such unit vector derivatives in r-θ system also. Since the angles are measured at the origin of both Cartesian and cylindrical coordinates, often students mix up the n-t and cylindrical coordinates. The resulting rates of change in angles must be separately tracked in theses coordinates. This helps (and gets reinforced) during general motion development in rigid body dynamics later.

The appendix shows 5 home work questions given on the HW set 1 in Fluids II to re-establish mathematical connectivity as discussed above. For each of these review questions, details of relevant mathematical recall are discussed individually or, in group mode (depending on the class size) during office hours. The first question reinforces discussion of a journal bearing question and the conceptualization of Couette flow in a cylindrical application, where students use $\frac{du}{dy} = \frac{\omega r}{t}$. Ask questions like "under what conditions is this linear approximation valid?", "what checks would you perform so that the plates may be considered infinite in size compared to the gap?", etc. In the second question on HW1 students are urged to review h = a + y sinθ type change of variables and choice of an infinitesimal strip to complete the problem rather than using the "I" formulas from a fluids text. Fluids II would need more integral reviews for flux terms as depicted on the later questions. The stream function evaluation and velocity profiles are all relevant recalls for this class. Finally, the last question recalls Reynolds Transport theorem for mass and momentum theorems on large control volumes. Bulk CV's set up the reviews on differential control volumes and derive the differential equations later for interpretation and understanding of each of the contributing differential terms. Powerful extensions using Taylor series and order discussions thereof lead to both the finite difference and finite volume methods in CFD later. The engineering Bernoulli equation which is used in the first fluids course is brought back in Fluids II to connect both inviscid Bernoulli equation and general convection equations later after the ideal flow topics. Connectivity is enjoyed by students (who take the Heat Transfer class concurrently) - knowing the fundamentals from Fluids II, and applying them in Heat Transfer class simultaneously for engineering designs. Application areas in Fluids II require understanding of boundary conditions, method of images and inverse design concepts. Some examples are discussed from each of these areas before velocity and thermal boundary layer concepts are introduced.

Connectivity – Step 3 (assessment and feedback)

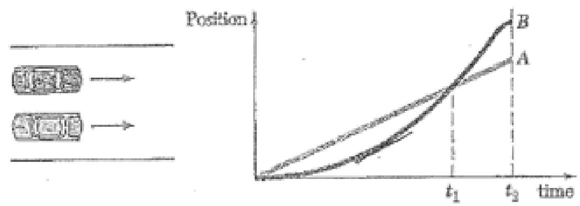
This section presents great success stories quoting benefits of mathematical connectivity. Rarely we experience any difficulty in implementing the steps mentioned above in a sequence of three courses – Dynamics, Fluid Mechanics and Fluids II. We present here encouraging performance data in two specific areas. After studying figure 3, we compare final examination performance on the use of slopes of curves as we go from Dynamics to Fluid Mechanics. After the introduction of n-t coordinates, students learn its connectivity to the Cartesian system. The slope of the velocity vector with the x-axis is the same as the slope of the tangent to the streamline learned later in Fluid Mechanics. The test performance on two questions from Dynamics are shown with test performance in Fluid Mechanics on figure 5 below. The first two questions were tested in 2013Spring, 2015Spring and 2016Fall final examinations with grades ranging between 51% and 66% correct respectively, while the last question was tested in Fluid Mechanics in 2017Spring with 80% of students answering the question correctly.

