Paper ID #45312

Understanding STEM Students' Conceptual Derivative Knowledge Through Analysis of Sub-concept Cognition

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Abstract. One of the most frequently used concepts by STEM majors during their undergraduate education is differentiation of mathematical functions. Investigating undergraduate students' differentiation knowledge through collected empirical data and determining weaknesses to improve pedagogical approaches to teach differentiation to STEM students are the main goals of this work. The conducted research's data collection procedure received Institutional Review Board (IRB) approval. Quantitative and qualitative data is collected from 20 STEM students at a university located at the Northeastern side of the United States. Written and video recorded information is collected and analyzed qualitatively and quantitatively to understand STEM students' ability to solve a research question related to derivative, absolute value, and trigonometric function concepts. The pedagogical techniques used in this work to analyze the collected data are Action, Process, Object and Schema (APOS) theory introduced in [1], and concept image and concept definition introduced in [16]. The written results indicated participants' elementary level ability to form a bridge between the derivative of the absolute value function and its image. The participants also had hard time to progress the solution step-by-step by using the building blocks of derivative of a composition function. These results indicated a need for a better concept image and concept definition coverage of the composition functions' differentiability in calculus education for improving their conceptual understanding of STEM majors.

Key words: STEM education, Derivatives of functions, APOS theory, Composition function, Concept image and concept definition.

1. Introduction

The derivative of mathematical functions is one of the central concepts in STEM applications, therefore investigating engineering students' ways to understand the derivative concept and ability to respond derivative related questions is an interest of STEM educators and pedagogical researchers. The nature of a calculus question with multiple sub-concepts can make it a difficult task to solve the problem for students, therefore a closer look at STEM students' missing conceptual knowledge through their responses to a complex calculus question and analyzing it pedagogically appears to be a necessity to improve teaching practices.

In this work, we use empirical data to analyze and evaluate engineering and mathematics students' comprehension of the derivative concept. The empirical data is collected from 20 undergraduate STEM majors who were enrolled to a STEM granting degree at a mid-sized university located on the Northeast side of the United States. This IRB approved research's participants are compensated money for their written responses to the following research question and the follow up video recorded oral interviews.

Is h(x)=sin|x| a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

The calculus sub-concepts used for evaluation included the following:

- Differentiability
- Function domain
- Composition of functions
- Absolute value and trigonometric function graphs

Two types of data were collected during the research period: The first type, quantitative data, consisted of pre- and post-interview written responses of the research participants. The pre-interview written response was the solution of the participant to the question prior to the video recorded interview. The post-interview response was the written response of the participant during the oral interview in the case the participant wanted to add or change the existing answer. The video recorded oral interviews were conducted to investigate the conceptual details of the written responses of the research participants' conceptual understanding related to differentiability, domain of the functions taking place in a composition function, and graphs of functions. The research team used two pedagogical methods to analyze the data that will be explained in the next two section, Action-Process-Object-Schema (APOS) theory as well as the concept image and concept definition. Asiala et al. [1] applied APOS to mathematical topics, and this theory was explained as the combined knowledge of a student in a specific subject based on Piaget's philosophy from 1970s. Participants' concept image and concept definition are investigated in this research to analyze their ability to establish a connection between the definition that they learned and the associated geometric structure. The qualitative data analyzed in this work consisted of the transcription of the video recorded interviews. Overall, qualitative and quantitative analysis of the data indicated participants' weaknesses in establishing a connection between the concept image and concept definition of the derivative concept while the main weakness in sub-concept knowledge was observed to be the absolute value function's geometric knowledge to determine the derivative of the sine|x| function.

The next section is devoted to outline a brief literature review of APOS theory used in undergraduate mathematics education. The third section contains research literature information on concept image and concept definition used for differentiation concept in mathematics. Data collection protocol followed for the conducted research and the details of the qualitative and quantitative data analysis is shared in the fourth section. The last section contains summary of the research results and conclusion remarks.

