

## Two Approaches to Optimize Formula SAE Chassis Design Using Finite Element Analysis

**Dr. Tanveer Singh Chawla, Western Washington University**

Dr. Chawla is an Assistant Professor in Plastics and Composites Engineering, Engineering & Design Department at Western Washington University, Bellingham, WA. His background is in solid mechanics and materials. Research interests other than in mechanics of materials include manufacturing, characterization and repair of fiber reinforced polymer composites, and diversity in STEM.

**Mr. Eric Leonhardt, Western Washington University**

I have been working to develop lower cost composite manufacturing processes for vehicles and built the Viking 40 and Viking 45 X Prize vehicles as demonstrators. Hybrid electric Viking 32 was built with funding from the Federal Highway Administration to demonstrate and develop carbon fiber honeycomb as an energy impact absorber—a development that was unique to Western. Viking 32 also became the world's first biomethane hybrid as we demonstrated "Cow Power to Horsepower". We have been developing a renewable fuel source, biomethane (a.k.a. renewable natural gas) since 2004. We built a compressed natural gas fuel station at the Vander Haak Dairy to help us develop our biogas upgrading technology. Funded by the Department of Energy, EPA, Paul Allen Family Foundation, Washington State Department of Agriculture, Whatcom Public Utility District, BP, and partnering with the Vander Haak Dairy and the Bellaire/Airporter Shuttle bus has allowed us to demonstrate some of the upgrading technology. We hope to develop the funding to complete the novel, pilot scale upgrading facility to produce up to 60,000 gasoline gallons of equivalent fuel energy. It is one of just a few facilities in North America. With students I helped develop a composite hood installation tool for Ford—the second use of composite tools in automotive assembly—and composite door molds for Bentley. I've also worked with students to develop lean manufacturing tools and jigs for PACCAR.

The Vehicle Research Institute operates as a technology development center that provides undergraduate students with opportunities for career specific training and research. Funding comes from a variety of sources including the Department of Energy, Department of Transportation, EPA, Paul Allen Family Foundation, BP, Washington State Department of Agriculture, Whatcom Public Utility District, Boeing, Janicki Industries, Northwest Porsche Club, Danner Corp. and Fluke. Past supporters include the Department of Defense, Fuji Heavy Industries (Subaru), PACCAR, Mazda, Ford, Bentley (parent company Audi), Alcoa, Conoco-Phillips, CNG Fuels of Canada, Chrysler, and DaimlerChrysler.

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## Abstract

This paper documents two approaches used by undergraduate students to design and optimize a steel space frame chassis using Finite Element Analysis (FEA) for the Society of Automotive Engineers' Formula SAE (FSAE) collegiate design competition. Junior level students in Vehicle Design I used CATIA V5's Generative Structural Analysis workbench to analyze their individual FSAE chassis designs. A tutorial is presented that allows a quickly modeled CAD wireframe to be analyzed within CATIA using FEA with beam elements. Senior vehicle design students in a course titled Introduction to Finite Element Analysis studied FEA using ANSYS Mechanical APDL. Five teams of senior students each performed a different type of optimization on the FSAE chassis design as the final course project. In both courses, students record predicted torsional rigidity and predicted chassis weight for their respective designs. The junior Vehicle Design I students measured torsional rigidity of two completed FSAE vehicles. Assessment of relevant student learning outcomes for each course and student feedback are presented.

## Introduction

This paper provides a tutorial for using the CATIA V5 Generative Structural Workbench to design a space frame. Junior level vehicle design students demonstrate the application of the process to their own individual FSAE chassis designs. In parallel, five groups of senior level vehicle design students used ANSYS Mechanical ADPL to perform different optimization strategies to improve the specific torsional stiffness of an FSAE chassis design. A literature review indicates three unique results of this work. First, the CATIA V5 wireframe approach has not been documented as a method to design a Formula SAE chassis. Second, this seems to be the first time that two class sections used FEA to design and optimize an FSAE chassis. The papers reviewed tend to be from individual or small groups from an SAE team and/or to satisfy a senior project or thesis requirement. Finally, results of specific chassis changes made by each group of seniors are documented. The authors believe these results are applicable to FSAE space frame chassis design generally. The results may be used to prioritize strategies for the FSAE chassis design process.

In Vehicle Design I, students practice the vehicle design process by designing a vehicle for a particular market. For fall 2017, the focus was for each student to design an FSAE vehicle. Student learning outcomes for Vehicle Design I are listed in Table 1 below:

Table 1. Vehicle Design I Student Learning Outcomes

Student Learning Outcome:	Assessment
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1.	Demonstrate an understanding of tire forces and characteristics.	Students calculate lateral loads and tire grip with respect to normal force and slip angle.
2.	Demonstrate the ability to design and fabricate a vehicle chassis.	Students design a CAD model of a vehicle and fabricate a scale model of the vehicle. Students use FEA to analysis chassis frame.
3.	Calculate weight transfer and estimate tire grip levels based on chassis parameters.	Students demonstrate calculations on worksheets.
4.	Demonstrate how to design a vehicle suspension system.	Students create kinematic models of the suspension system and integrate the resulting suspension design into their vehicle models.
5.	Calculate loads on suspension components.	Student demonstrate calculations on worksheets and using spreadsheets.
6.	Simulate chassis dynamics to estimate lap times.	Students demonstrate how changing vehicle parameters affect lap times in simulation software such as OptimumG or Bosch LapSim.
7.	Demonstrate how to select and size components in a braking system.	Students calculate tire normal forces under braking and then use a spreadsheet to select components for a brake system.

Course objectives and the resulting student learning outcomes for Introduction to Finite Element Analysis are focused on developing and demonstrating FEA skills and techniques. Assessment for the FEA course is based on the following ABET student learning outcomes.

- (i) Student outcome assessment (e) (an ability to identify, formulate, and solve engineering problems)
- (ii) Student outcome assessment (g) (an ability to communicate effectively)
- (iii) Student outcome assessment (k) (an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice)

## Background

SAE International's Formula SAE (FSAE) competition challenges collegiate students from around the world to design, build, and test an autocross racing vehicle that can be produced in 1000 units per year. This engineering competition attempts to educate students about vehicle engineering, product development, project management, testing, and marketing. Several versions of the competition are held around the world including venues in the U.K, Germany, Brazil, Japan, Australia, Italy, India, and the United States. Student FSAE teams must balance several competing requirements for a successful chassis design:

- Packaging
- FSAE Technical Regulations
- Safety
- Manufacturing
- Cost
- Knowledge/Team capabilities
- Technical/Performance requirements such as mass reduction, stiffness, and strength

The objective of the presented work is to improve students' ability to design spaceframe chassis utilizing FEA techniques.

## Literature Review

The authors reviewed many papers related to the analysis of Formula SAE space frame chassis using Finite Element Analysis techniques. Most of the papers provide an FEA analysis of an FSAE chassis and provide simulated torsional stiffness values. A few correlate analytical values with experimental values. Three papers provide an approach to determine how stiff to build an FSAE chassis—listed as source 1), 2), and 13) in Table 2 below. Most FEA is performed with beam elements unless noted.