1.1 The path of a particle is defined by $y = 0.5\,x^2$.
If the component of its velocity along the x-axis at x = 2 m is $v_x = 1$ m/s, its velocity component along the y-axis at this position is:

a) 0.25 m/s
b) 0.5 m/s
c) 1 m/s
d) 2 m/s

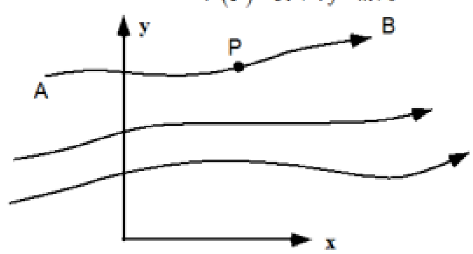
1. Two cars which are next to each other race down a straight road. The positions of each car from the current location are shown over time. Select the correct statement below

a. At time t_2 both cars have traveled the same distance
b. At time t_1 both cars have the same speed
c. Both cars have the same speed some time $t < t_1$
d. Both cars have the same acceleration some time $t < t_1$
e. None of the above



1. The point P is on the streamline passing through points A and B. The velocity at the point P is determined to be:

$$\vec{V}(P) = 3\hat{i} + 4\hat{j} \quad m/s$$



The slope of the streamline at point **P** is:

A) Zero
B) Infinity
C) 1.33
D) 0.75

Figure 5. Dynamics and Fluid Mechanics Connectivity

Also, we tracked the same area of streamlines as we move from Fluid Mechanics to the elective Fluids II as shown on the figure 6 below. This time the question is related to the topic of streamline evaluations as shown in the two quiz questions - first as a part of a set of MC questions, and then as a part of a stand-alone quiz of 10-minute duration on two different times. On each the class performance was better than 80% correct for the streamline evaluation, plus on the part (c) related to the ideal flows for the stand-alone quiz.

9. A velocity field is given by $\vec{V} = y\hat{i} - x\hat{j}$ (m/s), where, x and y are in meters. A streamline plotted at (2, 1) in this flow is given correctly by (You must show your work to get full credit)

(a) ln x = - ln y + ln 5, (b) $x^2 + y^2 = 5$, (c) xy = 5, (d) y = 5 x, (e) None of the above. (2 points)

Quiz No. 3

(Maximum Time: 10 min)

A water flow in the horizontal plane (x, y) is described by the velocity vector: **V** = y **i** – x **j** (m/s), where x and y are in meters.

(a) Find the equation of the streamline passing through the points (1, 1).
(b) Show the flow direction on the streamline in part (a).
(c) Is this flow irrotational? Verify your answer by a suitable check.

Figure 6. Streamline concepts carried over from Fluid Mechanics to Ideal Flows

Note that in the above two questions, the MC question asks for the details to get a full credit. The work load is not much beginning with the equation (available on the formula sheet) $\frac{dx}{u} = \frac{dy}{v}$.

The important task is to check if students carry out the integration and the constant evaluation correctly by fitting the point (2, 1) on the streamline. If they do not show the work, they get only one point, usually allocated for MC questions on a class quiz. They may also incorrectly perform the integration, simplify the ratio incorrectly, or, evaluate the constant incorrectly, which would be revealed from choices. If the class performance was not up to expectations, students would still get the opportunity to improve their score on the question by resubmitting the quiz. Details of the assessment leading to connectivity of ideal flow topics are discussed in references [3 - 5]. If the class size is small enough, alternate and accurate assessment may be performed using a stand-alone quiz as the Quiz No. 3 shows. As always, a prompt return of the original quiz is necessary so that students can relearn the missed topic over the approaching weekend.

Similar to the above examples, the focus area of vectors was traced on examples from Dynamics to Fluid Mechanics. In the absence of tensors, vectors are the primary focus for our ME students and the coverage is very broad beginning with coordinate systems, unit vectors, inner and outer products, vector calculus, etc. We have tested many detailed areas of importance individually, with positive results demonstrated due to reinforcement and connectivity. Here one such sample is shown on figure 7 below. For the first two questions taken from Dynamics, the focus is on component addition/subtraction to obtain relative velocity, but the second question uses the geometrical construction of a negative vector and parallelogram/triangle of vectors. The third example on the figure is from fluid mechanics involving dot products and its interpretation in finding fluxes of fluid momentum. Once again, the performance was better than 65% correct on each examination demonstrating that such instructional reinforcement is very effective (because ESCC considers 60% correct class performance on each MC question as acceptable).

