

2006-1206: TWO MATHEMATICS COURSES FOR ARCHITECTURE COLLEGE STUDENTS: FROM CONTEXT PROBLEMS TO DESIGN TASKS

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Abstract

This paper considers Mathematical Aspects in Architectural Design course in a college of architecture, which focuses on experiential learning activities in the design studio. The design process is tackled from three geometrical complexity directions: tessellations, curve surfaces, and subdividing space by solids. Mathematical needs in architecture design and relevant learning methods were selected from interviews with practicing architects and educational literature. The course evaluation was based on observations, attitude questionnaires, project portfolios and interviews. Portfolio's assessment criteria focused on the project contents, design solutions and mathematics applications. Results of the course follow-up revealed a variety of mathematically-defined complex geometrical shapes applied in students' design projects. The increased interest, challenge, motivation, self-learning and positive attitude towards application of mathematics in architectural design were demonstrated. The findings lead to the conclusion that continuous mathematics studies in the architecture education are required, in order to assimilate mathematical concepts and turn them into a practical tool in architecture design.

Introduction

Recent research has emphasized the value of mathematical thinking in architecture, particularly in geometrical analysis, formal description of architectural concepts and symbols, and engineering aspects of design. The studies call for accommodating authentic mathematical learning in architecture education¹⁻⁶.

The two general approaches to teaching mathematics in context are Realistic Mathematics Education (RME) and Mathematics as a Service Subject (MSS). In RME, the mathematics curriculum integrates various context problems which are experientially real to the student⁷, while the MSS approach considers mathematics as part of professional education and focuses on mathematical skills needed for professional practice^{8, p. 8}.

Our study utilizes the RME and MSS approaches to developing an applications-motivated mathematics curriculum for colleges of architecture. At the first stage we developed a first year mathematics course, based on the RME approach⁹. The two-year follow-up indicated the positive effect of integrating applications on motivation, understanding, creativity and interest in mathematics. However, from the analysis of 52 graduation design projects of students who studied the first year RME-based mathematics course, we found that the students did not apply mathematical knowledge acquired in the course in their architectural design projects.

This situation motivated us at the second stage of the study to develop a Mathematical Aspects in Architectural Design (MAAD) course based on the MSS approach. The MAAD course is given in the second college year. It relies on the first year mathematics course and offers mathematical learning as part of hands-on practice in architecture design studio¹⁰. The course focuses on the analysis of different types of geometrical forms used in architectural design.

This paper presents the concept of the MAAD course, its architectural design assignments, mathematical models and learning activities. The paper also reports students' achievements and attitudes towards the course.

Geometrical Forms

In developing the course subjects we considered the principles of classification of geometrical forms in architecture education. Salingaros¹¹ and Consiglieri and Consiglieri¹² formulated the principles of geometrical complexity of architectural forms and proposed studying them from elementary geometrical objects to their complex compositions. Grounded on these principles, our study dealt with three directions of geometrical complexity in architectural design. The course contents for each direction were selected from architectural education literature or recommended by the architects, as presented below:

1. Arranging regular shapes to cover the plain (tessellations)

Boles and Newman¹³ developed a curriculum which studied plain tessellations arranged by basic geometrical shapes with focus on proportions and symmetry. Applications of Fibonacci numbers and golden section in designing tessellations were emphasized. Frederickson¹⁴ studied geometrical dissections of figures into pieces and their rearranging to form other figures, using two methods: examining a shape as an element of the module, and examining a vertex as a connection of elements. Ranucci¹⁵ studied mathematical ideas and procedures of tessellation design implemented in Escher's artworks.

2. Bending bars and flat plates to form curve lines and surfaces (deformations)

Hanaor¹⁶ developed a course "The Theory of Structures" which focused on "the close link between form and structure, between geometry and the flow of forces in the structure". He pointed that distributed loads on bars and surfaces effect deformations which can be described by different mathematical functions. The curved surfaces defined by these functions are used to minimize deformation of structures under distributed loads and to express aesthetic principles. The reciprocal connection between form and construction characterizes Gaudi's approach to architectural design¹⁷. Gaudi systematically applied mechanical modelling to create geometrical forms and to examine their properties. He also created 3D surfaces such as paraboloids, helicoids and conoids by moving generator profiles "in a dynamic manner"^{18, p. 89}.

