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Transmission Line Analysis using PowerX

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Transmission Line Analysis using PowerX.

Introduction

This paper describes the software implementation of a Transmission Line Analysis module to an existing software application, named PowerX, which is used for educational instruction in the courses *EENG 350: Energy Systems* and *EENG 450: Power Systems Analysis* at Eastern Washington University (EWU). PowerX is a windows-based software application, written entirely in C#, which was originally developed under an internal grant from EWU to do research in the area of Synchrophasors for the Power Grid. Over time, more and more functionality was added to PowerX and eventually it started being used in classes like *EENG 350* and *EENG 450* in order to give the students a better physical feel for realistic numbers when doing numerical computations as well as displaying graphical representation of typical power system problems. The Transmission Line Analysis component developed for PowerX provides functionality for computing the resistance, inductance and capacitance of transmission lines for a variety of different line parameters. Given these quantities, PowerX provides the ability to analyze a transmission line using either the small, medium or long length mathematical model and subsequently compute power flow, power factor, efficiency and voltage regulation of a given transmission line.

Overview of PowerX

A variety of commercial software exists for analyzing power systems. These include ETAP[1], SKM[2] and EasyPower[3]. While these are all good tools for the working engineer, they may not necessarily be the best tool for the student just starting out and learning about power systems.

An example of an educational software application tied to a textbook for learning is PowerWorld Simulator. This tool is used both professionally and academically and discussed in the textbook by Glover et al. [4].

PowerX is similar to PowerWorld Simulator in that it is used for educational purposes but one aspect of PowerX is that it is also used to support independent studies for EE’s wanting to learn about writing software. Several students have done independent studies with the result of adding components to PowerX.

PowerX provides a variety of tools to analyze components of a power system. These tools include a complex calculator, a unit’s conversion tool, a per-unit tool, phasor diagram tool, one line diagrams, power flow analysis, transmission line analysis and symmetrical components [5].

The modules present in PowerX are mainly tied to the typical homework assignments the student will be assigned during a ten week quarter. The idea is to have the student complete the assignment by hand showing all their work. Then have the student check their answer using PowerX. Lastly, some changes are made to the original question such as "*what is the effect of varying these two parameters while these other parameters remain constant*". The student is then asked to characterize their answer by varying parameters within the appropriate tool in PowerX.

This paper will focus on the Transmission Line component within PowerX and how it is used with respect to student homework assignments in regards to improving student leaning.

Resistance of Transmission Line

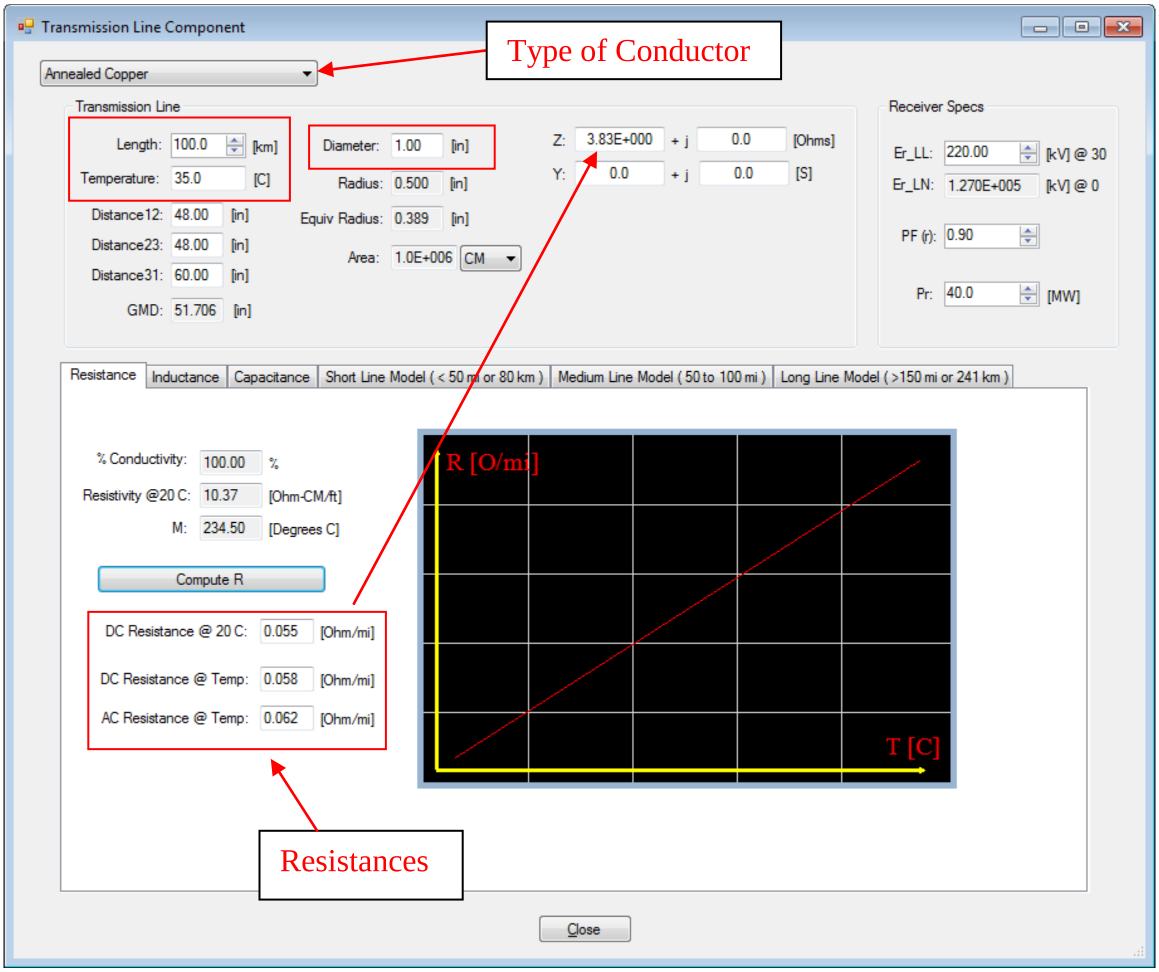
The AC Resistance of a transmission line is given as [6]

$$R_{ac} = \kappa R_{dc}$$

$$= \kappa\left(\frac{M+T}{M+T_{20}}\right)\left(\frac{\rho}{lA}\right) \text{ [Ohms/mile]}$$