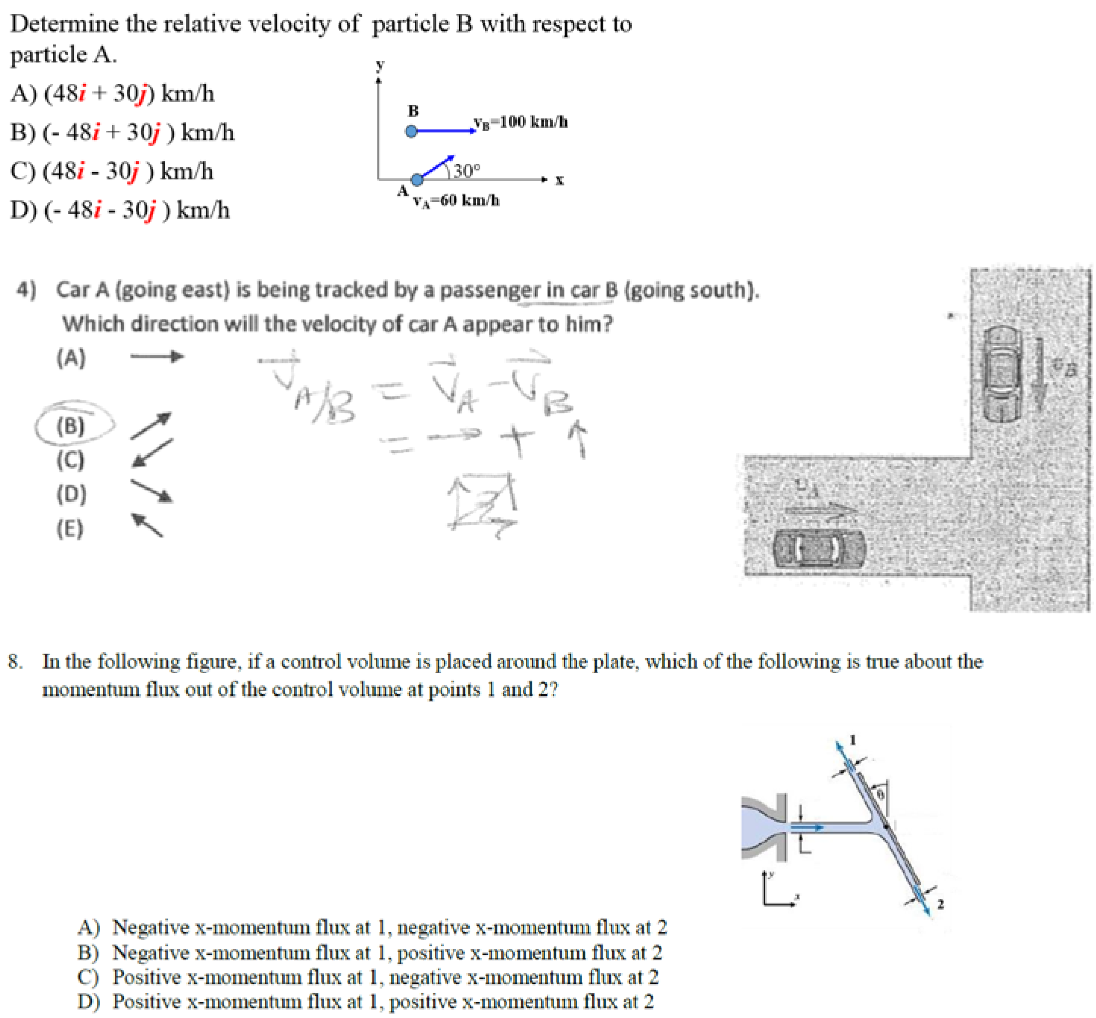


Figure 7. Use of vectors tested by Component method and parallelogram laws in Dynamics (2015 & 2016 Fall final exams), and component finding using dot products in Fluid Mechanics (2016 Spring Final)

