

## **Undergraduate Engineering Students' Representational Competence of Circuits Analysis and Optimization: An Exploratory Study**

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# Undergraduate Engineering Students' Representational Competence of Circuits Analysis and Optimization: An Exploratory Study (Evidence-based Practice)

## Background and Motivation

There is a long-standing interest and focus in educational research on electricity-related concepts, due to two essential reasons: (a) electricity is one of the central areas of science, technology, and engineering curricula at all levels of education, and (b) its concepts are particularly difficult to teach and learn because they are abstract and complex<sup>1</sup>. Therefore, both educators and students face several challenges throughout the learning process<sup>2</sup>. Students often develop their own conceptions of electricity, which may be in conflict with the formal science perspectives<sup>3</sup>. When these students' interpretations of scientific concepts are inconsistent with canonical interpretations, this phenomenon is variously referred to as misconceptions, naïve conceptions, or alternative conceptions<sup>4</sup>.

A correct understanding of formal concepts is a key element in developing the competences and skills needed for science, technology, and engineering students and professionals to problem solve or design. Engineers particularly rely on conceptual knowledge to make intuitive and educated inferences about the behavior of a system under specific circumstances without use of prototypes or complicated models, often due to time constraints<sup>5</sup>. Specifically in engineering education, concepts associated with electric circuits are particularly important because they are the foundation for other advanced topics and skills, such as the design of devices, circuits, and systems<sup>6</sup>. However, researchers such as McDermott and Shaffer<sup>4</sup> have identified that students are unable to apply formal concepts to an electric circuit. As a consequence, students may hold misconceptions about electric current, voltage, and resistance in forms of: (a) the implementation of formal concepts into electric circuits, (b) the relationship between representations and the formal concepts, and (c) the application of qualitative reasoning in electric circuits<sup>4</sup>.

One particular characteristic of misconceptions in science and engineering is that even after long periods of instruction, students may not demonstrate a significant improvement in their learning performance<sup>7</sup>. A main concern for educational researchers and educators has been finding ways to improve current learning techniques to consequently improve students' conceptual understanding. Examples of such strategies consist of including computational and laboratory activities, and components of cyberlearning tools used along with traditional materials.

This study explores the use of multiple student-generated and computer-generated representations as a feasible mechanism to improve conceptual understanding of electric circuits<sup>8-10</sup>. Thus, the guiding research questions are: *How effectively do students use multiple representations of electric circuits? And what is the relationship between students' conceptual understanding of circuits and their performance on a representational task?*

## Theoretical Framework

Model-based reasoning (MBR) is the theoretical framework that guided the design of the learning activity and its evaluation. MBR is one form of cognition that investigates how scientific representations are created from existing representations<sup>11</sup>. It has been documented that processes of reasoning taking place during MBR episodes can lead to concept formation, conceptual understanding or conceptual change<sup>11,12</sup>.

Models and modeling perspective (MMP) is one form of MBR that focuses on the process of creating representations used to express the studied concepts<sup>13</sup>. Models are conceptual systems expressed using

external notation systems and are used to construct, describe or explain other systems<sup>13</sup>. The ability to express, use and think about models in each medium would be representational competence<sup>14</sup>. Modeling in turn is the process of developing representational descriptions or models for specific purposes in specific situations<sup>15</sup>. This process depends on representational fluency, which is the ability to translate between and within different representations<sup>16</sup>. Representational fluency has been identified as a mechanism to build, describe and measure student conceptual understanding<sup>16-18</sup>. Other authors refer to this ability as representational transformation<sup>17</sup>, or representational literacy<sup>18</sup> as a broader category that contains representational competence as well.

In practice, MBR often consists of constructing artifacts and external representations allowing learners to make their understanding explicit by creating models, also called conceptual tools<sup>13</sup>. Such conceptual tools may include explicit descriptive or explanatory systems functioning as models designed specifically to reveal aspects about how students interpret specific problem-solving situations. MMP employs model-eliciting activities (MEAs) as thought revealing mechanisms,<sup>19</sup> in which students generate solutions that demonstrate their representational competence and fluency<sup>16</sup>. MEAs are problem solving activities that simulate real-world situations in which students develop, construct, describe or explain different representations<sup>16,19,20</sup>. The implications for the use of MBR as the theoretical framework for this study relate to using MMP as: (a) a guideline for the development of the learning design focused emphasizing representational competence<sup>16</sup> with MEAs, and (b) a mechanism to identify and assess student representational competence, via their produced representations<sup>18</sup> in the MEA.

