Turning Mesh Analysis Inside Out

Dr. Brian J Skromme, Arizona State University

Dr. Brian J. Skromme is a Professor in the School of Electrical, Computer, and Energy Engineering and was Assistant Dean of the Fulton Schools of Engineering at Arizona State University from 2011-19. He holds a Ph.D. in Electrical Engineering from the University of Illinois at Urbana-Champaign and was a member of technical staff at Bellcore from 1985 to 1989. His research interests are in engineering education, development of educational software, and compound semiconductor materials and devices.

Wendy M. Barnard, Arizona State University

Wendy Barnard is an Assistant Research Professor and Director of the College Research and Evaluation Services Team (CREST) at Arizona State University. Dr. Barnard received her Ph.D. from the University of Wisconsin-Madison, where she focused on the impact of early education experiences and parent involvement on long-term academic achievement. Her research interests include evaluation methodology, longitudinal research design, STEM educational efforts, and the impact of professional development on teacher performance. Currently, she works on evaluation efforts for grants funded by National Science Foundation, US Department of Education, local foundation, and state grants.

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Abstract

Elementary linear circuit analysis is a core competency for electrical and many other engineers. Two of the standard approaches to systematic analysis of linear circuits are nodal and mesh analysis, the latter being limited to planar circuits. Nodal and mesh analysis are related by duality and should therefore be fully symmetrical with each other. Here, the usual textbook approach to mesh analysis is argued to be deficient in that it obscures this fundamental duality and symmetry, and may thereby impede the development of intuition and the understanding of the nature of "mesh currents." In particular, the usual distinction between "inner" and "outer" meshes (if the latter is even recognized) is argued to be meaningless, as can be seen when drawing a planar circuit on the surface of a sphere. A generalized definition of a mesh is proposed that includes both inner and outer meshes on the same footing. Selection of a reference node in nodal analysis should be paralleled by the selection of *any* mesh to be the reference mesh in mesh analysis, which is always selected to be the outer mesh by default in the usual approach. All branch currents are shown to the difference of two mesh currents, and the zero of all mesh currents is now arbitrary just as it is for node voltages. Use of supermeshes is sometimes obviated by the new approach, and the analysis is sometimes simplified. This new approach has been used in two sections of a linear circuit analysis course in Fall 2019, and student survey data is presented to show preference for the new method over the usual textbook method. An interactive multiple-choice tutorial describing the new method has been integrated into a step-based tutoring system for linear circuit analysis.

1. Introduction

Elementary linear circuit analysis is one of the most widely taught gateway courses in virtually all engineering schools. For example, such a course was taught to 1364 students in 26 class sections in Summer 2019 through Spring 2020 at the author's institution alone. Such courses vary in that they may sometimes include topics in electronics or signal processing, but in general they tend to cover a well-established range of topics as outlined in many of the popular textbooks, e.g., [1-10]. The typical approach begins with electrical fundamentals and single loop and single node-pair circuits, which can be solved with elementary methods. For more complicated circuits, both nodal and mesh analysis are nearly always covered (based on systematic applications of KCL and KVL, respectively), though the former is more general in its ability to handle non-planar circuits and to be extended to numerical analysis of circuits via modified nodal analysis [11]. Both methods are however useful for hand calculations and to help develop conceptual understanding. In particular, mesh analysis can yield far fewer equations than nodal analysis when many elements are in series (at least if the concept of essential nodes is not used), just as nodal analysis is advantageous when many elements are in parallel. In the author's experience, students often prefer mesh analysis for the ease with which they can visualize loops to which KVL is applied as opposed to the closed surfaces to which KCL is applied. The arithmetic also involves multiplication rather than division and may avoid fractions, at least in textbook problems.