Where R_{ac} is the AC resistance of the line, R_{dc} is the DC resistance of the line, T is the current temperature, T_{20} is 20 degrees centigrade, M is the temperature constant in centigrade for this type of line, ρ is the resistivity of the conductor material, l is the length of the line and A is the cross-sectional area of the line.

The student will first select the type of conductor. The choices include Annealed Copper, Hard-drawn copper, Aluminum, Iron and Silver. Annealed Copper is shown selected in the Figure 1. The type of conductor will determine the value of conductivity, resistivity and the temperature constant.



Annealed Copper

Length: 100.0 [km]
Temperature: 35.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 3.83E+000 + j 0.0 [Ohms]
Y: 0.0 + j 0.0 [S]
Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

% Conductivity: 100.00 %
Resistivity @20 C: 10.37 [Ohm-CM/ft]
M: 234.50 [Degrees C]
DC Resistance @ 20 C: 0.055 [Ohm/mi]
DC Resistance @ Temp: 0.058 [Ohm/mi]
AC Resistance @ Temp: 0.062 [Ohm/mi]

Figure 1: Resistance of a Transmission Line

The student will then enter a variety of data including the length, temperature and diameter. The student clicks on the “Compute R” button and subsequently the DC resistance at 20 C, the DC resistance at specified temperature and the AC resistance at temperature are calculated. The resulting value of AC resistance is shown to be 0.062 ohms per mile. A plot is also shown displaying the relationship between Temperature and AC resistance. This value of AC resistance will subsequently be used in analyzing a given transmission line.

Inductance of Transmission Line

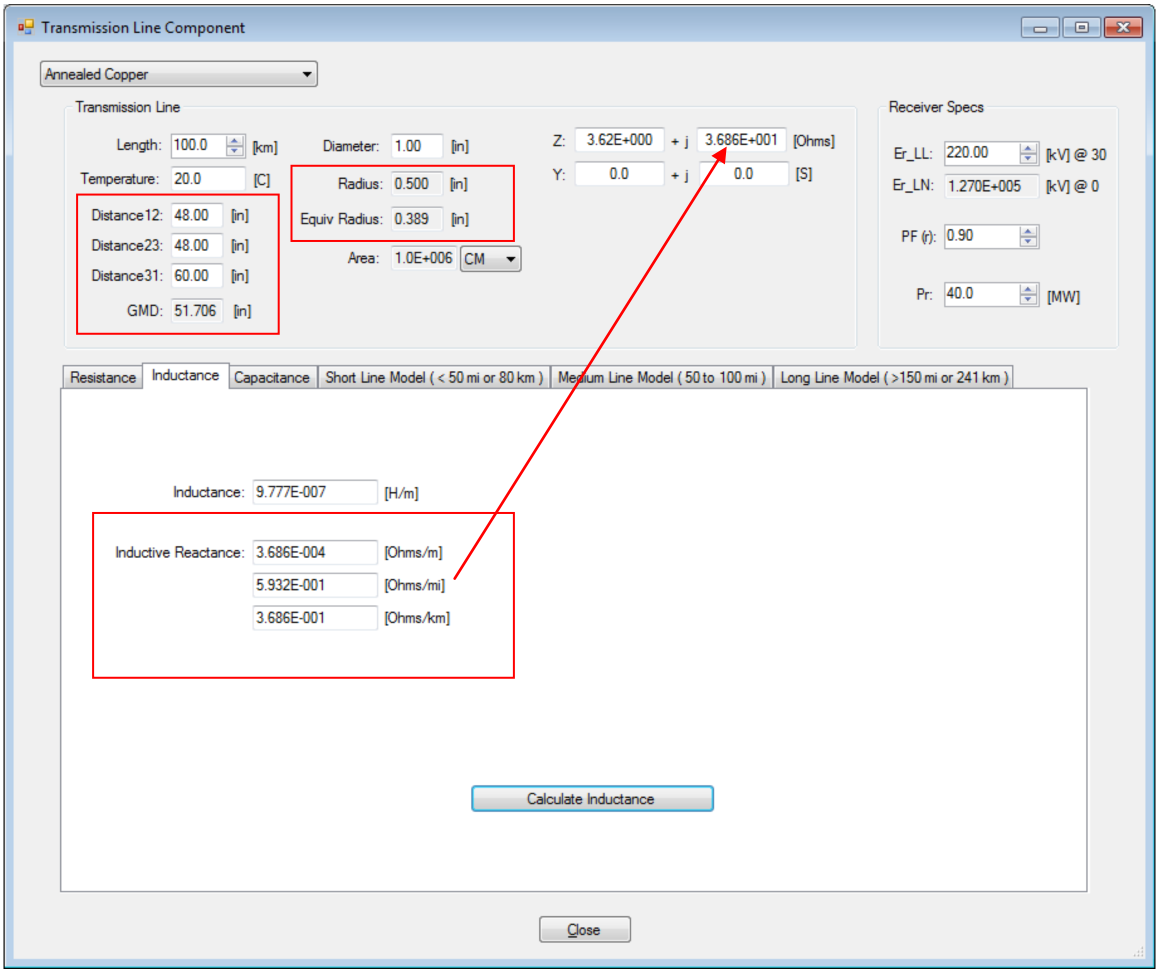
The Inductance, L_a, of a three-phase transmission line is given as [6]

$$L_a = 2x10^{-7} ln_e \left(\frac{GMD}{r'}\right) \text{ [H/m]}$$

where *GMD* is the geometric mean distance and is given as $[D_{12}D_{23}D_{31}]^{1/3}$, r’ is the equivalent radius and D_{ij} is the separation between the i^{th} and j^{th} line.