3. Intersecting solids (constructions)

Burt⁴ examined integrating and subdividing space by different types of polyhedral elements. He emphasized that this design method can provide efficient architectural solutions. Alsina¹⁹ considered the design of complex three-dimensional forms by intersecting various geometrical forms. He analyzed the use of these forms in Gaudi's creatures in order to achieve functional purposes such as light effects or symbolic expressions.

Case Study Framework

The Mathematical Aspects in Architectural Design (MAAD) course has been implemented in one of Israeli colleges as part of the architecture program which certifies graduates as practical architects.

The MAAD course consists of three parts corresponding to the three directions of geometrical complexity introduced in the previous section of this paper. Each part of the course includes the following components: mathematical concepts and methods with connections to architecture, practice in solving mathematical problems, and design projects. The 56-hours MAAD course outline is presented in Table 1.

Table 1. The MAAD course outline

Course subjects	Components	Instructional goals	Learning activities
1. Tessellations (16 hours)	A. Mathematical concepts of tessellations design	Understanding harmonic dimensions and their use in art, music, and architecture	Seminar presentations on golden section, Fibonacci sequence, logarithmic spiral, and applications.
	B. Practice in solving mathematical problems related to tessellations	Acquiring basic skills in analysis of proportions, symmetry, and drawing tessellations	Drawing logarithmic spirals and tessellation fragments, analyzing basic geometrical figures and their combinations
	C. Tessellation design project	Acquiring experience in tessellation design, modularity, differentiation, proportion and harmony	Designing a flat tessellation of a given floor surface as a combination of modules inspired by a certain metaphoric subject
2. Curved surfaces (20 hours)	A. Algebraic surfaces used in constructions	Identifying types of mathematical surfaces utilized in roof design of existing gas stations	Seminar presentations on roof surfaces such as the Sarger surface, hyper, and ellipsoid.
	B. Practice in drawing algebraic surfaces	Developing skills of drawing surfaces through calculating parameters and coordinates	Exercises of drawing algebraic surfaces (ellipsoid, cylinder, hyper, etc.) and calculating their volumes and areas.
	C. Gas station design project	Application of calculus to defining complex roof shapes for large span solutions	Designing and building a model of a gas station roof which answers the stability, and constructive efficiency criteria.
3. Solids Intersections (20 hours)	A. Algebraic solids and intersections	Defining algebraic solids, their features, parameters and intersections	Seminar presentations on solid intersections (cylinder, oblique cone, pyramid, sphere, and ellipsoid) in constructions.
	B. Practice in calculating solids intersections parameters	Developing skills of mathematical analysis of solids and intersections	Drawing algebraic solids, their sections and involutes. Calculating volumes, surface areas.
	C. Solids and intersections project	Acquiring the skill of analysis of composed structures using analytic geometry	Decomposing an existing architectural structure into solids and analyzing intersections. Building a model of the structure

The first column includes the three course subjects (tessellations, curved surfaces, and solids intersections). The second column details the above mentioned components for each of the course subjects. The third column contains instructional goals which directed mathematical

learning in each of the subjects. The fourth column describes learning activities towards achieving the objectives.

In all three subjects, the mathematics concepts were introduced in the form of student seminars. Each of the seminars was given by a number of students and included definitions of mathematical concepts and their applications in architecture. Hands-on activities and discussions were encouraged. The practice sections of the course focused on specific mathematical skills related to the subjects. The projects were performed as individual assignments, while the course meetings were dedicated to project guidance and studio discussions. Emphasis was put on the mathematical aspects of the project.

The tessellation project assignment was as follows:

Design a tessellation of a floor surface of $34 \times 55 \text{ m}^2$ by means of identical rectangular modules. The module should be a periodic combination of various geometrical figures. Define proportions and dimensions of the figures using golden section ratio and Fibonacci numbers. Develop a concept of the designed module choosing one of the following metaphoric subjects: a temple, kinder garden, political message, harmony with nature, and musical impression.