The mastery in ideal flow topics may further be traced using breadth and depth of our student interests. Two students took the initiative to prepare a set of nicely typed 550CD notes (Note: Fluids II used to be taught as EMEM550 – Transport Phenomena when RIT was on a quarter system) to be shared later with future batches. Two projects with papers were developed, both of which published a user-friendly MATLAB interactive environment for learning of ideal flows/aerodynamics [11]. Another student prepared a journal paper for the use of an advanced learning platform to prepare high accuracy CFD codes to study the modified equation approach for solving hyperbolic partial differential equations [12]. But these are highly motivated students that truly understood the approaches. *For struggling students, even the rigid body general motion questions in dynamics are not that straightforward. Therefore, various types of reinforcement are necessary*. At this time more faculty participation is necessary to fix such learning bottlenecks.

We close the present discussion with one example which illustrates how much effort is currently spent in exploring possible connectivity of mathematical topics. In a mechanical engineering program, students have an added advantage of observing applied examples and verifying mathematical models by experiments. Beginning with Statics, mathematical traces are recalled all the way to upper level courses such as ideal flows. *Dynamics receives a pivotal importance for this purpose.* This final example also points out a new trend [1] that is developing in student performance (which defies our concerted planning efforts).

One of the difficult conceptual areas that students should master at the end of Dynamics is the area of rigid body rotation. In deformable media, rotations are more complex and shear and rotational contributions are coupled by the same derivatives in planar flows. While vorticity $\Omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ *and, shear stress* $\tau_{yx} = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$ are taught during the first weeks in flow kinematics, more focus on the vortex motion may be desirable leading to aerodynamic and meteorological applications. Many questions of varying complexity may be constructed. Thus, Dynamics has a major focus in this area beginning with the instantaneous center of zero velocity [1], [6], [7].

We present below three morphed attempts from a focus question in Dynamics (Figure 8) and summarize its performance on a bar chart (Figure 9). About six years ago, a new topic of KD had newly been included in Dynamics syllabus by the ESCC faculty in addition to the usual Free Body Diagram (FBD). This particular example has two separate objects with mass (a disk and a block) but the mass of the disk is not provided explicitly in the question. Also, the disk is pinned at the center not allowing it to translate. When the block is released from rest, net torque using KD must include the moment due to the block's inertia force about the center, and add to it the moment due the couple $I_0\alpha$. In spite of a constant focus by 3 different ESCC teams from 2104 to 2018, the performance seems to have deteriorated after 2015.

12. A mass A hangs by a light cord wrapped around the disk of radius 100 mm. The mass moment of inertia of the disk about its central pin connection is $I_0 = 0.005$ kg-m^2 and the mass of block A is 1 kg. If the system starts from rest, the initial angular acceleration (rad/s^2) of the disk will be about (Hint: Sketch FBD & Kinetic diagram of the connected system before answering)

(A) 0 rad/s^2
(B) 10 rad/s^2
(C) 49 rad/s^2
(D) 65 rad/s^2
(E) 98 rad/s^2

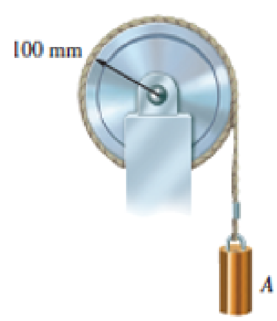


Figure 8. Focus question from a recent Dynamics final exam (see performance in figure 9)