The inductive reactance, X_l, is given as $2\pi f L_a$.

The transmission line is assumed to be a solid conductor. Future work will include implementing both stranded and bundled conductors.



Annealed Copper

Transmission Line

Length: 100.0 [km]
Temperature: 20.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 3.62E+000 + j 3.686E+001 [Ohms]
Y: 0.0 + j 0.0 [S]

Receiver Specs

Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

Inductance: 9.777E-007 [H/m]

Inductive Reactance: 3.686E-004 [Ohms/m]
5.932E-001 [Ohms/mi]
3.686E-001 [Ohms/km]

Figure 2: Inductance of a Solid Transmission Line

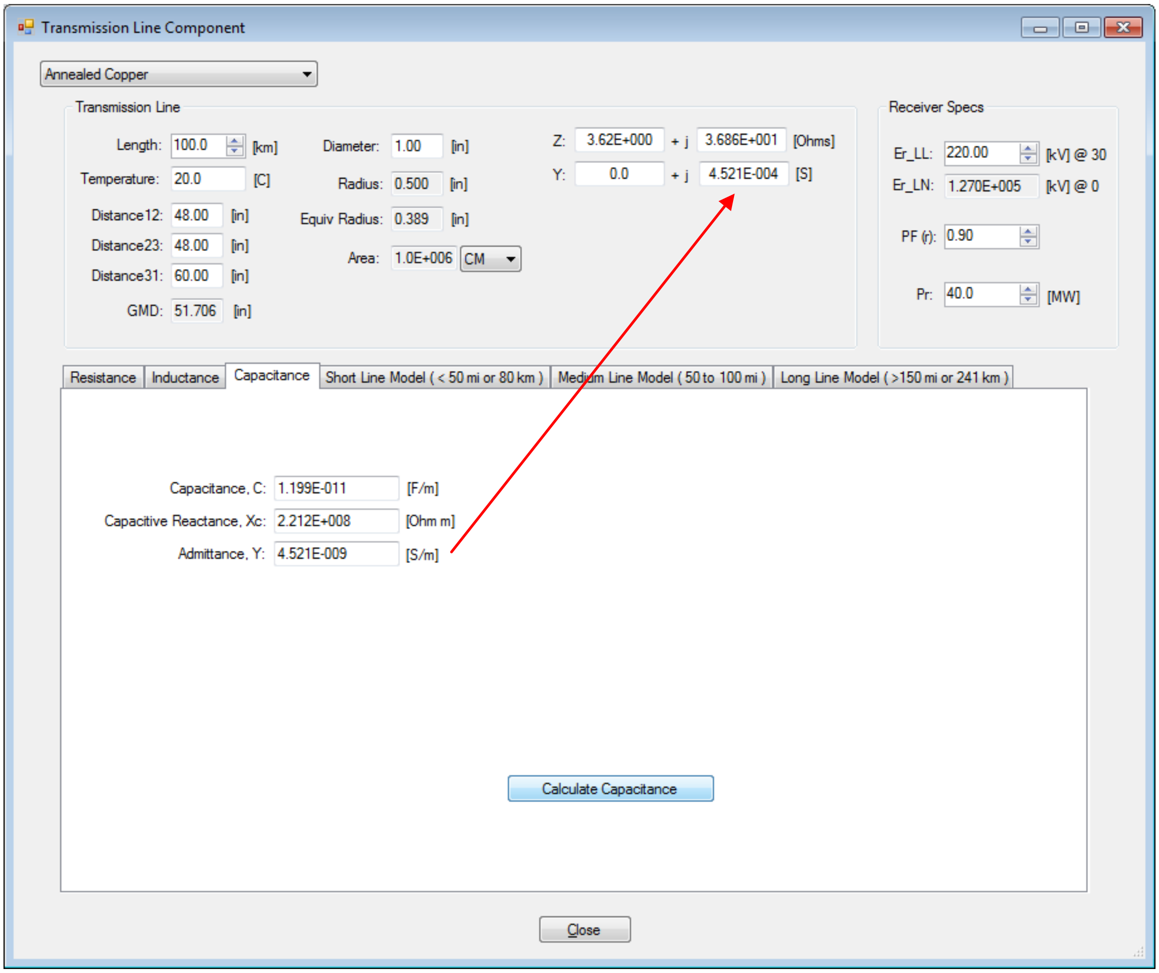
The student enters in the distance between the 3-phase conductors, $D_{12}, D_{23}, and\ D_{31}$, the radius of the conductor, r, and then clicks on the "Calculate Inductance" button. The inductance and inductive reactance are subsequently computed. This value of inductive reactance will be used in the complex impedance, Z, of the transmission line to be analyzed.

Capacitance of Transmission Line

The Capacitance-to-neutral of a three-phase transmission line is given as [6]

$$C_n = \frac{2\pi\varepsilon_o}{ln_e(\frac{GMD}{r'})} \text{ [Farad meters]}$$

Where *GMD* is the geometric distance defined previously, r' is the equivalent radius and ε_o is the permittivity of free space. The capacitive reactance, X_c, is given as $\frac{1}{2\pi f C_n}$. Similar to computing inductance, the student enters values for the separation between the conductors, D_{12}, D_{23} and D_{31} and a radius. The student then clicks on "Calculate Capacitance" and PowerX calculates the capacitance per line, the capacitive reactance and the admittance per phase.