The curved surfaces project assignment:

Design a plan of a gas station. Start from a zero level plan including access roads, parking, pumps, car wash, coffee shop, and an office. Design a top covering for the pumps area, or the roof of the coffee shop and office building. Find a design solution answering the stability, constructive efficiency, complexity and aesthetics criteria. The project stages are:

- *Identifying the project data*
- *Developing an architectural programme*
- *Defining design factors relevant to the project*
- *Generating alternative solutions*
- *Analyzing alternatives and selecting the solution*
- *Producing drawings, calculations, and a physical model.*

The solids intersections project assignments:

Select a known public building which was designed with solids intersections. The project requirements are:

- *Seminar on the building design process*
- *Mathematical analysis of the solids intersection*
- *Building a model which accurately presents solids intersection in the building*
- *Designing an additional functional module in the solids intersection area*

Methodology

The new MAAD course was implemented in the college in the 2003-2004 academic year by one of the authors (Maor) and attended by 26 second-year students. The goal of the course's follow-up was to examine mathematical learning in the architectural design studio. The study focused on the following questions:

- (1) What are the features of mathematical learning in the studio environment?
- (2) What is the effect of the proposed environment on learning mathematical concepts and methods?

The study used qualitative and quantitative methods, which analyzed architecture design experience and observed learning behaviour within the context of design studio. Data on the features of mathematical learning were gathered from:

- Interviews with experts
Two experienced practicing architects considered their professional activities and relevant mathematical concepts throughout the design stages. From these considerations we derived ideas of the design activities in the three course projects, of the design education features, and of mathematical needs in design.
- Architectural design and mathematics education literature
Relevant teaching methods using context, visualization, heuristic and intuitive reasoning, algorithmic analysis, and reflection were selected. Through integrating them with the ideas given by the architects we developed the concepts of learning activities in the course.

Data on learning outcomes and students' reflections were collected by:

- Design project portfolios
The design assessment criteria were based on the existing practice of studio evaluation and referred to the three following aspects: concept, planning/detailing, and representation/expression. The mathematics assessment criteria were: perception of mathematical problems, solving applied problems, precision in drawing geometrical objects, accuracy of calculations and parametric solutions. Frequencies and correlations of grades in design vs. mathematics evaluation grades were examined.
- Attitude questionnaire and interviews
The post-course questionnaire asked students to list the mathematical concepts studied in the course, give their opinion about its importance, and evaluate the learning subjects and methods. The in-depth interview with one of the students in the end of the course focused on his experience of applying mathematics in design before and in the course.

Analysis of results

Interviews and literature

Design stages

Two practicing architects were interviewed and described their professional activities and relevant mathematical concepts, throughout the design stages: concept design, data collection and analysis, design alternatives development, design criteria formulation, design solution selection, models and drawings producing and presentation, solution examining and revising.

Design criteria and related mathematical concepts

The criteria and mathematical concepts include the following: aesthetics, geometrical form, space division, proportions, functionality, culture, environment, symbolism, climate, geology, topography, construction rules and processes, flow of public, energy, materials, stability, durability, building limitations, efficiency, modularity, and accuracy.

Geometrical concepts in architectural design

As noted by the architects, when developing structural forms they use mathematical concepts such as reflection, symmetry, function, fractals, topological features, chaos, proportion, equality, identity, scale, algebraic surfaces, surface area, dimensions, volume, and

polyhedron. The literature sources mentioned in the geometrical forms section of the paper present numerous applications of these mathematical concepts in architecture design.

Mathematical aspects and course contents

The architects recommended to teach the mathematics concepts through architectural design projects which deal with curved surfaces, transformations, large structures and spans in airports, stadiums etc. They emphasized the importance of mathematical methods for obtaining accurate design solutions. They suggested introducing mathematics activities in the architectural design studio and focusing the activities on inquiry and learning discovery, design experiments, and critical discussions. The architects proposed to teach geometrical objects in the order of their geometrical complexity, from a point to a line, plane, surface, and 3D volume.

Design Project Portfolios

Each of the students in the MAAD course performed three projects and reported them in project portfolios. Our focus in evaluating student projects was on the correlation between students' achievements in design and in mathematics. Tables 2A and 2B present mean project assessment grades in the three course projects, following the design and mathematics criteria mentioned in the tables.