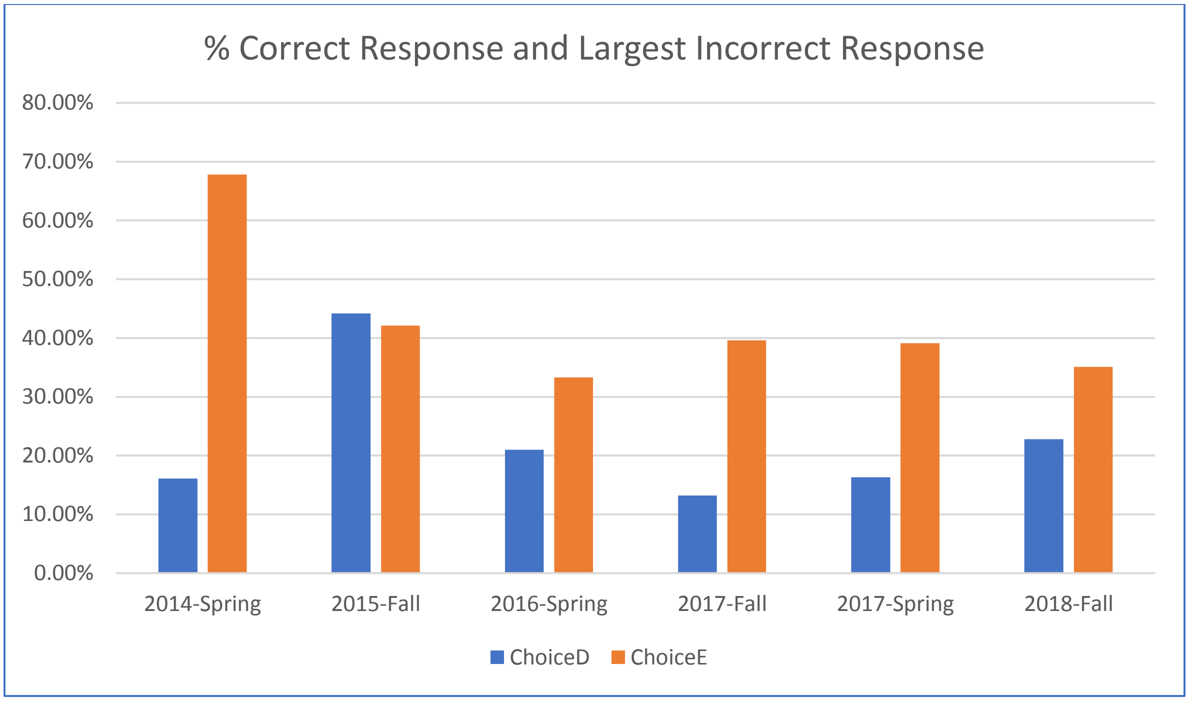


Figure 9. Percent correct and largest incorrect responses on the question in figure 8

Note that the question on figure 8 was morphed three times. In the first two years (2014 & 2015) the question simply asked to compute $(\Sigma M_k)_0$ as obtained from the KD in terms of the angular acceleration of the disk α. The choices given were:

(A) 0.005α, (B) 0.015 α, (C) 0.1α, (D) 0.01 α, and (E) 0 N-m.

For this version of the test, the correct choice was (B) and the largest incorrect choice given by students was (A). Therefore, on the figure 9 legend, replace the choices (D) and (E) by (B) and (A), respectively to make correct comparisons. The topic of KD was new on the syllabus, noted from [13]. So, the first response in 2014-Spring was understandable. After specially focusing on this area, the question was re-tested in fall semester final examination of 2015, when the responses improved to 44% correct, but still the incorrect answer (A) was chosen by 42% students. Incidentally, the choice (A) would be obtained if the mass of the block was neglected altogether. In three following final examinations after that, the question was posed in the current version (Fig. 8) but without the hint that is displayed on the question now. The correct and largest incorrect choices now are (D) and (E) respectively. After noticing not much improvement in correct responses, the question was posed in the present form (with the added hint) in 2018-Fall semester. However, the performance never seems to improve much even with the hint (since our target achievement level of 60% of the class being correct was never reached). The incorrect response (E) comes from neglecting the disk altogether as if the block fell independently under gravity. There may be several reasons for this lack of performance. Interviewing students after the test in 2018 revealed that they expected an easier question. Many students *did not even read the hint*. We plan to attempt using other means of reinforcement for this question in future. We have presented other examples and discussion of connectivity with engineering concepts in another publication in this conference [14].