Annealed Copper

Length: 100.0 [km]
Temperature: 20.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 3.62E+000 + j 3.686E+001 [Ohms]
Y: 0.0 + j 4.521E-004 [S]

Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

Capacitance, C: 1.199E-011 [F/m]
Capacitive Reactance, Xc: 2.212E+008 [Ohm m]
Admittance, Y: 4.521E-009 [S/m]

Figure 3: Capacitance of a Solid Transmission Line

The admittance, Y, is subsequently used in the long transmission line model. Typical homework would say something like "*what happens to L as the space between the lines increases from 1 [m] to 2 [m]*".

Analysis of Transmission Lines

The values of R, L and C, computed previously, are used to represent the transmission line impedance, Z, and admittance, Y. The student can now analyze the power flow of a transmission line. The student may choose to use either the short, medium or long transmission line model.

Short Transmission Line Model

The following figure shows the short transmission line model. It is assumed that the line is less than 50 miles (~80 km).

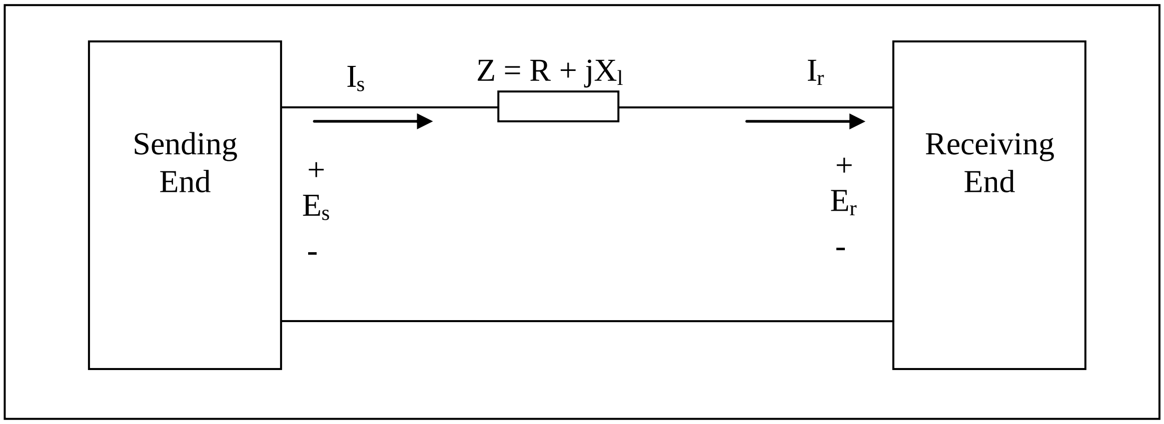


Figure 4: Short Transmission Line Model

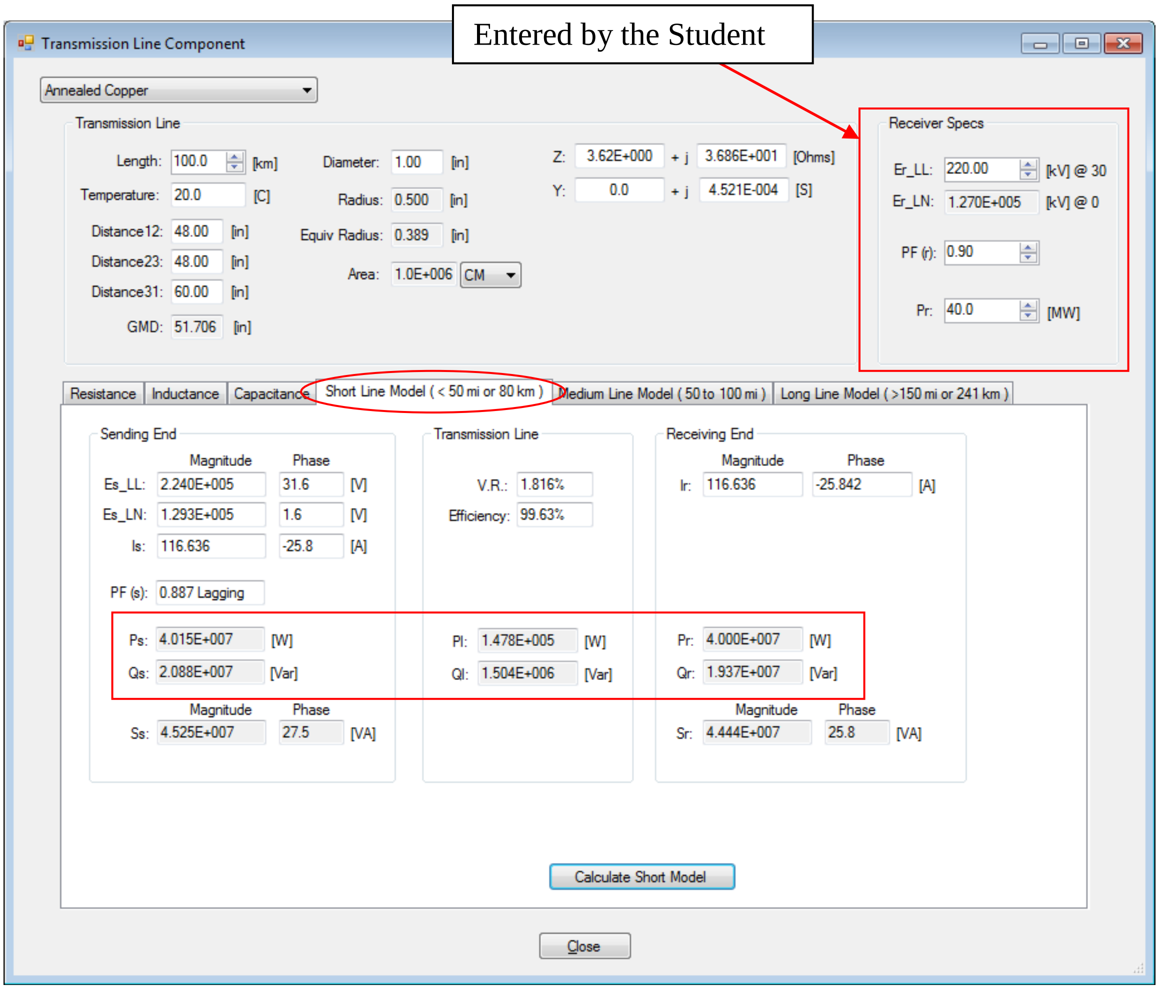
Z equals the complex impedance of the transmission line computed from the resistance and inductance values discussed previously. E_s is the sending-end voltage, E_r is the receiving-end voltage and $I_r = I_s$ is the line current.