Table 2A. Design assessment grades

Project 1	Subject expression	Conception & application	Geometrical variety	Graphic representation	Design grade
Mean	83.1	89.0	77.4	64.6	74.7
Project 2	Efficiency	Aesthetics	Functionality	Program quality	Design grade
Mean	76.9	85.6	73.5	87.5	78.9
Project 3	Structure selection	Geometrical form	Model aesthetics	Additional module	Design grade
Mean	90.0	68.0	76.3	77.5	77.0

Table 2B. Mathematics assessment grades

Project 1	Problems perception	Calculus application	Modularity application	Proportion harmony	Calculations	Drawings precision	Parametric solutions	Math grade
Mean	72.1	56.2	78.4	87.5	80.2	79.4	58.1	73.1
Project 2	Dimensions calculation	Surface parameters	Roof calculation	Building a model	Model precision	Model analysis	Geometrical complexity	Math grade
Mean	76.9	68.8	81.3	76.3	81.9	81.9	85.0	78.9
Project 3	Dimensions calculation	Use of parameters	Intersection calculation	Building a model	Model precision	Intersection analysis	Geometrical complexity	Math grade
Mean	73.8	68.8	77.5	75.0	80.0	77.5	82.5	76.4

Tables 2A and 2B reveal the following features:

1. The mean grades of the three projects are similar in design (74.7-78.9) and in mathematics (73.1-78.9). The project 1 grades were slightly lower than of projects 2 and 3. A possible reason is that in the project the students dealt with tessellations design for the first time.

2. Mean grades for the use of parameters (based on the assessment of parametric solutions for Project 1, surface parameters for Project 2, and use of parameters for Project 3) in the three projects were lower than for other mathematics criteria. These difficulties originated from the students' mathematical background.
3. Close correlation between the individual design and mathematics grades was found in project 1 ($\rho = 0.665$) and in project 2 ($\rho = 0.698$). This result indicates the tight integration of design and mathematical aspects of these course projects. In project 3 the correlation between the grades was lower ($\rho = 0.398$). A possible explanation is that project 3 does not include a design component, but focuses on analyzing existing structures.

Project Examples

Students' project portfolios included concept explanations, module drawings, module allocation plans, physical models, and descriptions of design and mathematical solutions. Here we present examples taken from the three projects performed in the course. The examples are a tessellation solution drawing (Figure 2A), a gas station roof model (Figure 2B), and an existing building model (Figure 2C).