Table 2. Examples of FSAE chassis design using FEA

Reference	Results	Comments
1) Deakin, et. al., 2000 [1]	Proposes that chassis stiffness be 80% of roll stiffness.	ANSYS FEA file analyzed using ADAMS Flex model with stiffness 1300 Nm/deg
2) Riley and George, 2002 [2]	Developed a relationship between desired chassis stiffness and ratio of suspension stiffness to wheel rate.	Compared experimental 949 Nm/deg to ANSYS FEA 1074 Nm/deg
3) Foroudastan and Campbell, 2005[3]	Evaluate hands-on vehicle design projects benefits for students	FEA is performed on chassis, but not as part of curriculum
4) Abrams, 2008 [4]	Evaluate stressed engine impact on chassis stiffness	COSMOS Works simulates three modes of vibration with and without tubes replacing engine
5) Singh, 2010 [5]	Detailed design process using FEA and modal analysis using linear tetrahedral elements	Documents improvement from 250 Nm/deg to 616 Nm/deg while reducing mass from 36 kg to 32 kg
6) Salter, 2011 [6]	Detailed design process using FEA Overview of historical frame design with Nm/deg data from race vehicles spanning 1935-1993	Provides a lateral load transfer vs. chassis stiffness graph ANSYS Mechanical APDL used to design 5728 Nm/deg chassis model
7) Limwathanagura, et. al., 2012 [7]	Validate chassis FEA result with physical testing	1006 Nm/deg FEA model vs. 955 Nm/deg experimental test
8) Gutierrez, Nuñez, and Quintero, 2013 [8]	FEA on FSAE composite body	
9) Jogi, et. al, 2014, IJRET [9]	Reviews past FEA work	
10) Jogi, et. al, 2014, IJESI [10]	FEA displacement results for chassis, applies ANSYS linear	Evidence of torsion analysis using FEA, no stiffness values reported, 90 kg chassis

	solver to non-linear front, side, rear, and rollover loading conditions	
11) Diaz, et. al., 2014 [11]	Complete FSAE design document	FEA results show reduction from 2856 Nm/deg to 2019 Nm/deg to reduce mass from 33 kg to 24 kg
12) Chambers, et. al., 2016 [12]	Documents design of a test stand for experimental testing of FSAE chassis	FEA chassis design 1491 Nm/deg vs. 1609 Nm/deg. experimental measurement
13) Velie, 2016 [13]	Uses vehicle dynamic simulation to show how reducing chassis stiffness requires more steering input Also used 10:1 ratio of chassis stiffness to roll stiffness	Compared chassis models with 2100 Nm/deg to 210 Nm/deg Asymptotic curve estimates required steer angle vs. chassis stiffness. Proposes 1900 Nm/deg design target to achieve 1550 Nm/deg experimental result
14) Chandan, Sandeep, and Vinayaka, 2016 [14]	Poissons and orthogonal continuity optimization applied to chassis optimization using FEA and Design of Experiment	Front, side, rear, and rollover impact analysis (static FEA), torsion ANSYS Mechanical ADPL shows 1463 Nm/deg, modal analysis performed

### Chassis Stiffness Background

Chassis stiffness determines how efficiently lateral loads developed during cornering may be apportioned to the front or rear of the vehicle. A four-wheeled, road-going vehicle develops lateral forces at the tire contact patch (i.e. where the rubber meets the road) as a function of tire design, construction, and the normal force on the tire. Because the vehicle's center of gravity is invariably above the lateral forces generated at the contact patch, a couple is created that is a function of the vehicle weight (mass\*gravity), height of the center of gravity, track width (distance between the tire centerline along an axle), and the lateral acceleration rate [15]. This couple removes a portion of the normal force on the inside tires and transfers this load to the outside tires.

On a four-wheeled vehicle using a conventional suspension, the lateral load transfer may be apportioned to the front or rear axle by changes including altering suspension geometry, the ratio of spring rates from front axle to rear axle, and an anti-roll bar—essentially a torsion bar that connects suspension members across an axle and resists the rolling of the sprung mass (i.e. the vehicle chassis) while the vehicle is cornering. The chassis engineer may change the front axle to rear axle roll stiffness to shift how much lateral load is apportioned to the front or rear axle. A stiff chassis is required to transmit these lateral loads both lateral and diagonally across the chassis [16]. This may be done to adjust for an imbalance in the normal force at the front and rear tires (think rear-engine, rear-weight biased vehicle or traditional front-drive, front weight-biased vehicle). This can also be done to affect small changes that improve the driver's ability to

control the vehicle or improves the overall grip of the vehicle, or some combination of these two, competing objectives. While preparing for the legal defense of GM and the Corvair's handling, Frank Winchell, Jim Musser, and Mauri Rose documented that with sufficient chassis torsional stiffness, the front to rear roll couple distribution could be adjusted (in part by adjusting roll stiffness) to achieve neutral handling on most vehicles [17].

If the vehicle chassis is perfectly rigid, changes in the roll stiffness of the vehicle will directly determine the apportionment of lateral load transfer to the front and rear axle [18]. If the vehicle chassis is relatively soft, the chassis will deflect and reduce the impact of an increase or change in roll stiffness. Some of the roll stiffness will effectively be absorbed by the chassis. Frank Winchell's GM team initially built a test vehicle with just over 400 Nm/deg of roll stiffness and just over 4000 Nm/deg of chassis stiffness with an under 26 kg aluminum monocoque chassis [19]. This 10:1 ratio of chassis stiffness to roll stiffness from the mid-1960's became a benchmark target for many years. This marked an improvement in understanding over most 1950's chassis design, when Formula 1 engineer Tony Rudd quoted chassis stiffness as only 325 Nm/deg [20].

Vehicle engineers wish to understand how stiff to build the vehicle [21] and what trade-off in increased structural and overall vehicle weight must be endured to achieve the desired stiffness. Thompson et al. performed an FEA analysis and found that increasing the FEA model chassis stiffness 130% over the baseline model stiffness of 13,469 Nm/deg resulted in an increase of 7.3% of effective roll stiffness at the modeled front suspension for a Winston Cup (nee Nextel or NASCAR series) car [22]. They found that for this class of vehicle a torsional chassis stiffness of 31,319 Nm/deg resulted in an effective front roll stiffness within 3% of the value from an infinitely rigid chassis [23].

The work in this paper seeks to provide students with a method of analyzing their chassis designs so that their design decisions may be more informed.

## **Vehicle Design I Course**

Students in Vehicle Design I follow a design process including capturing vehicle requirements, determining performance metrics, sketching and ideation, packaging, creating computer aided design models, and building a physical scale model of their designs. Vehicle Design I students are introduced to examples of tubular space frames (truss designs) and various forms of monocoque chassis including production stamped steel unit-body, aluminum riveted structures and composite chassis. More than fifty vehicles have been built within the program since 1972 so students have the opportunity to observe various construction techniques on prior vehicles.

Students perform four activities related to chassis torsional stiffness. These include an FEA tutorial, experimentally measuring torsional stiffness of two FSAE vehicles, building and testing 1/10 scale balsa chassis models, and analyzing their own designs using the FEA wireframe technique. The three activities related to FEA are explained below, followed by the quantitative results of each activity, and feedback from students related to each activity.