2. Action-Process-Object-Schema (APOS) Theory

Piaget and Garcia [14] had an influence on undergraduate mathematics researchers and therefore impacted mathematics and engineering teaching curriculums during 1990‘s. Students' conceptual view of the function was defined in [5] that relied on Piaget's study of functions in 1977 that helped forming action-process-object idea in mathematics education [15]. In 1996, APOS theory is applied in [1] to mathematical topics (mostly functions) and the theory is explained to be the combined knowledge of a student in a specific subject based on Piaget‘s philosophy. The main idea of APOS theory is to observe conceptual construction of subjects on sub-concepts and schemas and the construction of a specific concept depends on knowledge of other concepts. For instance, the differential of a composition function would require the conceptual understanding of functions, composition of functions, domain of a function, cusp point, and basic number knowledge. APOS theory was used for understanding undergraduate students' conceptual function knowledge with a calculus graphing problem in [4]. Cooley, Trigueros and Baker [7] focused on the thematization of the schema with the intent to expose those possible structures acquired at the most sophisticated stages of schema development. APOS theory was not determined to be appropriate for analysis of data in [6]. For a detailed review of the APOS theory see [10].

Understanding university students' conceptual derivative knowledge has been a focus point of several mathematics and engineering pedagogical researchers [3, 11, 13, 18]. Differentiation is observed to be taught as a rule rather than putting emphasis on conceptual meaning [13; pg. 242]; More sophisticated derivative concepts, such as chain rule, are observed to require conceptual calculus knowledge of students.

The triad stages of intra, inter, and trans are used to understand how first year calculus students construct the chain rule concept in [6]. First year calculus enrolled students attempted to provide an example to explain the chain rule rather than explaining how it works in [8]. Students' comprehension of function composition and the chain rule by studying how they use and interpret the chain rule while working in an online environment is investigated in [17]. Our goal in this study is to understand engineering and mathematics undergraduate students' conceptual derivative knowledge by observing the following:

- Ability to determine the domain of differentiability of a function.
- Ability to determine the differentiability domain of a composition function.
- Ability to apply the chain rule correctly.
- Ability to determine the domain on which the chain rule is applied.

APOS theory is briefly explained in [10] as follows:

- An action is a transformation of objects perceived as essentially external and as requiring, either explicitly or from memory, step-by-step instructions on how to perform the operation.

- When an action is repeated and the individual reflects upon it, the individual can make an internal mental construction called a process which the individual can think of as performing the same kind of action, but no longer with the need of external stimuli.
- An object is constructed from a process when the individual becomes aware of the process as a totality and realizes that transformations can act on it.
- A schema is individuals' collection of actions, processes, objects, and other schemas which are linked by some general principles to form a framework in individual's mind.

The collected data will be evaluated by using the following APOS classification similar to the one outlined in [21]:

- **Action:** The participants classified in this group were able to act on their knowledge to determine the derivative and graph of the sine function. These persons respond to the research questions during the pre- or post-interviews based on each sub-calculus concept.
- **Process:** The Action group members are classified in this group due to their ability to act on their knowledge to determine the derivative and graph of the absolute value function. The participants in this group respond to the research question during the pre- or post-interview period based on each sub-calculus concept.
- **Object:** The individuals classified in this group are members of the Process group that could compose the sine and absolute value functions by graphing the composition function.
- **Schema:** Object group members are classified to be in this group if they could justify how absolute value function impacts the differentiation and graph of the composition function. The participants are expected to reflect their Action, Process and Object knowledge in a comprehensive manner to be able to outline all the details on the composition function's graph and its derivative.

Next section is devoted to concept image and concept definition with its application on functions.