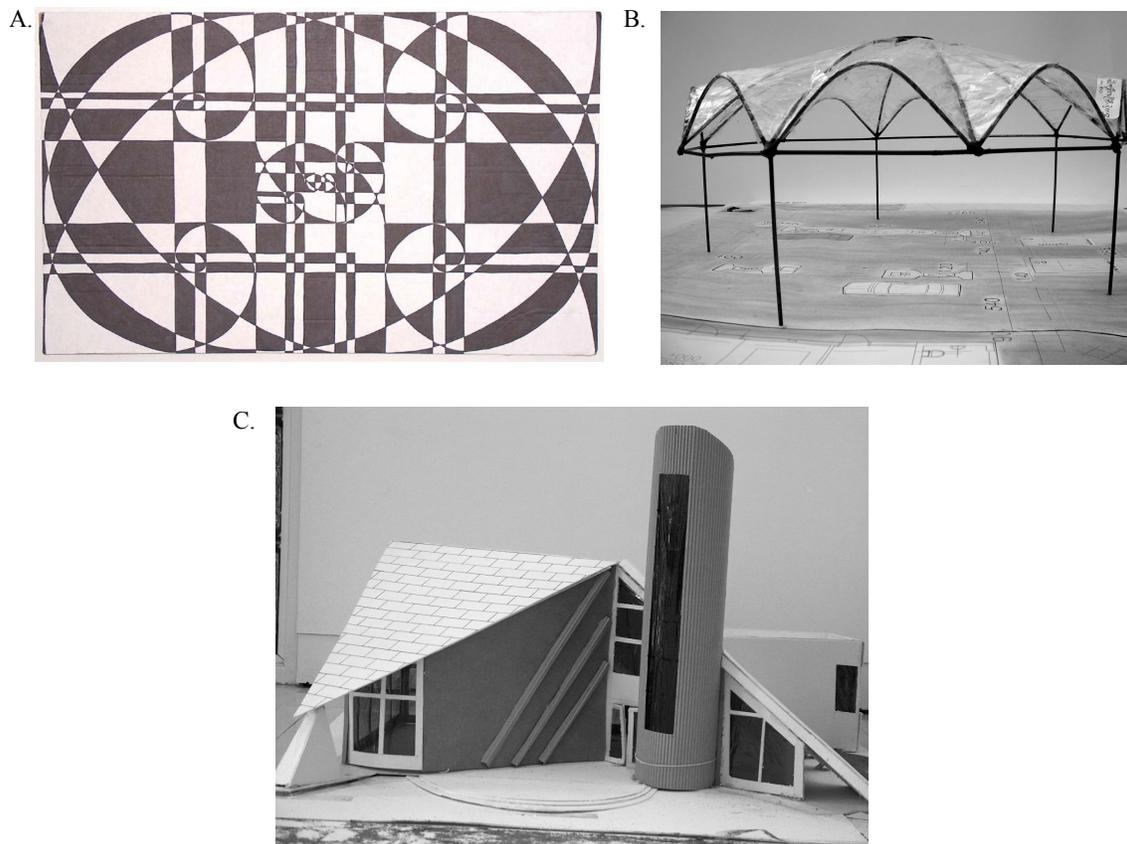


Figure 2. Examples of the three course projects: A. Tessellation design; B. Gas station roof design; C. Solids intersection in an existing structure

The tessellation drawing example presented in Figure 2A shows a solution developed by one of the students based on logarithmic spirals. The student combined drawing spirals with the golden section procedure and from her own curiosity defined their analytical parameters by measurements and calculations.

In the gas station roof example (Figure 2B) the student applied knowledge of algebraic surfaces and selected a solution based on the Sarger segment, which was suitable to cover the pumps area. This solution was precisely calculated and implemented in the physical model. When building the model, the student acquired experience of dealing with efficiency and construction stability factors.

When selecting a public building (a museum in the north of Israel) the student recognized the solids intersection part of the structure (cylinder-pyramid). She measured building's dimensions and extracted geometrical data from the architecture design plans. After mathematical analysis of these data the student constructed the precise model of the building (Figure 2C).

Attitude Questionnaire

The post-course questionnaire included four open questions. The examination of students' responses is based on categorizing the answers through context analysis and calculating their frequencies.

The first question evaluated students' opinions about the importance of the three course subjects (design of tessellations, curve surfaces, and solids' intersections) for their architectural studies. Answers to this question indicated that the majority of the students (93%) noted the high importance of the three course subjects. The students emphasized the course contribution to deeper thinking on the tessellation subject, understanding mathematical concepts applied in architecture, designing geometrical forms and connecting mathematics and architecture.

The second question asked every student to list mathematics concepts that he/she learned in the course. The answers are given in Table 3.

Table 3. Mathematical concepts learned in the course

Mathematics concepts	Frequency (%)
Proportions, sequences, logarithmic spirals, polygons, symmetry, harmonic division, algebraic surfaces and line intersections (polyhedral, cylindrical, spherical, elliptic, and conic)	90-100
Cartesian and polar coordinates, circles and arcs, exponential and logarithmic functions	70-89
Similarity of triangles, irrational numbers, geometrical dimensions	50-69
Fractals, trigonometric functions, derivatives	30-49
Parabolas, limits, radians, tangents, equations and inequalities, differentiability, vectors and matrices	Less than 30

Table 3 shows that the students in the course were exposed to a variety of mathematics concepts learned in class or on a need-to-know basis.

The third question related to the impact of the course on students' attitudes toward mathematics. In response to this question, the majority of the students (72%) noted that the course changed their attitude towards mathematics. Almost all these students (68%) expressed that the course roused their interest in mathematics and demonstrated its relevance to architecture. Some of the students recognized the challenge and even looked for new applications of mathematics in architecture by their own.

The fourth question asked students to evaluate instruction in the course. As found, 64% of the students think that the studio based instruction increased their motivation to learn mathematics, for 84% it stirred their interest, curiosity and was a challenge. The studio method enhanced students' creativity (60%) and opened a skylight to mathematics (68%).

Conclusions

Our study shows the positive change of students' ability to apply mathematics to design of architectural structures as a result of integrating mathematics and architecture design curricula. To implement this integration, we developed, implemented and evaluated the second year Mathematical Aspects in Architectural Design (MAAD) course. The course is based on the Mathematics as a Service Subject approach. It offers mathematical learning as part of hands-on practice in architecture design studio. The course deals with three directions of complexity in geometrical objects for architectural design: (1) arranging regular shapes to cover the plain (tessellations); (2) bending bars and flat plates to form curved lines and surfaces (deformations); (3) integrating and subdividing space by solids (constructions).

