AC 2011-217: UNCERTAINTY ABOUT UNCERTAINTY: WHAT CONSTITUTES "KNOWLEDGE OF PROBABILITY AND STATISTICS APPROPRIATE TO THE PROGRAM NAME AND OBJECTIVES" IN OUR PROGRAM ACCREDITATION CRITERIA

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Uncertainty about Uncertainty: what constitutes “knowledge of probability and statistics appropriate to the program name and objectives” in our program accreditation criteria

Abstract

EAC of ABET program accreditation criteria for Electrical, Computer, and similarly named engineering programs include the requirement that the program must demonstrate that graduates have knowledge of probability and statistics, including applications appropriate to the program name and objectives. In this paper, we examine what knowledge this might be and why this is a non-trivial question to answer. Probability and statistics appear in many places in the field of electrical engineering and the tools appropriate in one application can be quite misleading in another. Without a deep understanding of the underlying principles, students can easily become confused by the great variety of statistical techniques available to them and may form the impression that statistics is a bag of tricks with no underlying principles to provide guidance. Since professional statisticians still disagree about some of the most fundamental concepts in their field we must ask the question of how we can be sure that we are meeting this requirement for accreditation. Our goals are to identify patterns of topics in which probability and statistics are applied in electrical engineering and to note how the applications (and in some cases even the fundamental concepts involved) differ between these different topics.

Introduction

An important first step in the engineering design process is understanding the problem to be solved. Applying this process to the design of the engineering curriculum, we should start with an understanding of the tasks to be performed by our graduates and the skills they will need to perform them admirably. This paper attempts to accomplish this task for the specific area of probability and statistics, but we must also state what this paper does not attempt to do. It does not report results from surveying employers or recent graduates and it does not report results of a controlled experiment on alternative pedagogies. This paper does not provide a complete list of statistical topics that should be included in the curriculum and it certainly does not prescribe a specific approach to teaching probability and statistics. This paper is not intended as the final word on the topic but rather the beginning of a conversation.

Instead, we take the approach of inventorying the activities of an electrical engineer that involve the use of probability and statistics, and attempting to identify underlying commonalities in the skills needed to accomplish these tasks. As we follow a product from research through development into production and support, we find a number of applications of both probability theory and statistical techniques. We then examine these applications to determine what knowledge and skills an engineer must possess to accomplish them. Finally, we survey this list to suggest which concepts and techniques should be learned at the undergraduate level.

Data Analysis and Experimental Design

The first place we encounter statistical techniques is in the analysis of data. This occurs both in the laboratory of the R&D department and in the signal processing of any product that includes a sensor. We find the use of arithmetic means and experimental standard deviations in the reporting of measurements and their uncertainties. We also find probability distributions used to calculate confidence intervals for measurements. We may even find the use of least squared error curve fitting to estimate parameters and the use of r^2 and χ^2 measures of how well the resulting curves fit the experimental data[1]. Clearly, these are all techniques our graduates should know how to use, but how well should they know them?

While these data analysis techniques are ubiquitous, we also find more advanced tools used in data collection and analysis. Data analysis can also include techniques for clustering data and recognizing patterns. Most of these techniques have a probabilistic basis, but the added complexity comes from the use of additional mathematical tools, such as linear algebra, not additional concepts from probability or statistics. Experiments may be designed to detect correlations between variables or may be affected by undesired correlations. Statistical techniques can be used to design experiments that minimize uncertainty in parameter estimation and provide maximal data at minimum expense[2].

An Analysis of Data Analysis

Although statistical methods have been used to analyze scientific data since Gauss used them for the analysis of astronomical data in 1801, these techniques are often used without considering their validity or appropriateness. Cox argued in 1946 that if we wish to represent our degree of belief as a real number and remain consistent with logic, we are led inexorably to use probability theory[3]. However, alternatives to probability theory have been proposed and used for some applications[4,5] and the interpretation of probability theory as a measure of rational belief (often called the Bayesian viewpoint) is not the only interpretation of probability. More commonly, undergraduate texts view probability as a description of the relative frequency of different possible outcomes (the frequentist view) or in some cases as a purely theoretical structure derived from the Kolmogorov Axioms (the axiomatic view)[6]. While engineers may not need to know about these differences in the interpretation of probability theory, we as educators can benefit from the realization that the existence of these differing interpretations may help to explain some of the difficulties our students encounter in applying probability theory to engineering applications.