The well-known principle of duality (see, e.g., Refs. [2, 3, 12-16]) pervades the entire subject of circuit analysis, even if it is not always discussed explicitly. This property follows from the

underlying symmetry of Maxwell's equations. Dual sets of items include current and voltage, meshes and nodes, resistances and conductances, inductors and capacitors, series and parallel relationships, current sources and voltage sources, and short circuits and open circuits. Students cannot help noticing the symmetry between the current-voltage relationships for inductors and capacitors, the similarity of parallel and series RLC circuits, the correspondence of Thévenin and Norton equivalent circuits, and many other such things in circuit analysis. Thomas, Rosa, and Toussaint point out that students can use duality to help remember facts and theorems about circuits [17]. Rigorously, it is well known that one can construct an exact dual of any given planar circuit by replacing nodes by meshes and all of the other corresponding items listed above, and that the resulting dual circuit will obey the exact same differential equations as its dual [2, 3, 12-14]. Therefore, nodal and mesh analysis should be complete duals of each other [10]. Indeed, some works have mentioned that in constructing the dual of a circuit, the reference node should be mapped to the "outer mesh" [18], although usual definitions exclude the periphery of a planar circuit from being called a mesh at all.

Yet the typical prescriptions for carrying out nodal and mesh analysis are not symmetric. The steps listed for nodal analysis (including dependent sources) are typically:

1. Select a reference node, whose voltage is usually defined to be 0 V. This node can be any one in the circuit, but should ideally be connected to many circuit elements and to as many voltage sources as possible. Attach an (electronic) ground symbol to that node.
2. Identify and number the remaining nodes and assign them node voltages V_1, V_2, etc.
3. Write voltage constraint equations relating the difference in node voltages on either side of each voltage source to the value of that source (but the reference node voltage is just zero)
4. Write a Kirchhoff's current law (KCL) equation for each non-reference node not connected to a voltage source.
5. Form "supernodes" consisting of trees of voltage sources and their connecting nodes and write a KCL equation for each "non-reference" supernode (i.e., each supernode that does not include the reference node). (A "tree" of voltage sources includes all sources that are pairwise connected by a common node.)
6. Write an equation for each current or voltage controlling a dependent source in terms of node voltages.
7. Write an equation for each unknown ("sought") voltage, current, or power that one wishes to know about the circuit in terms of node voltages.
8. Solve the resulting system of equations for all such sought quantities.

(One could alternatively define additional unknowns for the current through each voltage source, at the expense of a larger system of equations, instead of using supernodes. The supernode method is however widely preferred for hand calculations.) Some books also draw a distinction between "essential nodes," (or "extraordinary nodes") that connect more than two circuit elements, and "nonessential nodes" (or "ordinary nodes") that connect only two, and use a variation of the above procedure [2, 6]. The author is not aware of any such distinction having been drawn for meshes, though it could be.

For mesh analysis (including dependent sources), however, the typical steps are not symmetric with the above:

1. Identify and number all (interior) meshes in the circuit and assign them mesh currents I_1, I_2, etc.
2. Write current constraint equations relating a single mesh current to the value of any "exterior" current source (i.e., one not shared between two adjacent meshes), and relating a difference of mesh currents to the value of any "interior" current source (i.e., one shared between two meshes).
3. Write a Kirchhoff's voltage law (KVL) equation for each mesh that does not include a current source.
4. Form "supermeshes" consisting of "trees" of current sources and their connected meshes and write a KVL equation for the periphery of each such supermesh. (A "tree" of current sources includes all sources that are pairwise connected by a common interior mesh.)
5. Write an equation for each current or voltage controlling a dependent source in terms of mesh currents.
6. Write an equation for each unknown ("sought") voltage, current, or power that one wishes to know about the circuit in terms of mesh currents.
7. Solve the resulting system of equations for all such sought quantities.

The lack of symmetry of these two approaches is apparent. The procedure for mesh analysis is missing the first step of nodal analysis entirely, and there does not appear to be any such thing as a reference mesh. Further, there is a distinction between "interior" and "exterior" current sources in step 2, and interior meshes and the outer mesh are in general treated very differently.

(We acknowledge that a small number of textbooks advocate generalized loop analysis as an alternative to the much more commonly used mesh analysis [1]. For planar circuits, however, this method has the distinct disadvantage that there are no longer exactly one or two loop currents in every element in opposite directions as there are in mesh analysis, and the equations are much less systematic in structure. It is therefore much easier to make mistakes.)