In the course follow-up study we used qualitative (ethnographic) methods, which observed learning behavior within the context of the design studio using observations and interviews, attitude questionnaire, and project portfolios.

Our observations showed that the students expressed curiosity and motivation to the project experience, and interest to deepen in mathematical subjects and their use. Assessment of students' activities in the projects indicated that the majority of them refreshed and practically applied their background mathematical knowledge. The correlation between design and mathematics grades showed the tight integration of the two subjects in the projects.

Analysis of the attitude questionnaires revealed the students' high positive evaluation of the course. The majority of the students noted that the course roused their interest in mathematics and demonstrated its relevance to architecture. The studio method encouraged students' creativity. Some of the students recognized the challenge and even looked for new applications of mathematics in architecture on their own.

References

1. Lewis, R. K., 1998, *Architect? A Candid Guide to the Profession* (Massachusetts: MIT Press).
2. Gilbert, H., 1999, Architect – engineer relationships: overlappings and interactions, *Architectural Science Review*, 42, 107-110.
3. Unwin, S., 1997, *Analyzing Architecture* (London: Routledge).

4. Burt, M. (1996). *The Periodic Table of The Polyhedral Universe*. Haifa: Technion – Israel Institute of Technology.
5. Lühr, S., 1999, "Math, logic, and symmetry: construction, architecture and mathematics". Available online at: <http://www.sckans.edu/math/paper3.html> (accessed 19 December 2005).
6. Williams, K., 1998, "Relationships between Architecture and Mathematics". Available online at: <http://www.leonet.it/culture/nexus/98/KimWilliamsintro.html> (accessed 19 December 2005).
7. Gravemeijer, K. and Doormen, M., 1999, "Context Problems in Realistic Mathematics Education: a Calculus Course as an Example", *Educational Studies in Mathematics*, 39, 112-129.
8. Howson, A., Kahane, J., Lauginie, P. and Turckheim, E., 1988, In: A. Howson et al (Eds.) *Mathematics as A Service Subject*, Cambridge: Cambridge University Press.
9. Verner, I. and Maor, S. 2003, "The Effect of Integrating Design Problems on Learning Mathematics in an Architecture College", *Nexus Network Journal*, vol. 5 no 2, <http://www.nexusjournal.com/Didactics-VerMao.html>.
10. Schoen, D. A., 1988, "The Architectural Studio as an Exemplar of Education for Reflection-in-Action", *Journal of Architectural Education*, 38(1), 2-9.
11. Salinger, N., 2000, "Complexity and Urban Coherence", *Journal of Urban Design*, 5, 291-316.
12. Consiglieri L., and Consiglieri V., 2003, "A proposed two-semester program for mathematics in the architecture curriculum", *Nexus Network Journal*, 5(1), http://www.nexusjournal.com/Didactics_v5n1-Consiglieri.html.
13. Boles, M., and Newman, R., 1990, "Universal Patterns. The Golden Relationship: Art, Math & Nature". Massachusetts: Pythagorean Press.
14. Frederickson, G., 1997, "Dissections: Plane and Fancy". Cambridge University Press.
15. Ranucci, E., 1974, Master of tessellations: M.C. Escher, 1898-1972, *Mathematics Teacher*, 4, 299-306.
16. Hanaor, A., 1998, "Principles of Structures", Oxford, Blackwell Science.
17. Alsina, K. and Gomes-Serrano J., 2002, Gaudian Geometry. In: D. Giralt-Miracle (ed.), Daudi. Exploring Form: Space, Geometry, Structure and Construction. Barcelona, Lunweg Editores, 26-45.
18. Alsina, K., 2002, Conoids. In: D. Giralt-Miracle (ed.), Daudi. Exploring Form: Space, Geometry, Structure and Construction. Barcelona, Lunweg Editores. 88-95.
19. Alsina, K., 2002, Geometrical Assemblies. In: D. Giralt-Miracle (ed.), Daudi. Exploring Form: Space, Geometry, Structure and Construction. Barcelona, Lunweg Editores. 118-125.