## Methods

The participants of this study were 25 predominantly sophomore engineering students enrolled in a linear circuit analysis course offered to electrical engineering students at Purdue University. Students were asked to complete a homework assignment guided by principles of MEA design (see Appendix A). The assignment was adapted from a textbook problem from DeCarlo and Lin<sup>21</sup>. It consisted of having students to first analyze an electric circuit, and then optimize it in order to be able to recharge a car's battery. Students were prompted to create and use multiple representations such as (1) diagrams and equations to identify the mathematical model of the problem, (2) simulations to represent the circuit behavior and its optimization and (3) computational tool (i.e., MATLAB) to program the model and evaluate the circuit. At the end, for each circuit (i.e., base and optimized) students were asked to explain their solution. This last question measured student conceptual understanding. In this way, students were asked to generate three different representational forms (i.e., graphical representation or diagram, mathematical representation, computational representation) and answer predicting questions to demonstrate their conceptual understanding.

Different forms of representations were analyzed qualitatively. Based on the categorization of each of the different representations a rubric was developed (see Appendix B). Students' representations were then assessed with the rubric, and scores were analyzed via descriptive statistics and a correlational analysis between representations. Rubric scoring ranged from one to four where scores below 1.5 were considered as low achievement, between 1.6 to 3.5 as moderate achievement, and over 3.6 as high achievement. An additional grading criteria of "no response" was also included.

## Results and Discussion

### *How effectively do students use multiple representations of electric circuits?*

Students were prompted to develop a total of five different representations as part of the assignment; two for the base circuit and three for the optimized circuit. Students, on average, developed 3 representations (mean=3.4, SD=1.2) and the overall score for those was rated as moderate (mean=2.9, SD=0.9). Overall

performance of students' conceptual interpretation of the circuit was identified as moderate (mean=2.0, SD=1.6).

Table 1. Descriptive statistics of students' representations for the base and optimized circuits.

Task	Diagram			Mathematical Representation			Computational Representation			Conceptual Understanding		
	Mean	SD	n	Mean	SD	n	Mean	SD	n	Mean	SD	n
<b>Base Circuit</b>	n/a	n/a	n/a	3.53	1.07	25	1.59	1.58	13	3.29	0.98	17
<b>Optimized Circuit</b>	3.08	1.04	25	1.16	1.67	9	1.08	1.47	13	1.80	1.71	15

The overall results, presented in Table 1, indicate that the students demonstrated a good understanding of the mathematical representation of the base circuit as well as a basic explanation of its behavior. However, for the case of the optimized circuit, students were able to represent the circuit at the beginning of the optimization process at a basic level, but failed to successfully represent its mathematical model as well as the computational model. For the optimized circuit, students were not able to provide an acceptable conceptual explanation of the optimized circuit.

A second analysis was performed with observations from those students who developed the two representations for the base circuit, and the three representations for the optimized circuit. To be included in this analysis, it was also required that students responded both of the conceptual questions. Table 2 depicts the descriptive statistics for the students who completed the analysis for the first circuit ( $n = 12$ ) and the optimized configuration for the second circuit ( $n = 5$ ). These five students also completed the analysis for the base circuit and therefore their results were analyzed separately.

Table 2. Descriptive statistics of students' representations for the base and optimized circuits for a subsample of students who completed all required representations.

Task	Diagram			Mathematical Representation			Computational Representation			Conceptual Understanding		
	Mean	SD	n	Mean	SD	n	Mean	SD	n	Mean	SD	n
<b>Base Circuit</b>	n/a	n/a	n/a	3.75	0.87	12	2.33	1.50	12	3.50	0.90	12
<b>Optimized Circuit</b>	3.08	0.45	5	3.4	0.89	5	3.4	1.41	5	3.6	0.89	5
<b>Base Circuit*</b>	n/a	n/a	n/a	4	0	5	3	1.41	5	4	0	5

\* Descriptive statistics for the base circuit but for only those students who completed the three representations for the optimized.

This analysis suggest that students who completed the five representations achieved moderate to high success in constructing the representations for meaningful use in their conceptual understanding.