## **FEA Tutorial**

Vehicle Design I students perform a roughly 20 step process to create and analyze the stiffness of a rectangular box within CATIA V5 that approximates the size of an FSAE cockpit. The box is 559 mm x 559 mm x 1016 mm. The box does not represent an FSAE rules compliant cockpit but rather a simplified structure to demonstrate the effect of triangulation. Students start first with an open wireframe box without triangulation. A join command fuses all of the line elements together within the wireframe. Students perform an FEA analysis using the Generative Structural Analysis workbench within CATIA V5. Steel is applied to the FEA model. The mesh element size used is 10.0 mm. Using the Model Manager toolbar, the 1 D Property is set to a beam element. A User Material is applied such that a tubular beam element is selected with a 12.7 mm outside radius and an 11.0 mm inside radius to represent FSAE rules compliant steel tubing with a 1.65 mm wall thickness. On one 559 mm x 559 mm face of the box representing the rear of the cockpit, the two lower most corners of the box are held with clamp restraints. These are applied to the wireframe (instead of adding them to the mesh) as required by the Generative Structural Analysis workbench. On the opposite face, 100 Nm moments are applied at each vertex of the 559 mm x 559 mm face. The moment is clockwise when viewing the cockpit box from the front face to the rear face. This approximates the process used to apply a torque to a scale model or full-scale physical chassis. An analysis of the box is performed. A screen shot of the deflection and maximum principal stress is saved to confirm and acknowledge the students' completion of each case study.

After each analysis is run and the results saved, the students add an additional diagonal and re-run the analysis. The tutorial guides the students to perform the addition of diagonals in a particular order. Students triangulate the full box, such that an actual "driver" can no longer climb into the cockpit opening. After results for the fully triangulate box are saved, students remove the cockpit opening triangulation and add an external "wing" or top plane triangle as an attempt to externally triangulate the box on one side. The analysis process is repeated and then a second external triangle is added to the other side. These external triangles arbitrarily feature a short side of 350 mm and a long side of 1016 mm running the length of the box. An additional triangulating member is added from the bottom most vertex at the rear of the cockpit box to the outer most vertex of the top plane triangle. This is illustrated in Figure 1 below. This figure also shows both cockpit sides, bottom, and rearmost face with triangulating members.

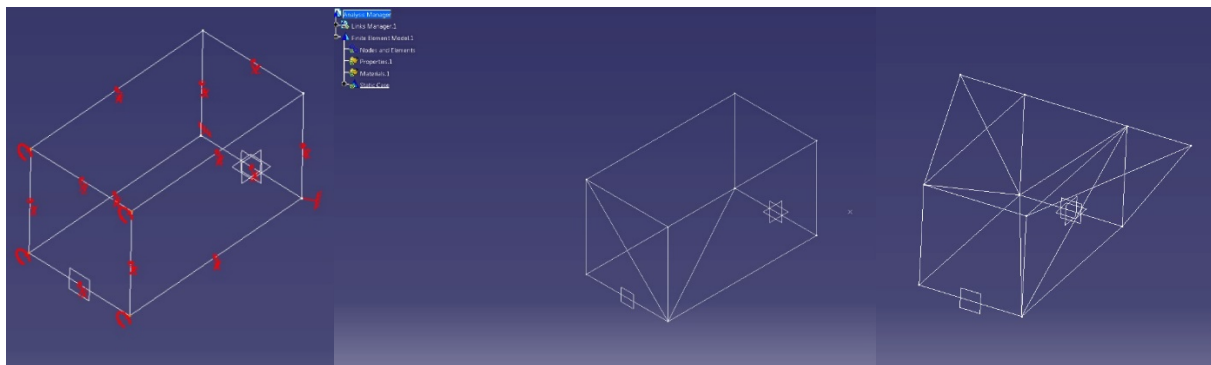


Figure 1. Wireframe cockpit CAD examples.

Table 3 records the results of the tutorial below. These results help determine whether or not students successfully completed the tutorial. The twist of the cockpit is calculated by dividing

the maximum deflection by the half-width of the chassis. The mass of the chassis is estimated using by using the measure function (caliper icon) on the join feature within the tree to estimate the total length of tubing. This was found to be within 99.98% of results calculated using trigonometry for every case study. This seem to validate the technique of estimating the total length of tubing in the model by measuring the join feature within the CATIA V5 design tree. A mass per mm of 1.376 g/mm was multiplied by the estimated length to calculate cockpit mass.

Table 3. FEA Tutorial results for each case.

Case #	Description Cockpit Geometry	Max Deflect. mm	Max Principal Stress MPa	Twist Deg	Stiff. Nm/deg	Stiff. to Mass Ratio Nm/deg-kg	Comments: Max. deflection recorded at vertices where moments are applied.
1	Wireframe box with no triangulation	3.26	+/- 0.623	0.669	598	50.9	
2	Add member to the front cockpit face where moment is applied	3.24	+ 0.618 - 0.626	0.664	602	46.9	Addition of this member adds no significant stiffness because twisting occurs at this face.
3	Triangulate top plane where driver enters cockpit	2.42	+0.931 -0.671	0.496	806	55.8	
4	Add member to the bottom of the cockpit	2.71	+1.17 -0.637	0.556	720	44.9	Deflection increases with addition of bottom member. Unexpected.
5	Add diagonal to the rear face of the cockpit	2.35	+1.28 -1.09	0.482	830	48.5	
6	Add diagonal to the side of cockpit.	1.84	+1.41 -1.22	0.377	1060	56.6	Select side of cockpit with vertices joined by diagonals
7	Add member to the other side face of cockpit	0.0364	+/- 1.49	0.007	53.5 kN/deg	2638	Cockpit box is fully triangulated



8	Remove the front face diagonal	0.0366	+1.41 -1.49	0.008	53.2 kN/ deg	2772*	Front face where moment is applied does not contribute to stiffness
9	Remove top face, cockpit opening diagonal.	1.37	+1.24 -1.39	0.281	1424	80.8	Driver can now “enter” cockpit. Significant drop in stiffness.
10	Add an external triangulating “wing” to one side of the top plane of cockpit opening	1.14	+1.19 -1.22	0.234	1711	83.5	External triangulation improves specific stiffness
11	Mirror external triangle	0.969	+1.22 -1.2	0.199	2013	86.2	External triangulation improves specific stiffness

\* This stiffness to mass ratio used a manual calculation for the tubing length and therefore mass estimate.

Eleven of twelve students were present to provide feedback on the tutorial. All of the students commented that the tutorial showed them how important triangulation is to improve stiffness. Several of the students observed how beneficial the external triangulation technique is to improve cockpit stiffness. One student felt that the external triangulation might not always be beneficial depending upon tubing size and location. Another student observed that depending upon the axis of rotation, tubes could be selectively added and removed to meet a stiffness target while reducing weight.

### Physical Testing

Following the FEA exercise, physical testing was performed on two actual FSAE vehicles to demonstrate to students how this test could be performed. It also allowed students to compare values of stiffness between the two vehicles (V54 and V58) to understand some of the challenges with this physical testing. Each vehicle was tested by placing the vehicle on a table. Each of the four tires was supported on a corner weight scale. Lasers were placed near the front and rear axle of each vehicle and pointed at a wall 11.8 m away. A jack was applied under the front tire scale closest to the wall where the lasers were projected. As the jack is lifted, a load is applied up to 454 N. The initial projections of the lasers are recorded, as are the laser projections after the load is applied. The difference between the displacement of each of the front laser projections and each of the rear laser projections is used to calculate the angle of twist of the chassis. The torque on the chassis is estimated as the applied force at the tire multiplied by the distance to the center of the chassis. The stiffness of the chassis is then calculated by dividing

the applied torque by angle of twist, Nm/deg. Because this particular test applied the force directly to the tire, the test represents the “hub-to-hub” stiffness of the vehicle. The spring and dampers were still attached to the vehicles and thus contribute to additional compliance in the measured values. Table 4 displays some of the results.