Symmetry is a crucial underlying principle in physics in general and is arguably fundamental to logical views of the world. It is pervasive in nature and often thought to be associated with notions of beauty as well. In the following, we discuss a modified approach to mesh analysis that seeks to highlight and preserve the beautiful symmetry and duality inherent in circuit analysis, rather than to obscure them, as we believe is the case in the conventional approach. The practical advantages of the more flexible symmetric method are pointed out, and the results of surveys of students who have been exposed to the new approach are presented.

2. What is the Proper Definition of a Mesh?

Typically, a mesh is defined in modern literature as a loop in a circuit that does not enclose any smaller loop [19] (though older references may use the words mesh and loop interchangeably to mean what are now called loops [15]). By this definition, the periphery of a planar circuit does not constitute a mesh, though some works refer to it as the "outer mesh" [9, 18, 20]. This distinction does not however appear to be logical. If a planar circuit is drawn on the surface of asphere, which Whitney proved is always possible [13], there is no meaningful distinction between "inner" and "outer" meshes [16]. The originally outer mesh now encloses a region of the surface just as the originally inner ones do. In fact, as Whitney also proved [13], a circuit
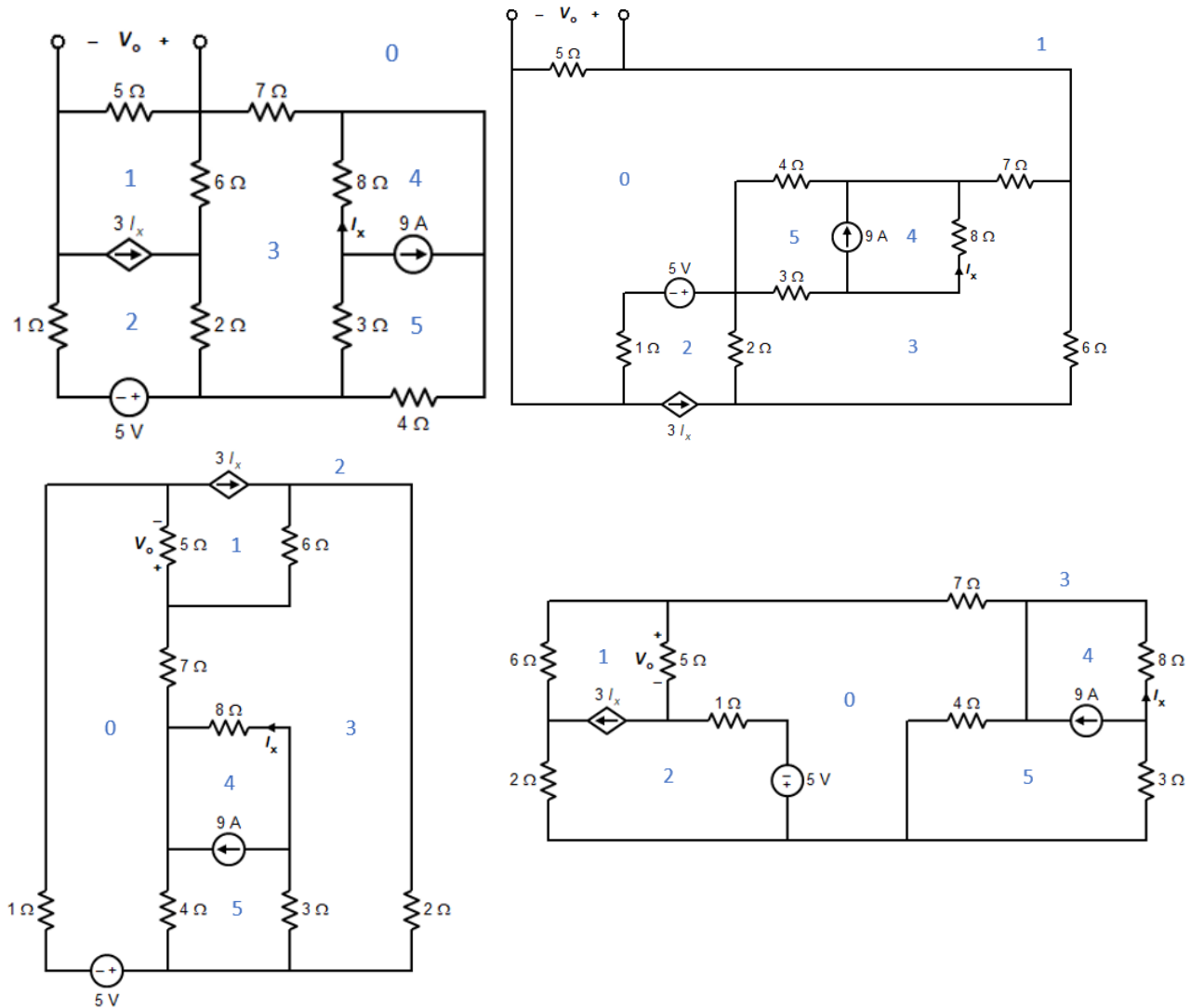
Fig. 1. The same circuit drawn with four of the six meshes chosen to be the outer mesh. Corresponding meshes are numbered the same on each diagram for clarity. Similar drawings can be done with the other two meshes as the outer mesh (not shown).