A parallel difficulty arises in the choice of statistics used to summarize data. Engineers frequently calculate the arithmetic mean of several measurements to reduce the uncertainty of their result and use the standard deviation of the measurements to provide an estimate of their uncertainty. Although they might be aware of other measures, such as medians and average absolute deviations, they are generally not aware of why means and standard deviations are used or under what conditions they might not be the best choice[7]. It is important that engineers know the limitations of their tools and their assumptions and this should apply to statistical tools as well. Although they should know that the normal distribution is commonly encountered, they should also know why and also that not all distributions are normal. They should know the different roles played by r^2 and χ^2 in evaluating a curve fit to experimental data and they should know how to correctly calculate confidence intervals for reported results.

Modeling and Simulation of systems and Signals

While data analysis is of the utmost importance in characterizing systems and validating one's understanding, a preliminary understanding must be developed before it can be validated by experiment. Models can be indispensable in developing such an understanding and once the model is validated, these same models can be used to leverage the results of experiment. While many of the models used in engineering are deterministic, we often find the need for stochastic models. For example, seek time for a disk drive will be dependent on the sequence of access requests, but this sequence cannot be perfectly known in advance. We characterize the distribution of requests and find the expected value of the seek time. This type of statistical model is used throughout electrical and computer engineering in such diverse applications as modeling thermal[8], shot and quantization noise; modeling reliability of complex systems[9]; or modeling of imperfect feedback in control systems[10].

To use statistical models in this way, the engineers must be able to choose a probability distribution that accurately represents their ignorance. This in turn requires either an understanding of the maximum entropy principle[11] or more simply, a familiarity with the probability distributions that arise in certain common situations**.** Unfortunately, in many probability and statistics classes students learn the mean, variance, and possibly the moment generating functions for common distributions, but never learn the applications in which they commonly appear, the reasons for those appearances, and the assumptions underlying them.

While some of these stochastic models can be quite simple, having a single random variable affecting the system in a linear way, complexity often appears and can appear in many forms. The engineer may need to account for how the value of a variable at one point in time affects its value at a later time. In these cases, an engineer should be able to model the autocorrelation and power spectrum of the signal. If random variables affect the system in non-linear ways, or if multiple random variables are present, developing a probability distribution for the output can be non-trivial.

In many cases, the model cannot provide analytical solutions and a simulation is used to produce useful results. In these cases, pseudo-random number generators sample the probability distributions of our random inputs and the probability distribution of the output is built up sample by sample. Thus in simulation, we take advantage of the fact that probability theory can be interpreted in terms of both ignorance and relative frequency. However, the use of pseudo-random number generators is not without peril and engineers using Monte-Carlo methods should be aware of the limitations of the pseudo-random number generators used in their simulations[12].

In developing communications, data storage, and encryption systems, engineers also need a model to quantify information. Shannon's work on entropy[13] ushered in a wealth of results in information theory that allow engineers to relate signal to noise ratios to the information capacity of communication channels and data storage systems. Engineers working in these fields must also be able to calculate expected bit error rates and to determine the redundancy needed to obtain acceptable failure rates.

Manufacturing Applications

In addition to the applications described above, engineers supporting manufacturing facilities find additional uses for statistics. Process capability studies are sometimes needed to determine if a manufacturing process can fabricate products that meet specifications with an acceptable yield. Once a manufacturing process is running, it is frequently monitored to detect problems before unacceptable merchandise is produced. This requires a knowledge of expected range of variability and a way of detecting when that range is exceeded or the variability of the process changes.