Table 4. Physical torsion testing results of V54 and V58 FSAE vehicles.

Vehicle Test	Applied Load N	Distance to Wall m	Track width mm	Applied Torque Nm	Chassis Twist Degree	Stiffness Nm/degree
V58 1	343	11.86	1316	225	0.26	865
V58 2	445	11.86	1316	293	0.33	884
V58 3	414	11.86	1316	272	0.26	1057
V54 1	440	11.84	1316	290	0.22	1310
V54 2	454	11.84	1316	299	0.31	953
V54 3	440	11.84	1316	290	0.31	925

These hub-to-hub stiffness results compare favorably with measured results of hub-to-hub stiffness of a Cornell vehicle at 995 Nm/deg [24], University of Southern Queensland vehicles from 2005 (485 Nm/deg) and 2004 (233 Nm/deg) [25], and the calculated (FEA) chassis stiffness of 616 Nm/deg from Chitkara Institute of Engineering and Technology [26].

V58 uses a composite monocoque for the driver’s cockpit bolted to a welded tubular steel sub-frame supporting the roll hoop, rear engine and suspension. V54 uses an entirely welded tubular steel chassis. Both use 25.4” diameter tubing that vary in thickness in response to FSAE rule requirements and local strength and/or stiffness requirements. The vehicles feature similar wheelbase dimensions and nearly identical track widths. V58 uses one combination pneumatic spring and damper unit per wheel while V54 uses a single conventional coil spring over damper per wheel. While performing the test, it was observed that V58’s pneumatic spring would not compress in a linear fashion. One outcome of this is observed in the variation between the applied load for V58 vs. the V54 test. The corner weight scales used to measure the chassis feature a resolution of 1 lb. (4.4 N). The four corner weights observed for the V54 test were much more consistent than the V58 values. As an additional 4.4 N was applied to the front left tire of V54, the addition was observed on the opposite corner tire and the left rear was observed to have a 4.4 N unit of force removed. V58 was tested first. After both vehicles were tested, students observed this difference and recognized that the V58 test would benefit from the use of a rigid link replacement for the pneumatic springs. Once the data was entered into a spreadsheet, it was observed that the V58 data has a smaller standard deviation than the V54 test, and this was not expected. In either case, many more data points should be collected from each test before a specific stiffness might be claimed for either vehicle. Even with using a nearly 12 m long triangle with a laser, the resulting angle changes are quite small and sensitive to how the test is carried out. The use of a jack to lift the unrestrained vehicle limits the amount of load that may be applied to increase the applied torque.

All of the students commented that they had learned how to conduct an experimental torsion test on a vehicle. A few students commented that the differences between the steel space frame and

carbon monocoque chassis are significant. A few mentioned comparing the experimental results to FEA results and results from scale balsa model testing that was performed. One student observed that the test procedure and fixture must be very stiff to improve the test results. None of the students commented on which of the two chassis—steel space frame versus composite monocoque—were stiffer.

### Wireframe FEA on Final Design

Eight of the students used the CATIA Generative Structural Analysis workbench to analyze their chassis designs. One student performed a full chassis FEA analysis within Hyperworks. The Hyperworks analysis is beyond the scope of the Vehicle Design I course. Most of the eight students followed the same process as outlined within the FEA tutorial. A few deviated by applying different moments or loads to produce a torque. Numeric results of the final models, when available, are displayed in Table 5 below. Most of the students applied their moments at the front bulkhead of the chassis, similar to how the tutorial was performed. Chassis 1 applied loads directly to each of the front suspension nodes. Chassis 11 applied 100 Nm moments to six nodes at the front foot box face. The remaining examples all applied 100 Nm moments to four vertices at the front foot box face. All of them clamped their models at the rear most bulkhead. Some of the students clamped the models at four nodes instead of two. The additional constraints effectively stiffen the chassis.

Table 5. Wireframe FEA Results on Final FSAE Chassis Designs

Chassis Number	Maximum Principal Stress N/m <sup>2</sup>	Maximum Deflection mm	Torsional Stiffness Nm/deg	Mass Estimate kg	Specific Stiffness Nm/deg-kg	Rear Bulkhead Clamp
1	3.61x10 <sup>5</sup>	0.156		78.3*		Two lower nodes
2	3.06x10 <sup>6</sup>	0.48	3337	65*	51.3	Two lower nodes
4	1.81x10 <sup>5</sup>	0.56	5477	55*,33**	108.5	Four nodes
5		0.89	1452	70*	20.6	Two lower nodes
7	3.46x10 <sup>6</sup>	0.81	1913	53*	36.1	Four Nodes
8	2.27x10 <sup>6</sup>	0.82	1931	50*	38.9	Four Nodes
11	1.19x10 <sup>7</sup>	1.73				
12		0.44				

\*Used FEA Tutorial approach \*\*Manually estimated using spreadsheets and surface models

In future courses, recording students' analysis throughout the design process would be beneficial to document students' progress. Chassis design one features external wing structures that added stiffness to the structure. Chassis seven has a similar feature, but none of the other designs have this feature. For chassis one, the low applied moment, and the relatively short wheelbase, below the FSAE legal minimum of 1524 mm, makes direct comparison between the other designs challenging. Mass estimates by students from designs four and seven compare favorably with

results from the FEA class models and design nine’s Hyperworks models. Mass from design one includes the lengths of the control arm members and a connecting “upright” length from the upper to the lower control arm outboard end. Stiffness was calculated by measuring the angular deflection at the front face of the foot box, measuring the distance from the applied torque to the chassis centerline, and then dividing the resulting moment by the angular deflection.

A weakness of this wireframe approach is that the beam elements are applied as a constant cross section tube to the entire model. With this approach, it is not possible to analysis a chassis with different diameter or tube wall thickness variation. The approach can help to determine where diagonal tubes should be placed, somewhat more quickly than building and importing a model into a dedicated FEA package.

Student feedback from using FEA to analyze their own chassis designs was much more varied than feedback from the tutorial and physical testing experiences. Many student comments related to observing how adding individual tubes or tube structures improved stiffness. Students compared their work to observe the impact of different approaches. Remaining comments were much more specific. One student observed that adding stiffening members also increased the maximum stress. The rollover bar was observed to have a large deformation during simulation. Poor cockpit stiffness was noted. Students commented that how the chassis was clamped in the rear affected the stiffness analysis. Another student noted that locating the input loads at the suspension nodes improved the accuracy of the analysis and that external cross bracing improved results. One student was motivated to learn Altair’s Hyperworks package and performed an optimization on the FSAE teams design for 2018. This student observed some of the same improvements as the senior level students and was also in daily contact with the senior students during the design process.