For the short transmission line model, the equations relating the sending-end voltage and current to the receiving-end voltage and current are given as follows [6]

$$E_s = I_s Z + E_r$$

$$I_s = I_r$$

The student will enter data into the “Receiver Specs” group box shown in Figure 5. These include the receiving-end voltage, power factor and active power absorbed by the receiving-end. Typical values for these are Er_LL equal to 220 kV, power factor of 0.9 lagging and active power absorbed of 40 [MW].



Annealed Copper

Transmission Line

Length: 100.0 [km]
Temperature: 20.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 3.62E+000 + j 3.686E+001 [Ohms]
Y: 0.0 + j 4.521E-004 [S]

Receiver Specs

Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

Sending End

	Magnitude	Phase	
Es_LL:	2.240E+005	31.6	[V]
Es_LN:	1.293E+005	1.6	[V]
Is:	116.636	-25.8	[A]

PF (s): 0.887 Lagging
Ps: 4.015E+007 [W]
Qs: 2.088E+007 [Var]

	Magnitude	Phase	
Ss:	4.525E+007	27.5	[VA]

Transmission Line

V.R.: 1.816%
Efficiency: 99.63%
Pl: 1.478E+005 [W]
Ql: 1.504E+006 [Var]

Receiving End

	Magnitude	Phase	
Ir:	116.636	-25.842	[A]

Pr: 4.000E+007 [W]
Qr: 1.937E+007 [Var]

	Magnitude	Phase	
Sr:	4.444E+007	25.8	[VA]

Figure 5: Computation for Short Transmission Line Model

Given these receiving-end parameters, the student then clicks on "Calculate Short Model" button. Various quantities at the sending-end, over the transmission line and at the receiving-end of the line are computed.

In the Sending End group box in Figure 5, the line-to-line voltage, line-to-neutral voltage, line current, power factor, active power, reactive power and apparent power are computed. In the Transmission Line group box, the voltage regulation, efficiency of the line, and active power and reactive power absorbed are computed. In the Receiving End group box, the line current is computed, which for this model is the same as the line current on the sending-end. The reactive power and apparent power at the receiving end are also computed. The phase angle of the voltage at the receiving end is always taken as the reference with a value of zero. Note that the phase angle at the sending end is computed to be 1.6 degrees. This is consistent with the fact that active power flows from high phase to low phase. Also, the transmission line is absorbing 147.8 [kW] of active power and 1.54 [MVars] of reactive power.

Medium Transmission Line Model

The medium transmission line model takes into account shunt capacitance which can be modeled as ½ of the admittance, Y, at the sending end and ½ at the receiving end of the line. This is illustrated in the following figure.

