

Understanding Loading in Feedback Amplifier Analysis

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Introduction

The application of negative feedback concepts to the analysis and design of electronic amplifiers is widely recognized as one of the most important subjects in electrical engineering curricula. Most electrical engineering students are exposed to feedback theory in courses primarily focused on systems and automatic controls. However, instructors teaching analog electronic circuits courses still devote considerable time explaining how to analyze feedback amplifiers because many practical aspects, such as the consideration loading effects, do not show up in other courses.

This paper presents some examples that illustrate some aspects of feedback amplifier analysis related to the correct use of input and output impedances. It is the author's experience that the relevant material normally found in undergraduate textbooks on microelectronics does not illustrate how to properly use these quantities. The objective of this paper is to provide educators with an additional tool to clarify these aspects of the subject.

Overview of the Feedback Method

An ideal feedback amplifier can be represented by the block diagram shown in figure 1. The quantities ω_s , ω_i , ω_o and ω_f represent the source, amplifier input, amplifier output and feedback signal, and can be voltages or currents. Amplifiers are classified according to the type of signal (voltage or current) being sampled at the output and mixed at the input. The four feedback configurations are shown in figure 2. The naming convention used in this article is summarized in table 1.

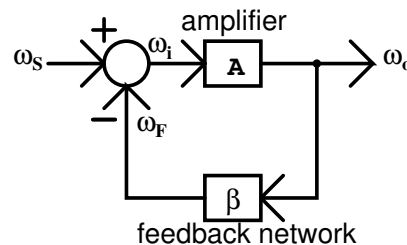


Figure 1: Ideal feedback amplifier, composed of a non-feedback amplifier and a feedback network.

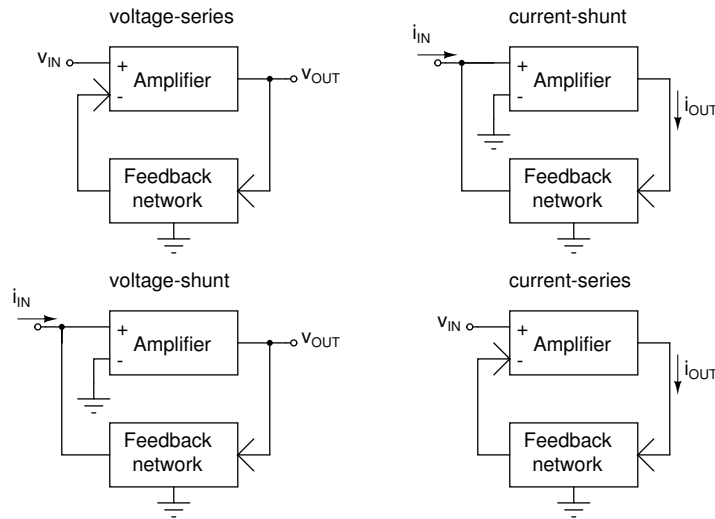


Figure 2: Feedback amplifier topologies.

Feedback Type	sampled signal	Feedback signal
voltage-series	voltage	voltage
current-shunt	current	current
voltage-shunt	voltage	current
current-series	current	voltage

Table 1: Feedback categories.

The feedback analysis method assumes that the blocks shown in figure 1 satisfy the following conditions:

1. the feedback network does not load the amplifier,
2. the amplifier does not load the feedback network , and
3. both amplifier and feedback network are uni-directional.

Provided that these conditions are satisfied, we can express the output in terms of the input

$$\begin{aligned} \omega_o &= A\omega_i = A(\omega_s - \beta\omega_o) \\ &= \frac{A}{1 + \beta A}\omega_s = A_f\omega_s \end{aligned}$$

or, equivalently,

$$A_f = \frac{\omega_o}{\omega_s} = \frac{A}{1 + \beta A} = \frac{A}{D} \tag{1}$$

where non-feedback amplifier and feedback network's gains are represented by A and β , respectively. The factor $D = 1 + A\beta$ is called the *desensitivity factor*.

Once the circuit components have been chosen, computer programs can be used to perform the analysis straightforwardly. They are, however, of limited use when the circuit must be designed. For all but the simplest cases only the feedback method provides a practical way of obtaining the insight needed for design. This method consists of associating parts of the circuit to A and β and then applying equation 1. Since A and β are *open-loop* quantities that involve no feedback, the use of this equation greatly simplifies the analysis of the circuit. This simplification is quite significant because it allows the hand analysis of circuits of considerable complexity and reveals the effect of different choices in a simple and intuitive manner.