### Vehicle Design I Outcomes

Following the physical testing and FEA tutorial, students continued to design their own individual FSAE chassis within Vehicle Design I. One student designed the FSAE chassis to be used in 2017-2018. All these students completed a computer aided design model of their chassis and a physical scale model of their chassis as part of the course outcome assessment. Nine out of twelve students utilized the FEA techniques taught during the tutorial to improve their chassis designs. Assessment results are tabulated in Table 6 below:

Table 6. Vehicle Design I Outcomes

Course Outcome:	Demonstrate the ability to design and fabricate a vehicle chassis.	Outcome Fall 2017	Comments
Current Assessment Description and Metric:	80% of students vehicle designs meet vehicle requirements	11 out of 12 students achieved, 92%	Required an FSAE rules compliant design, including

	assigned within course		cockpit opening templates.
	80% of students complete a CAD model of chassis.	12 out of 12 achieved, 100%	Students presented CAD model with suspension kinematics
	80% of students complete a scale chassis model	12 out of 12 achieved, 100%	
Proposed Assessment Metric:	80% of students successfully complete FEA tutorial	12 out of 12 students completed, 100%	Success implies obtaining correct values for each case study
	80% of students successfully measure torsional stiffness of chassis model	11 out of 12 students measured, 92% 11 out of 12 students reduced the std. deviation to 5% or less of the mean	Success implies reducing the measurement deviation with repeated measurements
	80% of students use FEA to improve chassis design	9 out of 12 students used FEA, 8 used wireframe FEA	Students will need more guidance and support to achieve this proposed goal.

### Introduction to FEA Class Project

As mentioned previously, a lot of research related to design of chasses of cars using FEA techniques has been carried out. Samant et al [27] designed a front double A-arm push rod suspension system for their formula student car. The design was then validated their design by static and kinematic analysis. Mat et al [28] designed an ‘Eco-Challenge’ student car space frame and performed FEA simulations under five varying loading conditions. In the Introduction to Finite Element Analysis class the final project for the students from the Vehicle Engineering concentration was based on optimization of the tubular chassis of the new FSAE vehicle Viking 59 (V59). The project was executed by eleven students who were split into five groups (four groups with two members and one group with three members). The FSAE has certain requirements pertaining to the design of the vehicle. These requirements revolve mostly around safety and innovation. Other than these requirements, the design of the vehicle also depends on manufacturing and budget constraints. As mentioned before, the chassis of the vehicle should be stiff but also light weight for better handling characteristics thus an optimal value of stiffness is required that does not cause increase in weight. The design of the chassis of V59 is based on its suspension. The points where the suspension mounts to the chassis cannot be changed. These were the considerations that students had to keep in mind while carrying out the optimization of the chassis. The optimizations were evaluated by determining the torsional rigidity of the chassis at each iteration.

The model of the V59 chassis in ANSYS Mechanical APDL was created using a key-point cloud imported from a chassis model in CATIA. These key-points were then connected by lines and meshed with the Pipe 288 elements to create the tubular frame. The limit of elements available influenced their size. An example of the chassis model is given in Figure 2. The initial design, material and the dimensions of the frame were the same as those of last year's FSAE car. Elements that represented a stressed engine were given a very high modulus as compared to the other elements that made up the rest of the frame (Figure 2.).

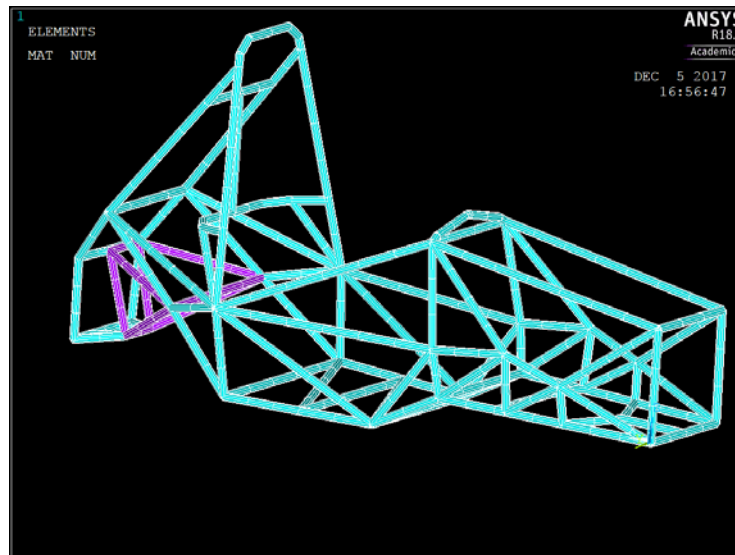


Figure 2. Chassis model of Viking 59 (Purple elements represent the engine)

Students assumed the rear bulkhead to be rigid and constrained it in all degrees of freedom (DOF). A torsional couple with 500lbs. each force was applied at the front suspension points by all students. The middle node on the top rung of the front bulkhead was used to determine the twist in the chassis for finding the torsional rigidity. The single bar on the underside in the front of the vehicle was constrained to prevent the chassis to move back or forth along the x-axis. The applied loads and constraints can be seen in Figure 3. Using these loads and constraints the students then conducted a mesh convergence study to find the optimal mesh size and type. Again, the limitation of the maximum number of nodes available in the academic version of ANSYS Mechanical APDL had a great influence on the mesh convergence study and some students were not able to fully justify their mesh size and/or type due to that. An example of the mesh convergence study is given in Figure 4. All the design changes were then carried out using their determined optimal mesh size and type.

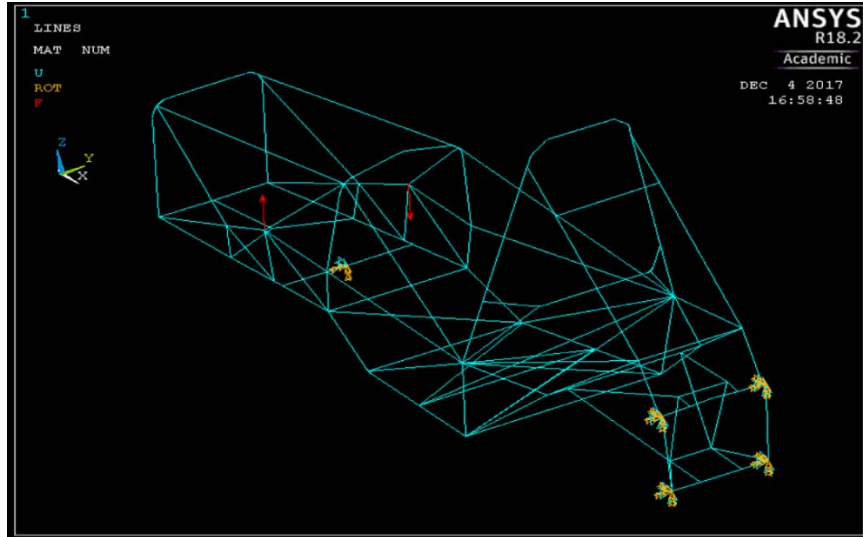


Figure 3. Loads and constraints applied on the chassis.