*What is the relationship between students' conceptual understanding of circuits and their performance on a representational task?*

Our overall results suggest a positive correlation between number of representations students developed, and their overall conceptual understanding ( $r = 0.53, p = .006$ ). In addition, the correlation between the quality of the representations students produced and their achievement in the conceptual questions was also analyzed. Table 3 depicts the correlations between students' representations and their conceptual understanding of the base circuit. Table 4 depicts the correlations between students' representations and

their conceptual understanding of the optimized circuit. Because not all students chose to construct all the representations, this analysis is limited to those students who developed the two representations for the base circuit, and the three representations for the optimized circuit.

Table 3. Correlation between representations and conceptual understanding of the base circuit ( $n = 12$ ).

<b>Base circuit</b>	<b>Computational representation</b>	<b>Conceptual understanding</b>
<b>Mathematical representation</b>	0.28	-0.17
<b>Computational representation</b>		0.53

This correlation analysis on Table 3 suggests that there is a positive correlation between the computational representation and the conceptual understanding. In contrast, the correlation analysis on Table 4 suggest strong relationships between students' representations, diagrams, mathematical and computational, and their conceptual understanding.

Table 4. Correlation between representations and understanding of the optimized circuit ( $n = 5$ ).

<b>Optimized circuit</b>	<b>Mathematical representation</b>	<b>Computational representation</b>	<b>Conceptual understanding</b>
<b>Diagram</b>	0.87	1	1
<b>Mathematical representation</b>		0.87	0.87
<b>Computational representation</b>			1

Finally, for those students who completed both sections of the conceptual questions ( $n = 14$ ), one for each circuit, it was identified a moderate positive correlation between the number of representations they developed and their conceptual understanding ( $r = 0.46$ ,  $p = 0.102$ ); and a strong correlation between the quality of their representations (as scored with the rubric) and their explanations in their conceptual understanding ( $r = 0.92$ ,  $p < .0001$ ).

Results from this exploratory study suggest that, when students developed accurate representations, students usually interpreted the behavior of the circuit accurately. This result suggests that students with a high level of representational competence may interpret formal concepts more deeply. It also indicates that the number and quality of students' representations are correlated with their conceptual understanding. This in turn may also indicate that developing multiple representations (representational fluency) could lead to deeper conceptual understanding. These findings may also suggest that students with deeper understanding of electric circuits are capable of producing more accurate representations. Further research is needed to identify the specific conditions where students benefit more from creating and translating between graphical representations.

The implications of this study relate to the evaluation of pedagogical approaches that can be integrated along with modeling and simulation practices at the undergraduate level. Findings of this exploratory study suggest that the use of MEAs<sup>13</sup> can serve as an appropriate pedagogical approach to not only support student conceptual understanding, but also to support student development of modeling and simulation skills. Specifically, this pedagogical approach can help students couple their disciplinary knowledge with the use of representational forms to build mathematical models and to solve engineering design problems. MEAs can engage students in iterative cycles of generation, mapping and revision of multiple representations that can develop their thinking through different stages of complexity in their

generated solutions<sup>13</sup>. Moreover, coupling MEAs with the use of computer simulations can further support student development of representational competence. Previous research has identified that the use of simulations for learning has resulted in students being more likely to generate more accurate representations after learning from simulations and corresponding learning materials<sup>22</sup>.

On the other hand, studies that have identified the effect of programming practices have suggested that exposing students to programming experiences may go beyond helping them develop other cognitive skills<sup>23</sup>. For example, when paired with systematic problem-solving instruction, students can acquire significant learning gains<sup>24</sup>. In our study, we exposed students to MATLAB programming with the intention to reinforce the understanding of the mathematical representation of the circuit behavior; however, further research is needed to identify specific benefits associated with exposing students to MATLAB programming experiences in addition to using computer simulations.

## Conclusion and Future work

The results of this exploratory study suggest that representational competence and fluency may have a positive impact on student conceptual understanding, and a multirepresentational approach may have a positive effect in supporting electric circuit education. Findings from this study are aligned from those to others who have identified the effect of the use of multiple representations in engineering education and its role in supporting conceptual understanding. Thus, the contribution of this study relates to the analysis of conceptual understanding in electric circuits education from a multirepresentational perspective and the integration of computational representations and modeling practices for problem solving.