The use of equation 1, however, requires the three previously listed assumptions to be satisfied. Usually real amplifiers fail to do this because normally the feedback network loads the amplifier. Also, the source is not ideal and is loaded by the amplifier, and external load is normally connected to the circuit. Thus in order to analyze the circuit using equation 1, the original amplifier is enhanced to include these loads and a new equivalent non-feedback amplifier is defined. Its gain is the one used in the basic feedback equation.

Nevertheless, the results obtained from feedback analysis are not exact because real circuits are only approximately uni-directional. However, in many cases the accuracy of the results is appropriate. Once the circuit is designed, practically exact results can be easily obtained from simulations.

Feedback Type	voltage-series	current-series	current-shunt	voltage-shunt
parameters	h	z	g	y
A_f	$\frac{A_V}{1+\beta A_V}$	$\frac{G_M}{1+\beta G_M}$	$\frac{A_I}{1+\beta A_I}$	$\frac{R_M}{1+\beta R_M}$
R_{if}	$R_i(1 + \beta A_V)$	$R_i(1 + \beta G_M)$	$\frac{R_i}{1+\beta A_I}$	$\frac{R_i}{1+\beta R_M}$
R_{of}	$\frac{R_o}{1+\beta A_V}$	$R_o(1 + \beta G_M)$	$R_o(1 + \beta A_I)$	$\frac{R_o}{1+\beta R_M}$
R_{11}	$\left. \frac{v_1}{i_1} \right _{v_2=0}$	$\left. \frac{v_1}{i_1} \right _{i_2=0}$	$\left. \frac{v_1}{i_1} \right _{i_2=0}$	$\left. \frac{v_1}{i_1} \right _{v_2=0}$
β	$\left. \frac{v_1}{v_2} \right _{i_1=0}$	$\left. \frac{v_1}{i_2} \right _{i_1=0}$	$\left. \frac{i_1}{i_2} \right _{v_1=0}$	$\left. \frac{i_1}{v_2} \right _{v_1=0}$
R_{22}	$\left. \frac{v_2}{i_2} \right _{i_1=0}$	$\left. \frac{v_2}{i_2} \right _{i_1=0}$	$\left. \frac{v_2}{i_2} \right _{v_1=0}$	$\left. \frac{v_2}{i_2} \right _{v_1=0}$

Table 2: Summary of feedback formulae.

To apply the feedback method, many authors^{1,2,3} use formulae similar to that shown in table 2. Because the feedback is a two-port network, hybrid parameters are used to determine the load it imposes on the amplifier.

The quantity A is identified with a different gain depending on the topology. The input and output are taken as a voltage or current depending on the variables subtracted and sampled by the feedback. Thus, if the feedback type is voltage-shunt, the trans-resistance $R_M = v_{out}/i_s$ would

take the place of the gain A . The feedback network's transfer function is then chosen as $\beta = i_f/v_{out}$, so that the loop gain $A\beta$ is unit-less, as expected.

Often, equivalent input and output resistances are calculated for the feedback amplifier. The relation between these and the corresponding non-feedback quantities is also shown in table 2 and depends on the topology. When the output voltage is sampled, the output resistance of the equivalent non-feedback amplifier is reduced by the desensitivity factor. If the output current is sampled the output resistance is increased. In the other hand, while subtracting a feedback voltage causes the input resistance to be increased by D , if the feedback signal is a current the input resistance is decreased.

A more detailed description of the application of feedback theory to amplifier analysis can be found in textbooks^{1,2,3,4} and articles^{5,6,7}. While the advanced feedback theory was developed by Bode⁸ in 1945, a book by Chen⁹ provides a recent exposition.

Effect of R_L and R_s

After the feedback method is applied, some textbooks propose moving the load R_L and the source's Thevenin resistance R_s "out" of the amplifier. After properly modifying the amplifier gain, the effect of changes in these quantities can then be easily estimated. There is, however, an aspect of this procedure that is easy to overlook and that can cause considerable confusion: only one, R_s or R_L , can be changed while the other must be kept at the value that was used when the feedback method was applied. It is easier to illustrate this point with the following example.