To summarize, the current trend of student performance reflects neglect of important mathematical concepts, and answering without proper technical considerations. If this trend persists for long irrespective of our instructional efforts, subjects requiring more in-depth deliberations would be difficult to deliver. Unable to recall relevant concepts required to solve a question during an examination, students usually guess or reply using a layman's approach [2]. It is therefore the responsibility of engineering educators to reinitiate interests and emphasize logical analysis without which applied mathematics may not be appreciated. We now know that poor performance in the question on figure 1 was due to erroneous integration methods used. Hopefully the methods and connectivity which were demonstrated here with ideal flow topics would be the starting point of a new instructional design for continuous connectivity of topics.

Conclusion

This paper presented mathematics as a common bond in developing ME thoughts. The approach seeks connectivity of mathematical topics/concepts to teach students engineering formulations related to ideal flows and CFD. This effort was necessary after faculty observed performance in ESCC examinations on mathematics related topics fell short of expectations. Positive results of applying the present approach were reported here for ideal flow applications. New solutions will be explored in future to address the challenges mentioned here.

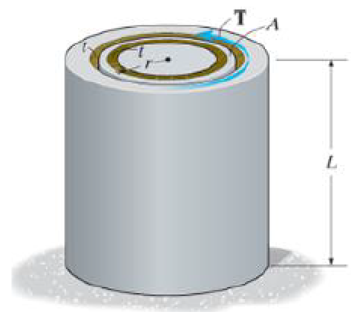
Acknowledgments

This study would not have been possible without active participation of all members of various ESCC instructional teams for the past twelve years. All participating faculty members and trained teaching assistants helped in designing, discussing, and evaluating examination archives from which examples are presented here. The author, who was also the ESCC coordinator in the past, gratefully acknowledges all their contributions. The author would also like to thank Drs. E. C. Hensel and R. Robinson for helpful discussions and support.

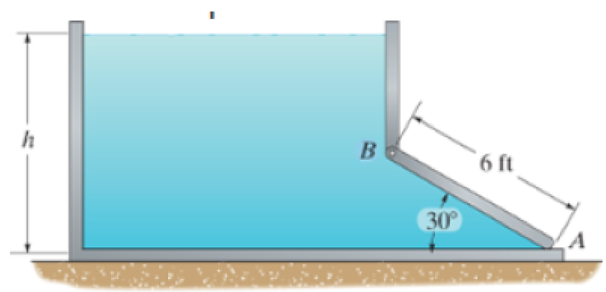
Appendix

The following home assignment set is designed to review several mathematical procedures and concepts for connectivity of Fluids II with previous courses.

1. The very thin tube *A* of mean radius *r* and length *L* is placed within the fixed circular cavity as shown. If the cavity has a small gap of thickness *t* on each side of the tube, and is filled with a Newtonian liquid having a viscosity μ, determine the torque **T** required to overcome the fluid resistance and rotate the tube with a constant angular velocity of ω. Assume the velocity profile within the liquid is linear.

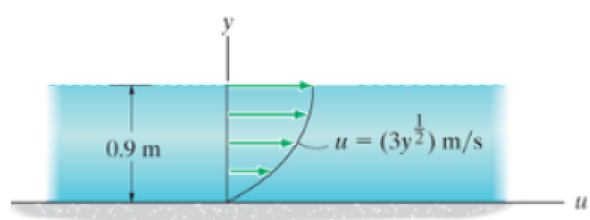


2. The uniform rectangular relief gate AB has a weight of 8000 lb and a width of 4 ft. Determine the minimum depth h of water within the container needed to open it. The gate is pinned at B and rests on a rubber seal at A.