(graph) can be re-drawn on a plane having any of the originally interior meshes become the outer mesh, as illustrated for one circuit in Fig. 1. This re-drawing can be accomplished by imagining that we "snip out" the interior of the mesh that is to become the outer one when it is drawn on a sphere, and then stretch out the remainder of the spherical surface to lie flat on a plane inside the cut we made. Alternatively, we can go directly from any of the drawings in Fig. 1 to any other by "stretching out" or enlarging a particular mesh that we want to become the outer mesh, then "folding" all other circuit elements underneath to fit inside that mesh, without breaking or changing any connections (i.e., turning the circuit "inside out.") It does not seem particularly logical to distinguish between meshes based on how the circuit has been drawn, given that this choice is arbitrary. We therefore propose the following new definition of the word mesh to place them all on an equal footing: (We assume that any loops of shorts have been removed from a circuit before applying this definition, as they can produce false meshes.)

A *mesh* is a loop that does not enclose any smaller loops, *or* that is not enclosed by or a portion of any larger loop in a planar circuit.

(As usual, we define a loop to be a closed path in a circuit that passes through at least one circuit element.) With this new definition, the smallest complete circuit has two meshes rather than one (its one interior and one exterior mesh). This observation yields a pleasing symmetry between single node-pair and single mesh-pair circuits, which was previously lacking.

The distinction between loops and meshes still remains somewhat arbitrary, however [21]. In the circuit at upper left in Fig. 1 (with outer mesh 0), for example, the 8 Ω and 9 A circuit elements are in parallel and could be switched with each other without changing the circuit or any electrical quantities therein. Yet the original mesh 5 involving the 3 Ω, 4 Ω, and 9 A elements would then become a loop, and the original loop around the combination of meshes 4 and 5, involving the 3 Ω, 4 Ω, and 8 Ω elements, would become a mesh. Meshes are therefore not generally invariants of a circuit with respect to the way it is drawn in the way that nodes are, at least when they contain elements in parallel sets. In nodal analysis there is no such ambiguity. However, switching the order of elements in a series set is the dual of switching elements in parallel, and does interchange the elements connected to any given node.

3. Revising the Mesh Analysis Procedure

Having defined the outer mesh to be a mesh like any other, it should be assigned a mesh current as well. If the interior mesh currents are all chosen to be clockwise, as is often done, the outer mesh current must be counterclockwise to be consistent [18, 20] (as is more obvious when looking at the circuit on a spherical surface). An example is shown in Fig. 2 for what is now denoted a three-mesh circuit. It is now evident that unlike the usual approach, *all* branch currents (labeled I_a, I_b, etc. in the figure) are now given by a difference of mesh currents, not just the "interior" ones. For example, $I_a = I_1 - I_0$, etc. This situation is now exactly analogous to that in nodal analysis, where all branch voltages are the difference of exactly two node voltages.