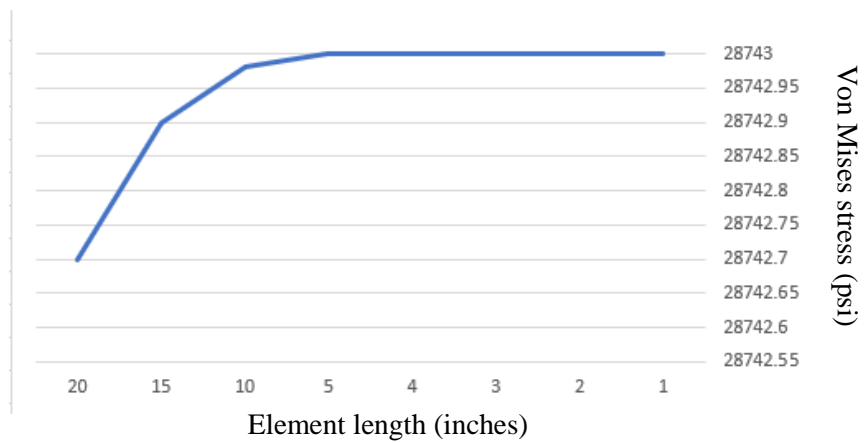


Figure 4. Mesh convergence study example.

The changes made to the original chassis design and the resulting changes in the revised chassis models are given in Table 7 to Table 11.

Table 7. Data for optimization of chassis for Group 1

Iteration	Total chassis mass (kg)	Percentage change in chassis mass*	Stiffness Nm/deg	Percentage change in stiffness*	Changes made to the chassis
0	33.39	NA	1098	0	Base model
1	33.39	0	1101	0.31	Triangulated parallel bottom brace

2	35.01	4.85	2119	93.00	Added V brace to top part
3	35.09	5.08	2149	95.75	Replaced V brace with X brace
4	32.02	-4.10	1043	-4.98	Removed cross brace 1
5	32.07	-3.96	1048	-4.52	Removed cross brace 2
6	30.70	-8.06	1026	-6.59	Removed both cross braces
7	32.33	-3.19	1067	-2.83	Triangulated bottom interior
8	34.10	2.12	1161	5.74	Added X brace above rear bulkhead
9	34.24	2.54	2058	87.49	Triangulated front
10	31.88	-4.54	2324	111.69	Combined 1, 6, 7 & 9 iterations
11	31.36	-6.10	2283	107.95	Combined 1, 6, 7 & 9 iterations and removed bottom cross brace

*\*All percentage changes are with respect to the base model.*

Table 8. Data for optimization of chassis for Group 2.

Iteration	Percentage change in chassis mass*	Stiffness Nm/deg	Percentage change in stiffness*	Changes made to the chassis
0	NA	1101	0.00	Base model
1	-3.82	1019	-0.82	Removal of seat back
2	-4.56	941	-1.60	Removal of bottom diagonal trusses
3	8.28	752	-3.49	Triangle Tubes added



4	9.98	783	-3.18	Triangle Tubes w/ supports
5	2.37	1569	4.68	One brace from bulkhead to roll hoop
6	4.73	1616	5.15	X brace from bulkhead to roll hoop
7	1.32	964	-1.37	One brace across front bulkhead
8	2.64	972	-1.29	X brace across front bulkhead
9	4.49	1547	4.46	"V" brace from front bulkhead to roll hoop
10	3.69	2383	12.82	One bulkhead-to-roll hoop and one front bulkhead brace (non-intersecting)
11	3.69	2383	12.82	One bulkhead-to-roll hoop and one front bulkhead brace (intersecting)
12	0.99	951	-1.50	One brace in front of rear bulkhead
13	4.67	2594	14.93	Intersecting front braces plus rear bulkhead brace

Table 9. Data for optimization of chassis for Group 3.

<b>Iteration</b>	<b>Total chassis mass (kg)</b>	<b>Percentage change in chassis mass*</b>	<b>Stiffness Nm/deg</b>	<b>Percentage change in stiffness*</b>	<b>Changes made to the chassis</b>
0	46.52	NA	1309	0.00	Base model (tube thickness 2.4 mm)
1	32.92	-29.24	895	-31.60	2.4 mm roll hoops, 1.65 mm side impact, rest is 1.24 mm
2	27.71	-40.44	722	-44.84	2.4 mm roll hoops, 1.65 mm side impact, rest is 0.89 mm
3	31.86	-31.51	881	-32.67	Rules legal, rest is 0.89 mm

4	32.04	-31.13	3953	202.01	Rules legal, rest is 0.89 mm, optimized tube placement
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Table 10. Data for optimization of chassis for Group 4.

Iteration	Total chassis mass (kg)	Percentage change in chassis mass*	Stiffness Nm/deg	Percentage change in stiffness*	Changes made to the chassis
0	40.10	NA	1093	-0.02	Base model
1	40.02	-0.21	1969	80.11	Cross brace at driver seat removed, long cross member added from front bulkhead to front roll hoop
2	41.98	4.69	2068	89.17	Triangulated front bulkhead to front roll hoop
3	41.96	4.64	2115	93.51	X brace added from front bulkhead to front roll hoop

Table 11. Data for optimization of chassis for Group 5.

Iteration	Total chassis mass (kg)	Percentage change in chassis mass*	Stiffness Nm/deg	Percentage change in stiffness*	Changes made to the chassis
0	35.70	NA	1561	0.00	Base model
1	33.30	-6.72	1299	-16.78	Simplified floor bracing
2	34.19	-4.24	2482	59.00	Added bracing to foot box with simplified floor bracing
3	34.53	-3.27	2948	88.85	Triangulated front bulkhead
4	36.09	1.09	3083	97.50	Added upper side impact members

As seen from the data in the tables shown above, students were able to increase the torsional stiffness from 15% to 200% using FEA and changing the design of the chassis. The corresponding increase in weight of the chassis was minimal. In some cases, the students were able to increase rigidity and decrease the weight of the chassis at the same time. The Introduction to FEA course has the course and program outcomes in accordance with ABET as given in Table 12.

Table 12. Course and program outcomes of Introduction to FEA

<b>Course Outcomes – Students will be able to:</b>		<b>Program Outcomes</b>
1.	Understand underlying concepts of simple finite element analysis.	(a), (k)
2.	Solve problems involving linear elasticity and heat transfer by performing engineering simulations using ANSYS.	(e), (k)
3.	Write a professional engineering report to present FEA results.	(g)
4.	Apply gained knowledge to execute self-directed projects involving finite element analysis of metals, plastics and composites to solve engineering problems.	(i), (k)

For the assessment of this project course outcomes 2, 3 and 4 were evaluated using the program outcomes (e), (g) and (k). The performance indicators used for evaluation and the results are given in Table 13 to Table 15. The results are expressed in four categories (unsatisfactory, developing, satisfactory and exemplary) along with the percentage of students on the project falling into the same.