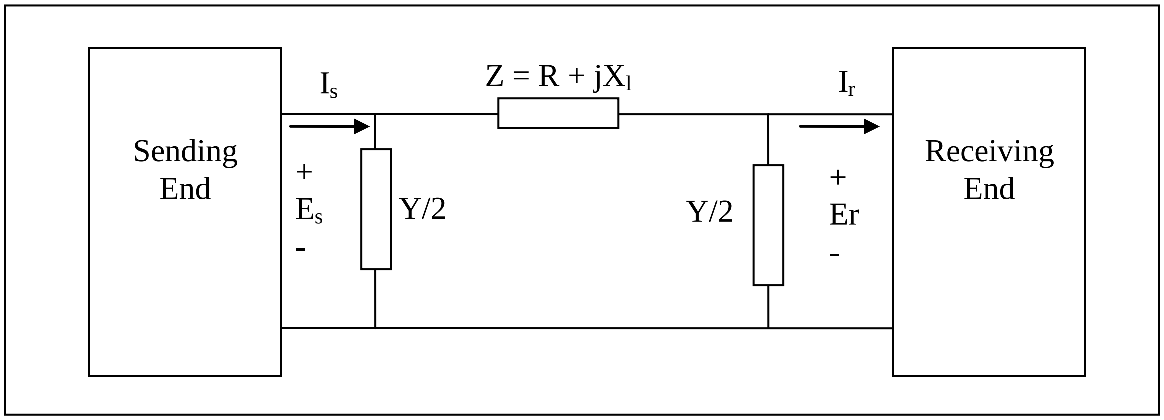
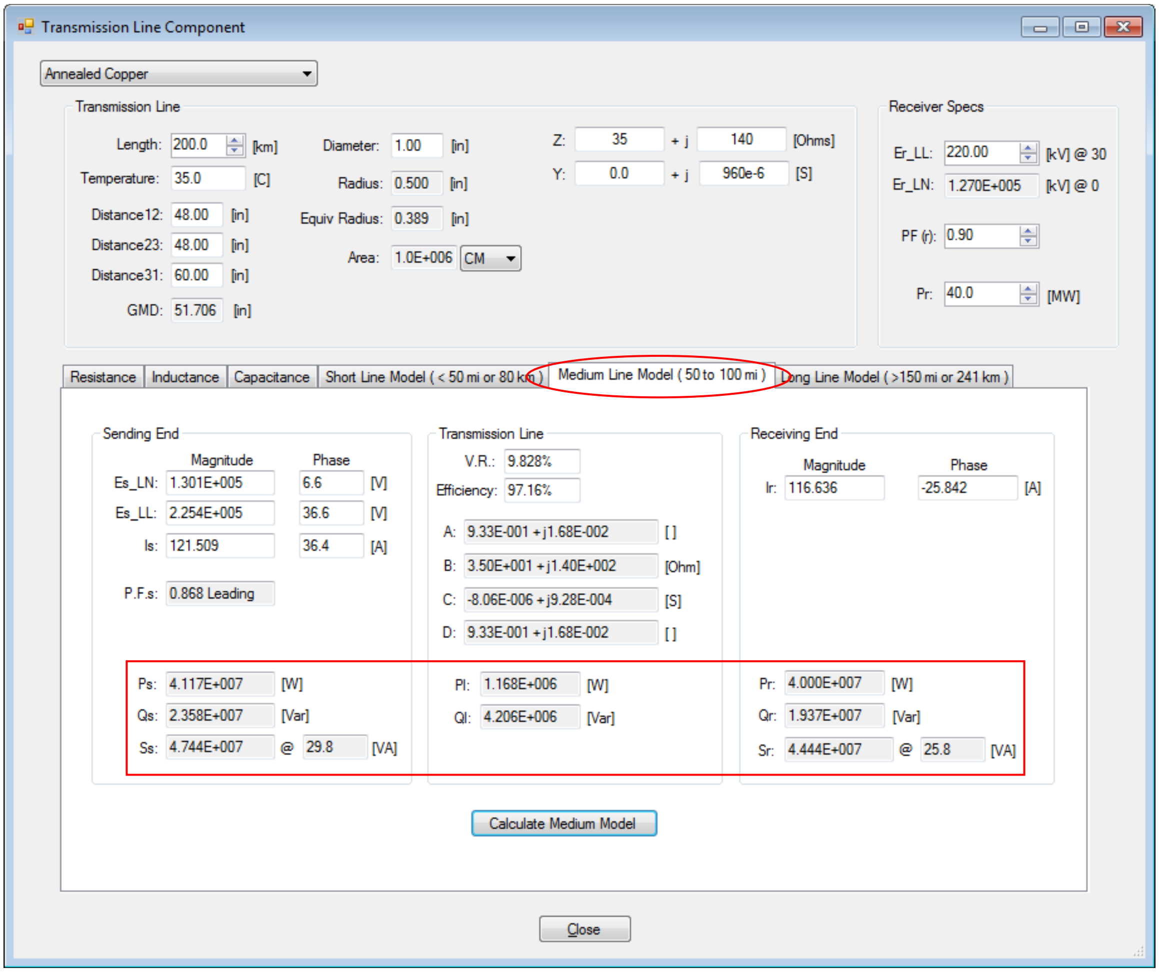


Figure 6: Medium Transmission Line Model

The sending-end voltage, E_s, and current, I_s, can be written in terms of the receiving-end voltage, E_r, and current, E_r, by using the following ABCD matrix [6].

$$\begin{bmatrix} E_s \\ \mathrm{I}_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_r \\ I_r \end{bmatrix}$$

where $A = D = \frac{ZY}{2} + 1$, $B = Z$, and $C = Y\left(\frac{ZY}{4} + 1\right)$ and Y and Z are the admittance and impedance of the transmission line as shown in Figure 7.



Annealed Copper

Transmission Line

Length: 200.0 [km]
Temperature: 35.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 35 + j 140 [Ohms]
Y: 0.0 + j 960e-6 [S]

Receiver Specs

Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

Sending End

	Magnitude	Phase	
Es_LN:	1.301E+005	6.6	[V]
Es_LL:	2.254E+005	36.6	[V]
Is:	121.509	36.4	[A]

P.F.s: 0.868 Leading

Ps: 4.117E+007 [W]
Qs: 2.358E+007 [Var]
Ss: 4.744E+007 @ 29.8 [VA]

Transmission Line

V.R.: 9.828%
Efficiency: 97.16%
A: 9.33E-001 + j1.68E-002 [ ]
B: 3.50E+001 + j1.40E+002 [Ohm]
C: -8.06E-006 + j9.28E-004 [S]
D: 9.33E-001 + j1.68E-002 [ ]

Pl: 1.168E+006 [W]
Ql: 4.206E+006 [Var]

Receiving End

	Magnitude	Phase	
Ir:	116.636	-25.842	[A]

Pr: 4.000E+007 [W]
Qr: 1.937E+007 [Var]
Sr: 4.444E+007 @ 25.8 [VA]