As branch voltages and currents are the only physically meaningful quantities, it is now evident that mesh currents have no real physical significance any more than node voltages do. One can add the same constant to all mesh currents without affecting any branch current, just as is well known for node voltages. The only reason that node voltages or mesh currents are even defined is to reduce the number of equations required to analyze a circuit compared to doing so using only branch voltages and branch currents. As is well known, the very act of defining unique node voltages guarantees that KVL is satisfied throughout the circuit, just as the very act of defining mesh currents guarantees that KCL is satisfied [2,
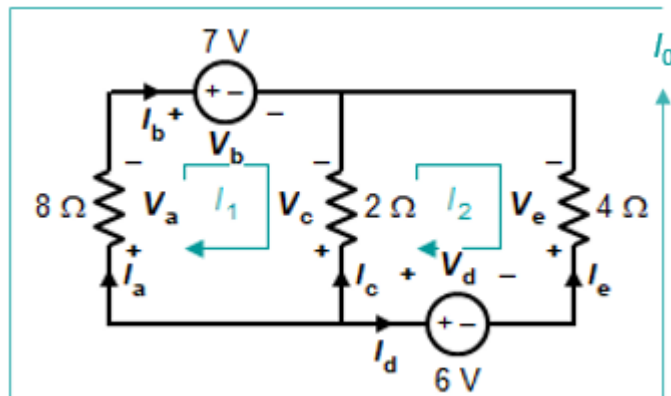

Fig. 2. A three mesh circuit with both inner and outer mesh currents defined with consistent directions.

22]. The conventional approach however suggests that mesh currents, being sometimes equal to the current in an external branch that conventionally is thought to have only one mesh current, have an absolute physical significance. A circuits handbook even suggests that a mesh current through an exterior branch can be uniquely measured with an ammeter [19], but in fact only *differences* in mesh currents (i.e., branch currents) can be measured, just as a voltmeter can only measure *differences* in node voltages. Whereas it is true that physical (branch) currents are absolute in a sense that physical node voltages are not, it becomes clearer in the new approach that neither node voltages nor mesh currents have any true physical significance at all and are purely fictitious quantities. (It is *not* only mesh currents in purely interior meshes that are fictitious, as asserted in Ref. [19].) This fact may help students to achieve a better understanding of what mesh currents really are.

Given that mesh current is now clearly relative rather than absolute, it becomes more apparent that a reference mesh needs to be selected in the same fashion that a reference node is selected in nodal analysis [16, 22]. The usual approach always (but usually without mentioning that it does so) takes the outer mesh to be the reference mesh by default, and conventionally its current is *defined* to be zero. (In principle it could equally well be defined to be any other constant value, but equations are obviously simplified by choosing that constant to be zero, just as the voltage of the reference node or ground is conventionally chosen to be zero, even though it could be chosen to be any other fixed value.) Once that definition is made, there is no need to show the current of the reference mesh, which explains why it is never shown in conventional mesh analysis. Yet, the outer mesh need not be the one we choose as the reference mesh. *Any* mesh can be so chosen, and the circuit does not need to be redrawn using that mesh as the outer mesh (which is often challenging to do in practice). If we choose one that is not the outer mesh, the outer mesh current must be shown and included in all calculations, but one of the inner meshes will lack a mesh current. In nodal analysis, a ground symbol is used to label the reference node. No such symbol now exists for a reference mesh. We therefore propose a new symbol called a "halt" symbol, consisting of a circle with an X inscribed in it, to denote that the mesh current of a particular mesh has been "halted" and set to zero. The halt symbol defines the zero of mesh current, just as the ground symbol defines the zero of node voltage. (This concept does not appear to be widely known. For example, a well-known circuits handbook asserts that "furthermore, no global reference exists for mesh currents as it does for node voltages." [19], p. 19-28) This symbol is placed in the interior of a mesh (or outside the circuit if the outer mesh is the reference mesh) and is not attached to any node. If mesh 2 in Fig. 2 is selected as the reference mesh, for example, the circuit would now appear as shown in Fig. 3.