Table 13. Student outcome assessment (e) (an ability to identify, formulate, and solve engineering problems)

<b>Performance Indicator (Student has the ability to .....</b> )		<b>Criterion used to evaluate</b>	<b>Results</b>
1.	identify problems with a quantifiable solution that can be approached systematically.	Performance of chassis quantified as torsional rigidity	Unsatisfactory: 0 Developing: 0 Satisfactory: 27.3% Exemplary: 72.7%
2.	select appropriate methods and techniques for solving the problem.	Ability to make chassis model from CATIA to work in ANSYS Mechanical APDL	Unsatisfactory: 0 Developing: 0 Satisfactory: 18.2% Exemplary: 81.8%
3.	correctly formulate the problem according to chosen solution method.	Used correct mesh size, elements, real constants and sections.	Unsatisfactory: 0 Developing: 0 Satisfactory: 0 Exemplary: 100%

4.	select appropriate values, ranges and bounds for variables and correctly use these in the formulation to obtain a solution.	Applied correct boundary conditions and loads to the model	Unsatisfactory: 0 Developing: 0 Satisfactory: 100% Exemplary: 0
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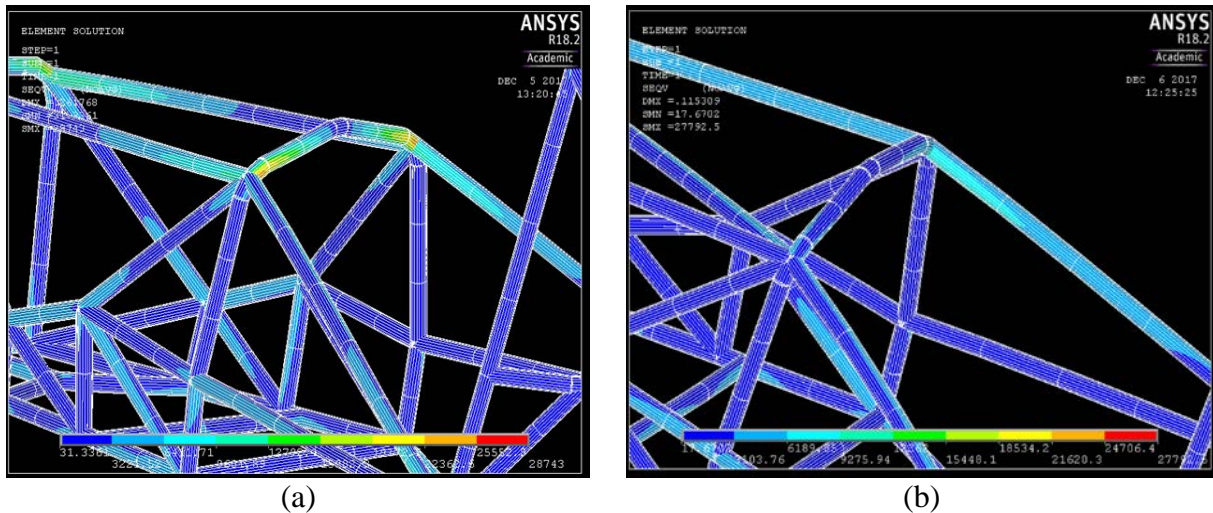


Figure 5. Example of FEA simulation of chassis. (a) high stress in front roll hoop in base model (b) reduction of stress in front roll hoop with triangulation

Table 14. Student outcome assessment (g) (an ability to communicate effectively)

Performance Indicator (Student has the ability to .....		Criterion used to evaluate	Results
1.	make effective use of available methods and tools	Quality of project presentation slides and project report	Unsatisfactory: 0 Developing: 0 Satisfactory: 81.8% Exemplary: 18.2%
2.	use the methods and tools in an organized and concise manner	Organization of slides and report	Unsatisfactory: 0 Developing: 0 Satisfactory: 81.8% Exemplary: 18.2%
3.	use the methods and tools with professionalism including grammar, spelling and usage	Delivery of presentation and language in report	Unsatisfactory: 0 Developing: 0 Satisfactory: 63.6% Exemplary: 36.4%
4.	use content and style appropriate to the audience	Content in presentation and report	Unsatisfactory: 0 Developing: 0 Satisfactory: 63.6%

			Exemplary: 36.4%
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Table 15. Student outcome assessment (k) (an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice)

Performance Indicator (Student has the ability to .....		Criterion used to evaluate	Results
1.	apply technology in design.	Ability to use ANSYS Mechanical APDL	Unsatisfactory: 0 Developing: 0 Satisfactory: 36.4% Exemplary: 63.6%
2.	apply technology in analysis or simulation.	FEA simulation of chassis model	Unsatisfactory: 0 Developing: 0 Satisfactory: 36.4% Exemplary: 63.6%
3.	use (and practical experience with) manufacturing processes for materials	Not evaluated	
4.	use technology in characterizing the properties of the designed product, process, or material to satisfy goals.	Analysis and discussion of properties of chassis	Unsatisfactory: 0 Developing: 0 Satisfactory: 72.7% Exemplary: 27.3%

### Student Feedback

Students taking the PCE 397A course provided specific comments about what they learned from FEA analysis. Students noted that the cockpit opening was the least stiff region. Triangulating the top of the foot box and the front of the foot box led to important stiffness gains. They noted that a single triangulating member to a bay provided significant gains while diagonal bracing provided minor specific stiffness gains over the singular member. They noted that tube thickness did not have a significant impact on chassis stiffness. The students were also asked to rate (Table 16.) how well they thought they had gained abilities related to the ABET outcome (e). Comparing Table 13. with Table 16. an interesting observation was made. In case of all performance indicators except for 1., students rated themselves lower than what they had been rated by the instructor based on their project report.

Table 16. Student responses to questions related to student outcome (e).

Performance Indicator (Student has the ability to .....	Questions asked	Student Responses
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1.	identify problems with a quantifiable solution that can be approached systematically.	Rate how well you were able to analyze and quantify the torsional rigidity of the chassis?	Unsatisfactory: 0 Developing: 0 Satisfactory: 50% Exemplary: 50%
2.	select appropriate methods and techniques for solving the problem.	Rate your ability to make the chassis model from CATIA to work in ANSYS Mechanical APDL	Unsatisfactory: 0 Developing: 25% Satisfactory: 75% Exemplary: 0
3.	correctly formulate the problem according to chosen solution method.	Rate your ability to use the correct mesh size, elements, real constants and sections.	Unsatisfactory: 0 Developing: 12.5% Satisfactory: 62.5% Exemplary: 25%
4.	select appropriate values, ranges and bounds for variables and correctly use these in the formulation to obtain a solution.	Rate how well you were able to apply correct boundary conditions and loads to the model	Unsatisfactory: 0 Developing: 50% Satisfactory: 50% Exemplary: 0

## Conclusions

An FEA tutorial using a wireframe analysis of chassis stiffness was introduced to students to improve their understanding of truss structures and to provide them with an approach to analyze their own chassis designs. All of the students successfully completed the tutorial. All of the students demonstrated a key course learning objective by designing and building a chassis for the FSAE vehicle competition. Eight of the twelve students used the wireframe technique to analyze their wireframe models.

The use of FEA to analyze the chassis model will require additional support and guidance to ensure that all students have the capability to improve the stiffness to mass ratio. In addition, the specific process used to evaluate the FEA models requires improvement. Deflection, for example, will need to be measured at a consistent location on the chassis. A logical location would be the suspension mounts. The key metric of specific stiffness was calculated for the FEA course and the wireframe FEA analysis.

The authors observe that VHCL 360 students found the front of the foot box bay did not require triangulation. This was an observable difference between the PCE 397A students and the VHCL 360 students. This occurred because the PCE 397E students applied their loads directly to the front suspension mounting points, while restraining the chassis at the rear suspension nodes. The VHCL 360 students did not all have suspension nodes clearly defined both in the FEA tutorial and with their own chassis designs so the torsion loads were applied to the front of the foot box. Since the front foot box face was being rotated about the longitudinal chassis axis, a diagonal member had no effect and in fact reduced specific stiffness.