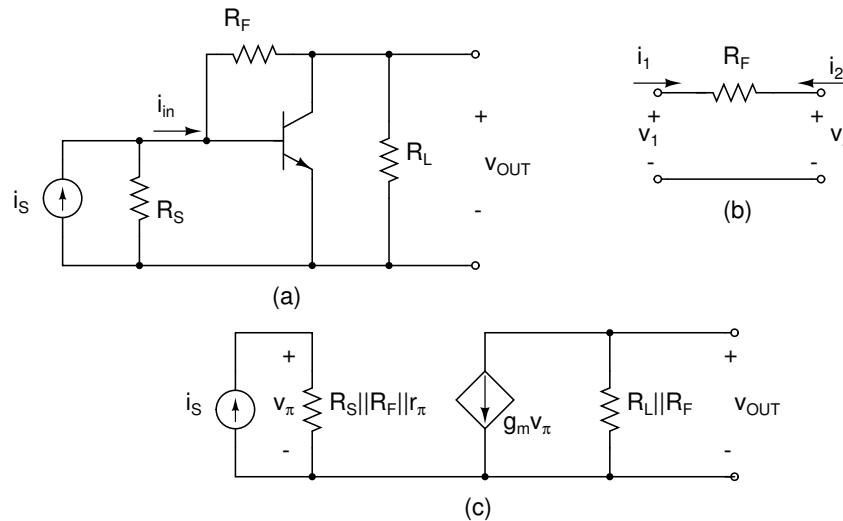


Figure 3: Voltage-sampling, current-mixing circuit for example 1. (a) Feedback amplifier; (b) Feedback network; (c) Non-feedback amplifier equivalent circuit.

A shunt-shunt feedback amplifier, in which the output voltage is sampled and a feedback current is subtracted at the input, is shown in figure 3. This circuit is interesting because its simplicity

allows an exact analysis to be carried out. The feedback circuit consists only of resistor R_F .

Application of the appropriate formulae from table 2 yields

$$R_{11} = R_{22} = R_F; \quad \beta = -\frac{1}{R_F}$$

The two resistances represent the loading imposed by the feedback network on the amplifier. The amplifier model is enhanced to include these loads to determine the non-feedback gain. This gives place to the circuit shown in figure 3(c), which can be readily analyzed to obtain

$$A = \frac{v_o}{i_S} = -g_m \times (R_L || R_F) \times (R_S || R_F || r_\pi) = -g_m R_o R_i$$

where $R_o = R_L || R_F$ and $R_i = R_S || R_F || r_\pi$ represent the non-feedback amplifier output and input resistances, respectively.

The corresponding feedback quantities are thus

$$A_f = \frac{v_o}{i_S} = -\frac{g_m R_o R_i}{1 + g_m \frac{R_o R_i}{R_F}}$$

$$R_{if} = \frac{R_i}{1 + g_m \frac{R_o R_i}{R_F}}$$

and

$$R_{of} = \frac{R_o}{1 + g_m \frac{R_o R_i}{R_F}}$$

Observe that the feedback gain can be expressed in two alternative forms:

$$A_f = -g_m R_o R_{if}$$

and

$$A_f = -g_m R_{of} R_i$$

To find the resistance seen by the independent current source and its input resistance, the latter is subtracted from R_{if} in the same way it was added i.e. in parallel;

$$R_{if}' = \frac{1}{\frac{1}{R_{if}} - \frac{1}{R_S}}$$

Notice that this quantity is of interest only if we want to explore the effect that different source resistances have on the gain. To do this, the gain must be expressed in terms of the current i_{in} shown in figure 3(a). We obtain the modified feedback gain

$$A'_f = \frac{v_o}{i_{in}} = A_f \frac{R_{if}'}{R_{if}} = -g_m R_o R_{if}'$$

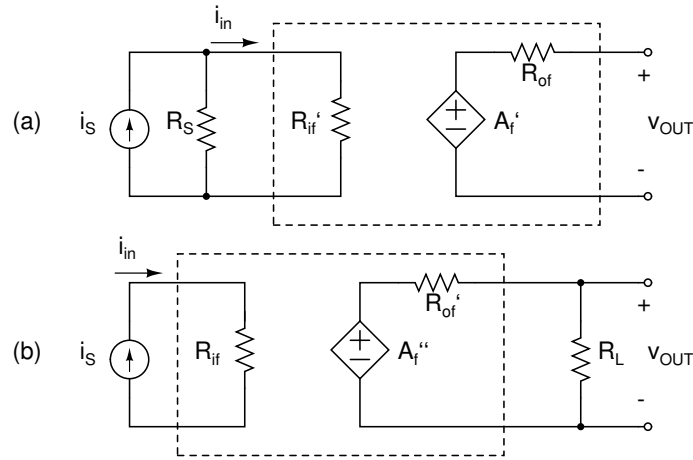


Figure 4: Removing R_S and R_L out of the feedback amplifier model.