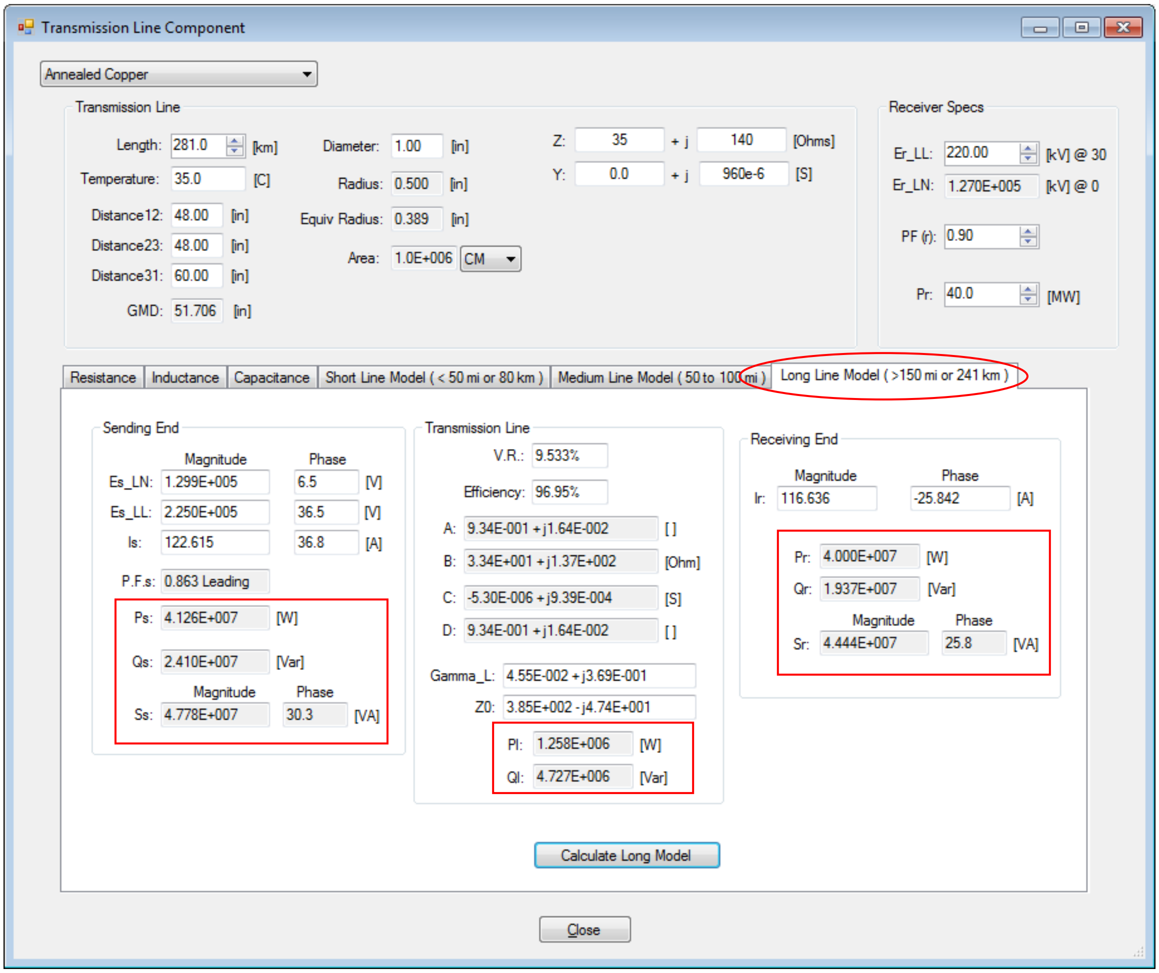
Figure 7: Computation for Medium Transmission Line Model

Similar to the short transmission line model, the student will enter data into the "Receiver Specs" group box. These include the receiving-end voltage, power factor and active power absorbed by receiving-end. Typical values for these are Er_LL equal to 220 kV, power factor of 0.9 lagging and active power absorbed of 40 [MW]

The student clicks on the "Calculate Medium Model" button and subsequent values are computed for the sending-end, transmission line and receiving-end just like for the small transmission line model. One difference is that here, the A, B, C, D parameters are computed. Note that the transmission line absorbs 1.168 [MW] of active power, 4.26 [MVars] of reactive power and there is a voltage drop from the sending-end to the receiving end of 225 [kV] to 220 [kV] at the receiving end. Also, note that the phase difference between the sending-end voltage and receiving-end voltage is 6.6 degrees. This is consistent with the fact that active power flows from high phase to low phase and active power is in fact absorbed by the transmission line.

Long Transmission Line Model

The long transmission line model is similar to the medium model but takes into account propagation delay. It is typically used for lines longer than 150 miles. It also relates the sending-end voltage and current to the receiving-end voltage current through the use of an ABCD matrix where $A = \cosh(\gamma l)$, $B = Z_o sinh(\gamma l)$, $C = sinh(\gamma l)/Z_o$ and $D = \cosh(\gamma l)$.[6]



Annealed Copper

Transmission Line
Length: 281.0 [km]
Temperature: 35.0 [C]
Distance12: 48.00 [in]
Distance23: 48.00 [in]
Distance31: 60.00 [in]
GMD: 51.706 [in]
Diameter: 1.00 [in]
Radius: 0.500 [in]
Equiv Radius: 0.389 [in]
Area: 1.0E+006 CM
Z: 35 + j 140 [Ohms]
Y: 0.0 + j 960e-6 [S]

Receiver Specs
Er_LL: 220.00 [kV] @ 30
Er_LN: 1.270E+005 [kV] @ 0
PF (r): 0.90
Pr: 40.0 [MW]

Sending End

	Magnitude	Phase	
Es_LN:	1.299E+005	6.5	[V]
Es_LL:	2.250E+005	36.5	[V]
Is:	122.615	36.8	[A]

P.F.s: 0.863 Leading
Ps: 4.126E+007 [W]
Qs: 2.410E+007 [Var]

	Magnitude	Phase	
Ss:	4.778E+007	30.3	[VA]