Students of the Introduction to FEA course were successfully able to change the designs of the chassis and improve its torsional rigidity. The groups used different methods to carry out the chassis optimization. FSAE team now has FEA data and designs for both the current V59 FSAE chassis and future student race cars.

## References

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- [1] A. Deakin, D. Crolla, J. P. Ramirez, and R. Hanley, "The Effect of Chassis Stiffness on Race Car Handling Balance," in *Proceedings of the 2000 SAE Motorsports Engineering Conference and Exhibition, SAE Motorsports Engineering Conference and Exhibition*, November 13-16, 2000, Dearborn, Michigan, SAE Technical Paper Series 2000-01-3554.
- [2] W. Riley and A. George, Design, "Analysis and Testing of a Formula SAE Car Chassis," in *Proceedings of the 2002 SAE Motorsports Engineering Conference and Exhibition, SAE Motorsports Engineering Conference and Exhibition*, December 2-5, 2002, Indianapolis, Indiana, SAE Technical Paper Series 2002-01-3300.
- [3] S. Foroudastan and I. Campbell, "Student Projects: Hands-On Experience with Mechanical Engineering Technology," in *Proceedings of the 2005 American Society for Engineering Education Annual Conference and Exhibition*, June 12-15, 2005, Portland, Oregon.
- [4] R. Abrams, "Formula SAE Race Car Analysis: Simulation & Testing of the Engine as a Structural Member" presented at FISITA World Automotive Congress (Student Congress Section), September 14-19, Munich, Germany, F2008-SC-00.
- [5] R. Singh, "Structural Performance Analysis of Formula SAE Car," *Jurnal Mekanikal*, No. 31, pp. 46-61, Dec. 2010.
- [6] J. Salter, "Analysis and Manufacture of 2011 REV Formula SAE Vehicle Chassis," Five year project dissertation, School of Mechanical and Chemical Eng., Univ. of Western Australia, Crawley, WA, Australia, 2011.
- [7] T. Limwathanagura, C. Sithananun, T. Limchamroon, and T. Singhanart, "The Frame Analysis and Testing for Student Formula," *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*, vol. 6, no. 5, pp. 998-1002, 2012.
- [8] J. Gutierrez, A. Nuñez, D. Quintero, "Development of a Formula SAE Body," B.S. thesis, Florida International Univ., Miami, Florida, United States, 2013.
- [9] N. G. Jogi, A. P. Take, Y. Asolkar, S. M. Aftab, "Review Work on Analysis of F1 Car Frame Using ANSYS", *International Journal of Research in Engineering and Technology*, vol. 3, no. 4, pp. 215-217, April 2014.
- [10] N. G. Jogi, P. R. Shahade, A. K. Kaware, "1Analysis of Formula Racing Car Frame Using ANSYS", *International Journal of Engineering Science Invention*, vol. 3, no. 6, pp. 22-33, June 2014.
- [11] A. Diaz, O. Fernandez, R. Gonzalez, C. Ramos, "FSAE 2015 Chassis and Suspension Final Report," B.S. thesis, Florida International Univ., Miami, Florida, United States, 2014.
- [12] A. E. Chambers, S. A. Rodriguez, R. M. Walsh, D. Wootton, "Development of a Test Stand for Determining the Torsional Rigidity of a Formula SAE Space Frame," *Proceedings of the 2016 American Society for Engineering Education Annual Conference and Exhibition*, June 26-29, 2016, New Orleans, Louisiana.
- [13] Hubbard D. Velie, "Chassis Torsional Rigidity Analysis for a Formula SAE Racecar", unpublished white paper, Dept. of Mechanical Engineering, University of Michigan, Ann Arbor, MI, United States, 2016.
- [14] Chandan S. N., Sandeep G. M., and Vinayaka N., "Design, Analysis and Optimization of Race Car Chassis for its Structural Performance", *International Journal of Engineering Research & Technology*, vol. 5, no. 7, pp. 361-367, July 2016.



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- [15] W. Milliken and D. Milliken, *Race Car Vehicle Dynamics*. Warrendale, PA: SAE International, 1995, p. 679.
- [16] W. Milliken and D. Milliken, *Race Car Vehicle Dynamics*. Warrendale, PA: SAE International, 1995, p. 676.
- [17] W. Milliken and D. Milliken, *Race Car Vehicle Dynamics*. Warrendale, PA: SAE International, 1995, pp. 444-445.
- [18] W. Riley and A. George, "Analysis and Testing of a Formula SAE Car Chassis," in *Proceedings of the 2002 SAE Motorsports Engineering Conference and Exhibition, SAE Motorsports Engineering Conference and Exhibition*, December 2-5, 2002, Indianapolis, Indiana, SAE Technical Paper Series 2002-01-3300.
- [19] W. Milliken and D. Milliken, *Race Car Vehicle Dynamics*. Warrendale, PA: SAE International, 1995, p. 453.
- [20] T. Rudd, *It was Fun*. England: Patrick Stephens (Haynes Publishing Group), 1993.
- [21] W. Riley and A. George, "Analysis and Testing of a Formula SAE Car Chassis," in *Proceedings of the 2002 SAE Motorsports Engineering Conference and Exhibition, SAE Motorsports Engineering Conference and Exhibition*, December 2-5, 2002, Indianapolis, Indiana, SAE Technical Paper Series 2002-01-3300.
- [22] L. Thompson, P. Soni, S. Raju, H. Law, *The Effects Chassis Flexibility on Roll Stiffness of Winston Cup Car*, Racing Chassis and Suspension Design, C. Smith, Ed. Warrendale, PA: SAE International, 2004, pp. 93-105.
- [23] L. Thompson, P. Soni, S. Raju, H. Law, *The Effects Chassis Flexibility on Roll Stiffness of Winston Cup Car*, Racing Chassis and Suspension Design, C. Smith, Ed. Warrendale, PA: SAE International, 2004, pp. 93-105.
- [24] W. Riley and A. George, "Analysis and Testing of a Formula SAE Car Chassis," in *Proceedings of the 2002 SAE Motorsports Engineering Conference and Exhibition, SAE Motorsports Engineering Conference and Exhibition*, December 2-5, 2002, Indianapolis, Indiana, SAE Technical Paper Series 2002-01-3300.
- [25] J. Salter, "Analysis and Manufacture of 2011 REV Formula SAE Vehicle Chassis," Five year project dissertation, School of Mechanical and Chemical Eng., Univ. of Western Australia, Crawley, WA, Australia, 2011.
- [26] R. Singh, "Structural Performance Analysis of Formula SAE Car," *Jurnal Mekanikal*, No. 31, pp. 46-61, Dec. 2010.
- [27] Y. S. Saurabh, S. Kumar, K. K. Jain, S. K. Behera, K. Kalita, "Design of Suspension System for Formula Student Race Car," *Procedia Engineering*, vol. 144, pp. 1138-1149, 2016.
- [28] M. H. Mat, A. R. Ab. Ghani, "Design and Analysis of 'Eco' Car Chassis", *Procedia Engineering*, vol. 41, pp. 1756-1760, 2012.