This expression is the one we would have obtained by excluding the source resistor from the feedback analysis, and is independent of R_S . Thus, different values of R_S can be explored. What one has in mind is the situation modeled in figure 4(a).

In a similar fashion, we might want to explore the effect of different values of R_L . Following similar arguments we can “take out” the load resistance from R_{of} in parallel,

$$R_{of}' = \frac{1}{\frac{1}{R_{of}} - \frac{1}{R_L}}$$

and modify the expression for feedback gain to get

$$A_{f''} = A_f \frac{R_{of}'}{R_{of}} = -g_m R_i R_{of}'$$

which is independent of R_L . The model obtained is shown in figure 4(b).

The models shown in figure 4(a) and (b) allows the exploration of different values of R_S and R_L and produce the correct results. What might not be obvious is that, as the above procedure shows, this technique allows for **only one** of these parameters to be modified at a time. We can study the effect different values of R_S have on gain, but while doing so the load must be the one originally used in the feedback analysis! Alternatively, different loads can be studied but only for the original value of R_S .

Voltage or Current Output

The load resistance also plays an important role in the analysis of some amplifiers that can accept more than one “classification.” In these circuits the quantity being sampled by the feedback can be taken to be either a voltage or a current. Each selection leads to a different procedure. Since the

circuit is exactly the same irrespective of our choice, the results must be equivalent. Proper identification of the "load" is necessary in order to show this equivalency.

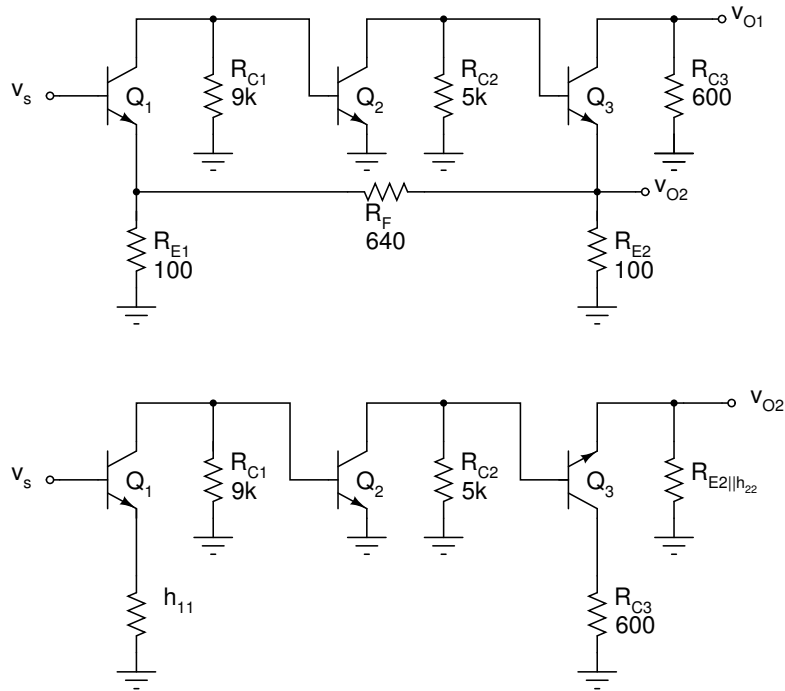


Figure 5: Circuit analyzed by Sedra and Smith¹.

An example of this is provided by the circuit shown in figure 5. This circuit is analyzed by Sedra and Smith¹, taking Q_3 's collector voltage as the output. Since the feedback network is attached to the emitter, the sampled signal is proportional to the output current. Thus these authors classified the circuit as a current-sampled, voltage-mixed feedback amplifier. However, we can also define the output voltage at Q_3 's emitter, so the standard procedure would lead us to conclude that the feedback circuit samples the output voltage rather than the output current.

Quantity	Sedra and Smith	this article
β	11.9Ω	$\frac{10}{74}V/V$
A	$20.7A/V$	$1820V/V$
R_{11}	88.1Ω	86.5Ω
R_{22}	88.1Ω	740Ω
A_f	$83.7mA/V$	$7.37V/V$
R_{of}	$35.6k\Omega$	0.138Ω

Table 3: Results of the analysis of the circuit shown in figure 5.