One limitation of this study is the number of participants enrolled in the study. Regardless of the limited sample size, the results of this exploratory study provide insights into evidence-based practice for improving conceptual understanding of electric circuits by fostering student representational competence and fluency. Additionally, this research contributes to the analysis of the integration of computational representations and modeling practices in engineering problem solving process.

## Acknowledgements

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# Recharging a Car's Battery

*Analysis and Optimization of an Electric Circuit*



**Figure 1 Hybrid car**

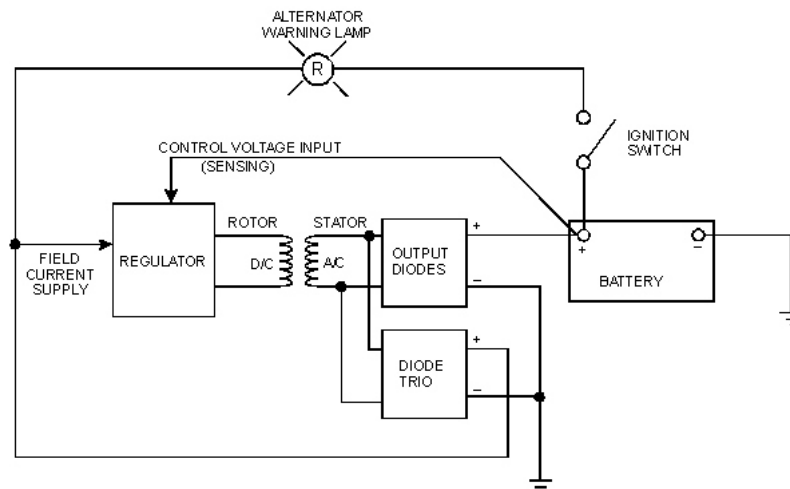
A car battery charger circuit consists of an alternator, a regulator and the battery itself. When the engine is running, the alternator generates a current to feed the vehicle electric charge. But before using it, the regulator drives the voltage to working levels for the lights, radio, windows and to charge the battery. If any of the components does not work properly, the battery will not be charged and you will ended up with a dead battery. In that case, you will have to use another way to turn on the car, such as a jump-start or push-start.

## Introduction

Electric circuits are very common in our diary life. Every appliance and electronic device have them, for instance a fridge, oven, cellphone and cars. Generally, all cars have a battery that turns the engine on and powers other systems like the lights. Cars also have a power generation circuit to recharge the battery, thereby avoid running out of battery or having a dead battery. Even though batteries can do more than turning on some vehicles, they can also power them through electric engines, making an electric or a hybrid car.

## Background:

A car battery charger circuit consists of an alternator, a regulator and the battery itself. When the engine is running, the alternator generates a current to feed the vehicle electric charge. But before using it, the regulator drives the voltage to working levels for the lights, radio, windows and to charge the battery. If any of the components does not work properly, the battery will not be charged and you will ended up with a dead battery. In that case, you will have to use another way to turn on the car, such as a jump-start or push-start.



**Figure 2: Electric circuit of a car**



## The Challenge:

The battery of your car suffered a sudden death by the sub-zero North wind and a faulty alternator. Unable to fight the elements, you wait a few days hoping for a thaw, which eventually comes. You replace the alternator. Then, using your roommate's car, you attempt a jump-start. Nothing happens. You let it sit for a while with your roommate's car running juice into your battery for 20 minutes. Still, nothing happens. Why won't your car start?



Figure 3: Jump-start

## Task 1:

Consider the circuit depicted in Figure 4. Notice that your “dead” battery is labeled as “V<sub>0</sub>”. Your roommate's battery is labeled 12V. Each battery has an internal resistance of 0.02 Ω. The starter, labeled “R<sub>Load</sub>,” has an internal resistance of 0.2 Ω. The starter motor requires 50A to crank the engine. Find the minimum value of voltage V<sub>0</sub> needed before the starter can draw 50A and work.