It should be noted in Fig. 3 that some branch currents now appear to be determined by a single mesh current. For example, $I_e = I_2$. Yet in reality, they are still differences in mesh
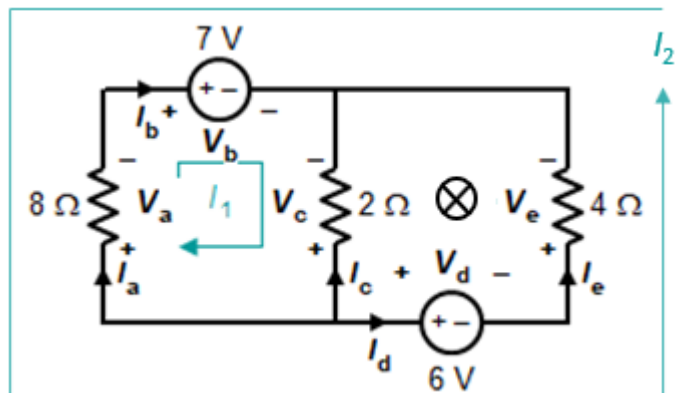

Fig. 3. The circuit of Fig. 2 after selecting its mesh 2 as the reference mesh (and then renumbering the meshes). The outer mesh current is maintained.

currents; it is just that one of those mesh currents has been set to zero. An exactly analogous situation applies to branch voltages in nodal analysis.

Fig. 4. A circuit in which choosing meshes 2 or 5 as the reference mesh could considerably simplify the resulting mesh equations.

There is no obvious advantage in Fig. 2 to selecting mesh 2 as the reference mesh. Yet, we know in nodal analysis that we can often simplify the resulting node equations if we choose a reference node that is connected to a large number of circuit elements. Moreover, choosing a reference node that is connected to a one or more voltage sources sometimes eliminates the need to define a supernode, which may be confusing to beginning students. It also simplifies the form of the corresponding voltage constraint equations. A similar principle applies in choosing a reference mesh. It should be chosen to be one containing many elements, and if possible one or more current sources to minimize or eliminate the need for supermeshes. One does not normally write a KVL equation for the "reference supermesh" (a supermesh that includes the reference mesh) or reference mesh in mesh analysis, just as no KCL equation is normally written for the reference supernode or node in nodal analysis.
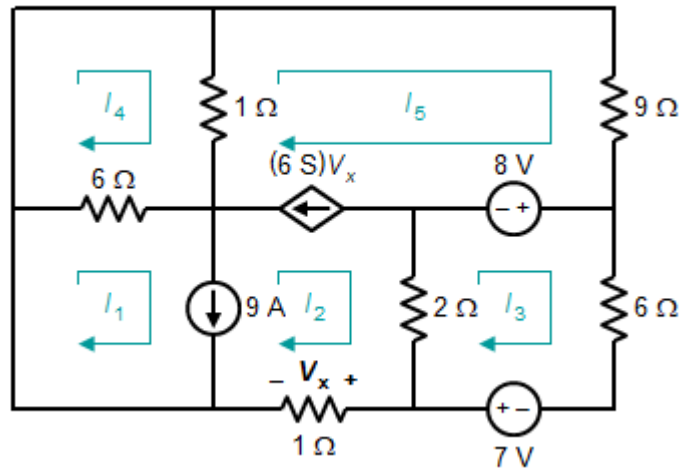
For example, for the circuit in Fig. 4, it would be clearly advantageous to choose either mesh 2 or 5 as the reference mesh rather than the outer mesh. If the outer mesh is chosen by default as the reference mesh, one will need to form a supermesh consisting of meshes 1, 2, and 5, yielding a KVL equation with nine terms, in addition to KVL equations for meshes 3 and 4. Further, neither of the current constraint equations directly yields a mesh current. Choosing mesh 2 as the reference mesh however eliminates its nine-term KVL equation entirely and only one term is added to the KVL equation for mesh 3 (and none for mesh 4). Further, simple equations now directly give both I_1 and I_5 directly as current source values, eliminating two more equation terms. The algebraic solution process will be greatly simplified. These advantages do not of course obtain in every problem, but the outer mesh can still be selected as the reference mesh whenever it is simplest to do so. Choosing a different reference mesh shifts the values of all mesh currents in the circuit by the mesh current of what will be the new reference mesh with respect to the old reference mesh (just as choosing a different ground in nodal analysis shifts all node voltages by the node voltage of what will be the new reference node with respect to the old reference node).