Transmission Line
V.R.: 9.533%
Efficiency: 96.95%
A: 9.34E-001 +j1.64E-002 [ ]
B: 3.34E+001 +j1.37E+002 [Ohm]
C: -5.30E-006 +j9.39E-004 [S]
D: 9.34E-001 +j1.64E-002 [ ]
Gamma_L: 4.55E-002 +j3.69E-001
Z0: 3.85E+002 -j4.74E+001
Pl: 1.258E+006 [W]
Ql: 4.727E+006 [Var]

Receiving End

	Magnitude	Phase	
Ir:	116.636	-25.842	[A]

Pr: 4.000E+007 [W]
Qr: 1.937E+007 [Var]

	Magnitude	Phase	
Sr:	4.444E+007	25.8	[VA]

Figure 8: Computation for Long Transmission Line Model

The student clicks on the "Calculate Long Model" button and subsequent values are computed for the sending-end, transmission line and receiving-end just like for the small transmission line model. One difference is that here, in addition to the A, B, C, D parameters, the propagation constant, γl, and characteristic impedance, Z_o, are also computed. Note that the transmission line absorbs 1.258 [MW] of active power, 4.727 [MVars] of reactive power and there is a voltage drop from the sending-end to the receiving end of 225 [kV] to 220 [kV] at the receiving end. Also, note that the phase difference between the sending-end voltage and receiving-end voltage is 6.5 degrees. This is consistent with the fact that active power flows from high phase to low phase and active power is in fact absorbed by the transmission line.

PowerX Usage Scenario

This section describes the typical usage scenario of using PowerX in a classroom environment. The student is given a homework problem and required to solve the problem by hand showing all their calculations. Then the student will solve the same problem using PowerX verifying that they got the correct answer. Then the student will be asked a variety of questions so as to learn how varying typical parameters will affect the system. For example, typical questions might be as follows:

- *What is the effect on increasing the diameter of the transmission line on the watts and vars dissipated in the transmission line? Use PowerX to justify your answer.*
- *What is the effect of a decreasing power factor at the receiving-end on the drop in voltage over the transmission line? Use PowerX to justify your answer.*

The student will then make the appropriate changes and run the computation in PowerX and then try to describe the effect from a physical point of view. All the while the student is exposed to realistic numbers and hopefully developing a sense of what a realistic value would mean in this type of problem.

Student Assessment

Student assessment was done by comparing grades from one quarter where PowerX was not used (Spring 2013) with another Quarter where PowerX was used (Spring 2014). The raw data representing the students test scores on transmission line materials is presented in Table 1.

	Population	**Exam1**	**Exam2**
Without PowerX (Spring 2013)	A	50,72,72,96,100,86, 100, 90, 80, …94, 100 $\hat{\mu}_{A_1} = 87.18$ $S_x = 13.48$ $n = 22$	43, 75, 76, 55, 74, 74, 81, 65, 30, 93, …73, 74 $\hat{\mu}_{A_2} = 68.77$ $S_x = 16.13$ $n = 22$
With PowerX (Spring 2014)	B	77, 76, 64, 61, 70, 54, 84, 53, 86, …63, 72 $\hat{\mu}_{B_1} = 72.28$ $S_x = 10.75$ $n = 29$	64, 86, 74, 81, 84, 53, 86, 60, 92, 91, …71, 84 $\hat{\mu}_{B_2} = 78.67$ $S_x = 10.33$ $n = 29$

Table 1: Assessment Data

Statistical hypothesis testing[7] was used to compare whether the average on the two exams differed between the courses. The first hypothesis test compared the average grade on exam one for population A (without PowerX) to population B (with PowerX). The results are given as follows:

H_o: $\mu_{A_1} \geq \mu_{B_1}$ (PowerX did not improve grades on Exam #1)
H_a: $\mu_{A_1} < \mu_{B_1}$ (PowerX did improve grades on Exam #1)

This is a typical lower tail t-test with the test statistics computed as follows:

$$t_c = \frac{\hat{\mu}_{A_1} - \hat{\mu}_{B_1}}{s_p} = 4.47$$

At a 95% level of confidence, with a t-value of -1.96, we fail to reject $H_{o.}$ From this we conclude that PowerX did not increase the average value of Exam #1 between the two classes. This may be due to the fact that Exam #1 was mainly concerned with computing R, L, and C of a transmission line and the student was able to learn that material on their own without much help from PowerX.

The second hypothesis test compared the average grade on exam two for population A (without PowerX) to population B (with PowerX). The hypothesis is given as follows:

H_o: $\mu_{A_2} \geq \mu_{B_2}$ (PowerX did not improve grades on Exam #2)
H_a: $\mu_{A_2} < \mu_{B_2}$ (PowerX did improve grades on Exam #2)

The test statistic for this test is computed as follows:

$$t_c = \frac{\hat{\mu}_{A_2} - \hat{\mu}_{B_2}}{s_p} = -2.61$$

The value of -2.61 will result in rejecting Ho, at a 95% level of confidence, just barely. So for the case of the second exam, there appears to be a little improvement in the exam scores. This makes a little sense since more of the material on the exam was related to the transmission power flow problems and this material typically caused more problems for the students.

Future Work

Future work on PowerX involves implementing a DC Machines component that will examine the properties of Series, Shunt and Compound Motors/Generators. Also, the implementation of a Gauss-Seidel tool to solve for the power flow in a small power system consisting of multiple busses and transmission lines is currently under development. Students are being encouraged to get involved with this work in the form of independent studies or senior capstone projects. Such a project would require the student or team to develop a significant component in PowerX to include design, development, testing and documentation of their work.

Conclusion

This paper presented an overview of a software application called PowerX that initially started out as a research tool and eventually made its way into the classroom to help students get a better understanding of power systems problems and solutions to these problems. For the most part, student response has been very positive and assessment data shows a little improvement in student performance. Development on PowerX will continue with the goal of implementing more functionality related to topics in relevant courses.

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