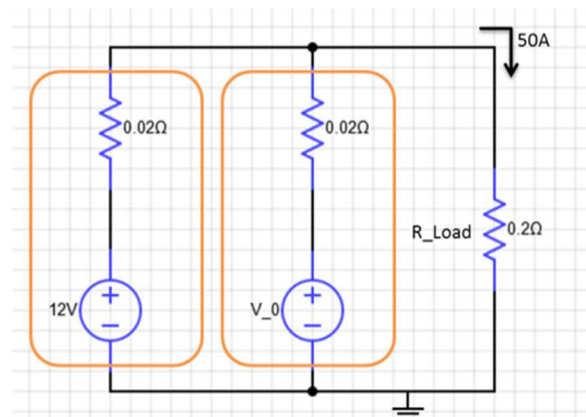


Figure 4: Jump-start circuit

## Task 2:

Optimize the circuit in Figure 4 so the minimum value of voltage V<sub>0</sub> needed is 0V (zero volts). You can find help by running a DC analysis in a simulation tool such as Circuit Sandbox that can be accessed at the link below. In your response, please provide the equations and its solution for this task and the circuit diagram found for the optimized configuration. Circuit Sandbox can be found following this link:

[https://6002x.mitx.mit.edu/courseware/6.002\\_Spring\\_2012/Overview/Circuit\\_Sandbox/](https://6002x.mitx.mit.edu/courseware/6.002_Spring_2012/Overview/Circuit_Sandbox/)

A user guide on how to use Circuit Sandbox can be found here:

<https://6002x.mitx.mit.edu/wiki/view/InteractiveLaboratoryUsage>

## Task 3:

Use MATLAB to solve the equations derived from Task 1 and substitute the constants to find the voltage  $V_0$ , then find an equation for the current  $I$  on the load ( $R_{Load}$ ) and graph  $I$  vs  $V$ , increasing the voltages from 0V to 12V. Repeat the steps with the equations of Task 2. In your response, please provide the MATLAB code and the plots.

HINT: This task could be accomplished by using the symbolic package of MATLAB or solving the system of equations in matrix form.

## Task 4:

Based on the MATLAB implementation and the plots, answer the following questions:

On the initial configuration (Task 1):

- How does the current change as the voltage of the dead battery increases from 0V to 12V?
- Based on the last question and the graph generated in Task 3 for the circuit of Task 1, does the current always achieve the goal of starting the engine for each value of voltage from 0V to 12V?

On the optimized configuration (Task 2):

- How does the current change as the voltage of the dead battery increases from 0V to 12V?
- Based on the last question and the graph generated in Task 3 for the circuit of Task 2, does the current always achieve the goal of starting the engine for each value of voltage from 0V to 12V?
- Please explain what the possible downsides to this optimized design are.

## Credits:

- MATLAB is a copyright of MathWorks.
- Main task: DeCarlo, R., and Lin, P. M. (2009). Linear Circuit Analysis: Time Domain, Phasor, and Laplace Transform Approaches (3rd ed.) (p.142, Problem 3.2). Dubuque, IA: Kendall Hunt Publishing.
- Figure 1: <http://auto.howstuffworks.com/hybrid-car-pictures.htm#page=8>, (09/14/14)
- Figure 2: <http://alternatorparts.com/understanding-alternators.html>, (09/14/14)
- Figure 3: <http://blog.cochran.com/wordpress/index.php/jump-start-car-battery/>, (09/14/14)
- Figure 4: Done with Circuit-Sandbox from MIT,
- [https://6002x.mitx.mit.edu/courseware/6.002\\_Spring\\_2012/Overview/Circuit\\_Sandbox/](https://6002x.mitx.mit.edu/courseware/6.002_Spring_2012/Overview/Circuit_Sandbox/), (09/14/14)

**Appendix B: Rubric derived from a qualitative analysis of students' responses**

<b>Scoring</b>	<b>Advanced (4)</b>	<b>Proficient (3)</b>	<b>Basic (2)</b>	<b>Below basic (1)</b>
<b>Diagram</b>	Correct diagram	Appropriate diagram not clear	Appropriate diagram but has some errors	No correct diagram
<b>Mathematical representation</b>	Good circuit analysis and good mathematical development	Good circuit analysis but with calculation error(s)	Good circuit analysis but with modeling error	No evidence of analysis or evident conceptual misunderstanding.
<b>Computational representation</b>	Good computational representation and good outcome	Good computational representation but the outcome has some errors (programming or mathematical)	Good computational representation but the outcome is incorrect	There is a computational representation but the outcome is incorrect due to a conceptual error
<b>Conceptual understanding</b>	Correct conceptual understanding (Three answers are correct)	Proficient conceptual understanding (2 answers are correct)	Basic conceptual understanding (1 answers are correct)	No correct conceptual understanding (Three answers are wrong)