Similarly, both end products and components purchased from vendors might be inspected to verify that they meet specifications. Given the variability of the measurement process this requires the use of statistics to develop inspection tolerances that provide assurance of product quality. Selecting these tolerances requires more than just statistics. It is an economic decision based on the relative costs of accepting defective products and rejecting acceptable ones. Hence it is an application of decision theory and needs to include a measure of risk such as a loss function and not just probability[14]. It should be noted that the other cases where we needed to select an estimator could be examined from this perspective also. For example, one reason for choosing the mean as an estimator is that it minimizes a loss function equal to the mean squared error. If instead we were to choose average absolute error as our loss function, we would find that the median is a preferred estimator.

Generalizations

Examining this list of engineering applications of probability and statistics, we can identify a set of skills in probability and statistics that must be mastered in order to accomplish them.

1. Calculate sample means and experimental standard deviations for data sets
2. Find confidence intervals for measured values, selecting an appropriate distribution for the measurement process.
3. Use least squared error fitting techniques to find best fit parameter values for a given data set and model and the uncertainty of the fitted parameter values, the quality of the model fit, and the uncertainty in predicted values for the model.
4. Calculate the covariance matrix for a data set of multiple variables and the correlation between pairs of variables and use this information to find principle components for classification.
5. Choose an appropriate probability distribution family to describe an unknown quantity, including appropriate use of the assumption of normality and estimation of parameter values.
6. Choose an appropriate autocorrelation or power spectrum for a stochastic process.
7. Evaluate the reliability of a simulation including the number of samples and the randomness of the pseudo-random number generator used.

8. Calculate the entropy of a signal and the mutual information between two signals. And find the information capacity of a communications channel or storage device.
9. Find an acceptance threshold that minimizes a specified loss function.
10. Design experiments that minimize uncertainty in parameter estimation and that allow efficient determination of the strength of influencing factors.

Note that some applications such as statistical process control are not explicitly included in this list. This is not because they are not important, but because they are specific examples of these more general skills. For example, developing the limits for an $\bar{x}$, σ chart one simply finds the appropriate confidence intervals for the variables. Rather than requiring additional statistical skills, this requires the engineer to be cognizant of the need for statistics in this application.

Given this list of skills that electrical engineers should be expected to possess, an engineering faculty might decide which topics must be covered at the undergraduate level to ensure that graduates are capable of completing the more basic tasks and learning the more complex ones. A typical list of undergraduate topics might be:

1. Frequentist and Bayesian interpretations of probability
2. Probability distributions for continuous and discrete quantities and joint distributions
3. Calculating moments of a distribution and expected values of functions of distributions
4. Probability distributions that arise in commonly encountered engineering applications
5. Maximum entropy distributions for common forms of ignorance
6. Correlation between random variables
7. Autocorrelation and power spectrum of a stochastic process.
8. Finding the distribution for a function of one or more variables either analytically or computationally.
9. Estimating distribution parameters from experimental data and criteria for choosing estimators.
10. Loss functions and why a quadratic loss function is commonly used.
11. The use of mean, median and mode as estimators and the loss functions they minimize
12. The use of standard deviation, range, and average absolute deviation as estimates of variability.
13. Least squared error fitting techniques
14. R squared and chi squared measures of quality of fit and uncertainty of parameters.
15. Calculating confidence intervals to quantify the uncertainty of a result.
16. Calculating p-values of an appropriate distribution to estimate the significance of a result.
17. The concept of entropy and its role in information theory.

Other lists could also be generated based on a department's objectives and the degree to which they expect their graduates to understand how their statistical tools work.

Conclusions

While different schools may have different objectives, the method presented here should provide a template for developing an answer to the question of what statistical knowledge is appropriate to those objectives. By starting with the range of statistical applications their graduates will encounter in practice, engineering schools can work backwards to determine the skills needed at graduation and the topics covered in various classes. This is very different from simply requiring a statistics class and hoping that the right topics are covered. Faculty should be able to trace the statistical topics covered in their curriculum back to skills needed by their graduates and program evaluators should look for such linkages. Of course, graduates will vary in their statistical abilities and this complicates the problem of assessment. Since some of our graduates eventually join the faculties of engineering schools and take part in program development, our list of engineering applications of statistics is incomplete. Assessment of statistical skills is itself a statistical skill.

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