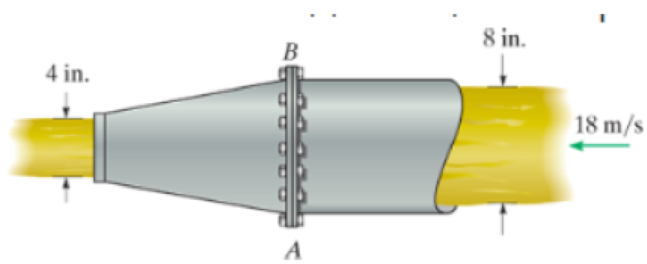


3. A flow field is defined by u = (3x) ft/s and υ = (6y) ft/s, where x and y are in feet. Determine the equation of the streamline passing through point (3 ft, 1 ft). Draw this streamline.

4. The liquid in the rectangular channel has a velocity profile defined by u = (3y1/2) m/s, where y is in meters. Determine the average velocity of the liquid. The channel has a width of 2 m.



5. Oil flows through the 8-in.-diameter pipe with a velocity of 18 ft/s. If it discharges into the atmosphere through the nozzle, determine the total force the bolts must resist at the connection AB to hold the nozzle onto the pipe. Take $\gamma_0 = 55$ lb/ft^3.



Home Work Set 1 due before 3 pm on 8-31-18 [8]

References

[1] A. Ghosh, “Formative Assessment using Multiple Choice Questions in Statics and Dynamics”, Paper No. IMECE-66304, ASME International Mechanical Engineering Congress and Exposition, November 11 – 17, Phoenix, Arizona, 2016.

[2] A. Ghosh, “Foundations of Statics – An Assessment Study and Feedback Implementation”, Paper No. IMECE-66302, ASME International Mechanical Engineering Congress and Exposition, November 11 – 17, Phoenix, Arizona, 2016.

[3] A. Ghosh, “Motivation Building Strategies of Mathematics Instruction for Undergraduate Students in Mechanical Engineering”, Paper ID #23730, ASEE Annual Conference and Exposition, Salt Lake City, Utah, 2018.

[4] A. Ghosh, "Teaching Formulation Skills in an Upper Level Fluid Mechanics Course", Paper No. IMECE-63989, ASME International Mechanical Engineering Congress and Exposition, November 11 – 17, Denver, Colorado, 2011.

[5] A. Ghosh, “Development of Analytical Skills through Cooperative Learning”, Paper No. IMECE-12947, ASME International Mechanical Engineering Congress and Exposition, November 13 – 19, Lake Buena Vista, Florida, 2009.

[6] A. Ghosh and E.C. Hensel, “An interpretive assessment of engineering science core courses”, Paper No. IMECE-12939, ASME International Mechanical Engineering Congress and Exposition, November 13 – 19, Lake Buena Vista, Florida, 2009.

[7] A. Ghosh, "Use of Multiple-Choice Questions as an Assessment Tool in Dynamics", Paper No. IMECE-63987, ASME International Mechanical Engineering Congress and Exposition, November 11 – 17, Denver, Colorado, 2011.

[8] R. C. Hibbeler, *Fluid Mechanics*, Pearson Education, 2018.

[9] A. H. Shapiro, “Vorticity – Parts I & II”, National Committee for Fluid Mechanics Films, MIT Press, 2008.

[10] A. Ghosh, “Analysis of a Feedback Assessment Loop in Engineering Sciences Core Curriculum”, Paper IMECE-66486, International Mechanical Engineering Congress & Exposition, San Diego, California, USA, Nov. 15-21, 2013.

[11] A. Ghosh, and C. Pantaleon, “Teaching Computational Fluid Dynamics Using MATLAB” Paper IMECE-66458, International Mechanical Engineering Congress & Exposition, San Diego, California, USA, Nov. 15-21, 2013.

[12] C. Pantaleon, and A. Ghosh, “Taylor series expansion using matrices: an implementation in MATLAB”, *Computers & Fluids*, Volume 112, 2 May 2015, Pages 79–82, 2015.

[13] R. C. Hibbeler, *Engineering Mechanics – Dynamics*, 13th Edition, Pearson, 2013.

[14] A. Ghosh, ASEE Annual Conference and Exposition, Tampa, Florida, 2019 (in preparation)