The circuit is the same no matter how we define the output. Therefore the results of these two

approaches must be consistent. Table 3 shows the results obtained by analyzing the circuit as a voltage-sampled, voltage-mixing amplifier, and compares them those obtained by Sedra and Smith.

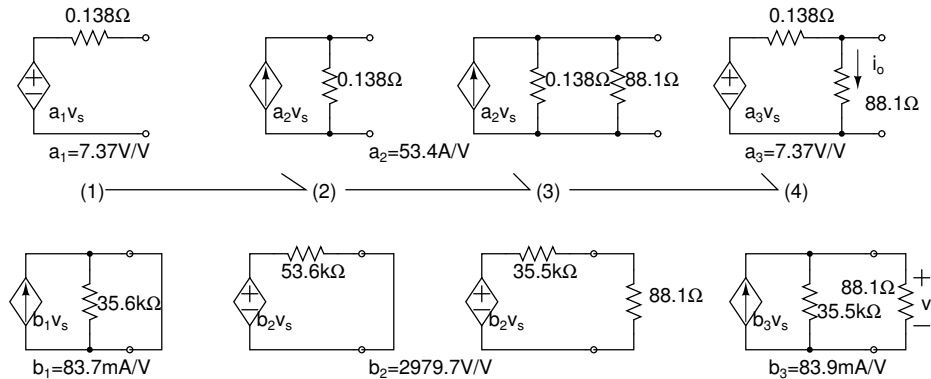


Figure 6: Transformation of results.

Figure 6 shows how one result can be converted into another by “moving out” the “load.” Because the feedback knows nothing about the “real” load connected at the collector, we must use $R_{E2} || (R_F + R_{E1})$ as the “load.” It is important to notice that when the current-series type is used, taking this “load” out of the feedback amplifier can not be handled as in example 1 because it modifies β . This can be illustrated by considering a change in R_{E2} to 200Ω . If the amplifier is treated as the voltage-series type, then we need only to replace the 88.1Ω resistor for one equal $200\Omega || (640\Omega + 100\Omega) = 157\Omega$ in the top of figure 6. Because this leaves the A and β (and therefore $A_f = v_o/v_s$) virtually unchanged, the trans-resistance becomes $i_o/v_s = 46.9mA/V$. However, if the circuit is treated as of the current-series type, this modification changes β to 21.28Ω , D to 441.5 , R_o to 212Ω , R_{of} to $93.6k\Omega$ and A_f to $46.9mA/V$, in agreement with our previous result.

This shows that care must be exercised to identify the “load” after the feedback method has been applied. An additional example of a similar situation can be illustrated using the circuit shown in figure 3. Supposed that the requested gain is i_c/i_s instead. The previously described feedback method requires us to first determine v_{out}/i_s . To convert this result we must use the correct relationship between collector current and voltage,

$$i_c = v_{out} / (R_L || R_{22}) = v_{out} / (R_L || R_F)$$

These examples are relevant because they illustrate that the resistance used to relate voltage and current is an *open-loop* quantity. The output resistance R_{of} is part of the amplifier model, but it is not used to relate external voltages and currents.

Conclusion

This paper discusses examples that clarify the effect of load and source resistances when the feedback method is employed to analyze amplifiers. These examples complement the material

presented in textbooks by illustrating some subtle aspects of the analysis, and can be used to clarify some potentially confusing details of the application of feedback theory to amplifiers.

It is shown that, after the feedback method is applied, the load R_L and the source's Thevenin resistance R_s can be taken "out" of the amplifier. Proper modification of the amplifier gain permits to study the effect that different values of these components have. However, only one component (R_s or R_L) can be modified at a time, while the other must be kept unchanged at the value that was used when the feedback method was applied. Examples that illustrate the correct calculation of a load resistance to relate output voltage and current have also been presented.

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Biographical Information

Manuel Toledo-Quiñones finished his BS the Electrical Engineering Department at UPRM in 1979, his MS in Applied Physics at the University of Massachusetts in 1989, and his Ph.D. in the Electrical, Computer and Systems Engineering Department at Boston University in 1995. He joined the University of Puerto Rico in 1998, where he has been teaching undergraduate courses in electronics. He is currently involved in projects in the areas of micro-sensors, machine vision, and digital watermarking.