A step-based tutoring system known as Circuit Tutor now incorporates an introductory tutorial for students on this new approach to mesh analysis [21]. Unfortunately, the randomly generated examples and exercises in that system do not yet support using this method, as the relevant parts of the software were written prior to conceiving of the new approach and considerable changes

will be required. It is planned to add support for it in the future. The entire tutorial system is freely available (via the first author) to instructors who wish to use it.

The revamped approach to mesh analysis can now be summarized as follows:

1. Select a reference mesh, whose mesh current is usually defined to be 0 A. This mesh can be any mesh in the circuit, including the outer mesh, but should ideally include as many circuit elements and as many current sources as possible. Place a halt symbol within that mesh.
2. Identify and number all remaining meshes in the circuit (possibly including the outer mesh) and assign them mesh currents I_1, I_2, etc. If used, the outer mesh current should point counter-clockwise (opposite to the interior mesh currents).
3. Write current constraint equations relating a difference of mesh currents to the value of each current source (but the reference mesh current is just zero).
4. Write a Kirchhoff's voltage law (KVL) equation for each non-reference mesh that does not contain a current source.
5. Form "supermeshes" consisting of "trees" of current sources and their connected meshes and write a KVL equation for the periphery of each such supermesh. (A "tree" of current sources includes all sources that are pairwise connected by a common mesh, whether interior or exterior.)
6. Write an equation for each current or voltage controlling a dependent source in terms of mesh currents.
7. Write an equation for each unknown ("sought") voltage, current, or power that one wishes to know about the circuit in terms of mesh currents.
8. Solve the resulting system of equations for all such sought quantities.

Note that the above procedure is now fully symmetrical with and dual to that for nodal analysis, as it should be.

4. Experience in Using the New Approach

The first author used this method for the first time in two sections of a linear circuits class in Fall 2019 with ~60 students in each. He assigned the interactive tutorial in Circuit Tutor mentioned above and explained the new method in detail in lecture. Students generally appeared to be quite receptive to it. An anonymous survey was carried out by an independent evaluation team to assess student opinions regarding this method. The results of two survey questions are shown in Fig. 5. A majority of students (85%) reported the new approach was somewhat or much better than the traditional approach of using their textbooks to solve problems. Moreover, 60% stated that they would recommend future students learn the subject using the new approach.

The students were also asked to share comments about the new approach to teach mesh analysis. Evaluating the open-ended comments made by students, 14 were classified as favorable towards the new method, 3 as unfavorable, 5 as neutral, 3 as "no opinion," and 2 as irrelevant. Some illustrative favorable comments included:

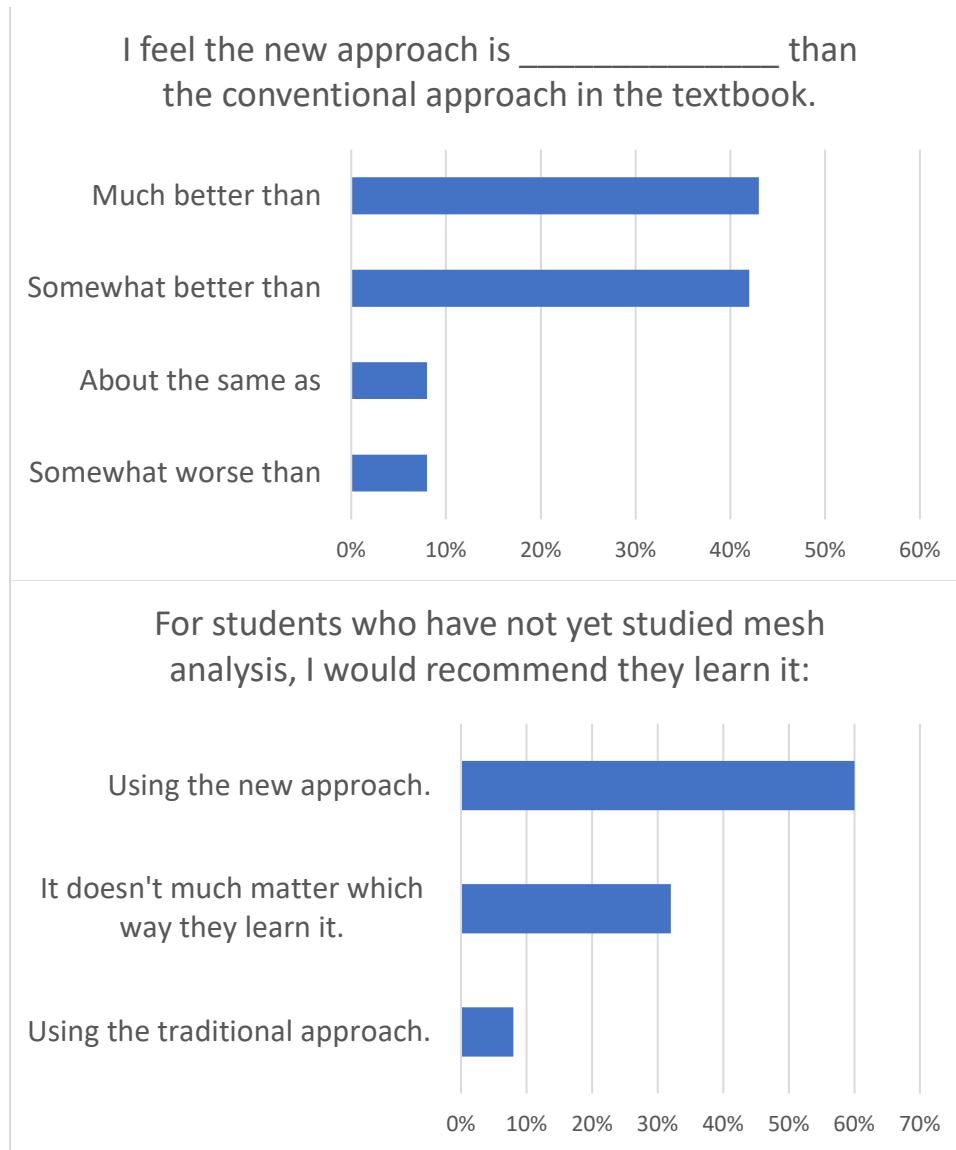- *Tying mesh and nodal analysis together helped my comprehension and made retention easier*

Fig. 5. Combined results of a student survey for two sections of a linear circuit course in Fall 2019.

- *It's useful in cases in which there are current sources within the circuit but not on the outer mesh, which can make the problems easier to solve in avoiding using supermeshes*
- *The more similar the steps are to nodal analysis, the harder they will be to mess up. Teaching mesh analysis in this way has been extremely beneficial due to how similar they look through the techniques learned in class.*
- *I felt it was easier to understand than the traditional method.*

Of the three students who reported unfavorable comments, it was stated that the program was confusing. For example, as one student said, "*Having a reference mesh was a bit confusing to me.*"

One limitation reported that could be linked to unfavorable comments about clarity, was that exercises in any format need additional support. As stated by one student,

- *As long as practice opportunities are provided on how to determine where to put the reference mesh, the new method will work well. The lack of practice discouraged me from picking a reference mesh that wasn't the outer loop due to my worries of if I am abiding by passive sign convention or not.*

A limitation was that neither the textbook nor the homework (Circuit Tutor) exercises supported the new method. Having such support would be helpful while students become more accustomed to using it. Some students were observed using this method on mesh analysis problems on exams. The method is being used again in Spring 2020, and more detailed evaluation is planned. One planned improvement suggested by student comments is to use concrete examples such as Fig. 4 to better illustrate the value of the method in some problems for reducing the number and complexity of the mesh equations. A seminar is also planned at the author's institution to introduce other instructors to the new approach and hopefully stimulate its adoption.

5. Conclusions

A new procedure for mesh analysis has been developed that greatly enhances and emphasizes the symmetry and duality of nodal and mesh analysis. It is shown that any mesh in a circuit can be selected as a reference mesh, increasing the flexibility of the analysis and its similarity to nodal analysis. It is hoped and believed that this method will improve student intuition with regards to electrical circuits, though quantitative assessment of that outcome will be difficult. The new method can reduce the complexity of the equations in mesh analysis in a significant fraction of cases. Student survey responses were generally quite supportive of teaching this subject using this new approach, with 85% finding it superior to the traditional one.

6. Acknowledgments

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