3. Concept Image & Concept Definition

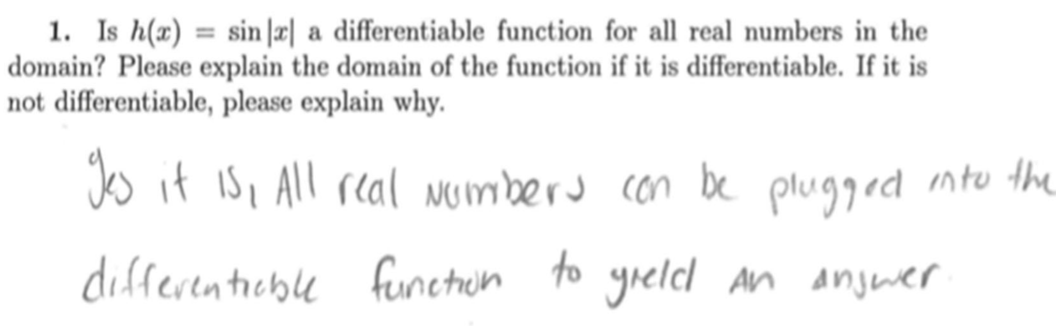
The pedagogical investigation on understanding undergraduate students' function concept is not explored until the 1970s. The formal definition of a mathematical concept is usually displayed in an abstract form and this definition plays an important role in applications of the abstract concept in real life settings. On the contrary to the abstract understanding of mathematics, many engineering applications require an extensive understanding of the geometry (or visual understanding) of the abstract concept. One way of analyzing undergraduate students' conceptual mathematics knowledge can be through the analysis of matching between their concept image and concept definition correspondences. The concept image and concept definition approach of Dreyfus & Vinner [9] is one of the techniques applied in mathematics education.

The concept image and concept definition of functions in mathematical education research is used in several studies. Aspinwall et. al [2] investigated on students' uncontrollable mental imagination as a part of graphical connections between a function and its derivative. A geometric approach is introduced by Hershkowitz and Vinner [12]; however, the most extensive research in the undergraduate curriculum was done by Dreyfus and Vinner [9] in which they defined the concept image and concept definition of functions based on their research with undergraduate students. Additional function and associated graph related pedagogical research is conducted in [19-25] and we refer to these articles for the interested readers. In this work, collected written questionnaire data didn't target the concept image and concept definition of the differential of a function directly, however the video recorded interviews included questions on participants' knowledge on specific domain of differentiation and perception on reflecting this knowledge to solve the research question. The concept image and concept definition are used in conjunction with the APOS theory to analyze the depth of participants' conceptual chain rule understanding. The concept image and concept definition methodology will be used to investigate undergraduate engineering students' ability to interrelate function images when differentiability is considered along with the given two functions.

4. Data Collection Protocol & Analysis of the Data

Research participants of this work are 20 undergraduate STEM students who were either enrolled or recently completed (i.e., 1 week after the completion of the course) the first two courses of a 12-credit calculus sequence consisting of three courses at a midsized Northeastern university in the United States. The research was conducted during a semester and received IRB approval due to its' data collection nature. The research team consisted of three undergraduate engineering students and a tenure-track professor that worked together on IRB approval and data collection. All research participants are compensated for their participation to the written and oral interviews. The rest of this section is devoted to the research participants' responses and analysis of the data.

One of the challenges that students faced was understanding the conceptual image of functions and be able to match them with their definitions. Participant 1 with the written response displayed in Figure 1 below relied on the image of the composition function sin|x| and responded accordingly. The justification of this participant was much different than a typical response to the research question.



1. Is $h(x) = \sin|x|$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

Yes it is, All real numbers can be plugged into the differentiable function to yield an answer

Figure 1. Response of participant 1 depending on the graph of the function plugged into a calculator.

The oral interview helped to further understand the participant's conceptual understanding of the differentiation and mental construction of the interrelated concepts. The response of the participant was based on the function to have "no limits where it gets bounded."

P 1: …for the first question you said it is a differentiable function. Can you please explain your answer?
S: It's differentiable for all numbers because I plotted in my calculator to see the function and it just keeps going.
P 1: I see.
S: Yeah, there is no limits where it gets bounded.
P 1: Is absolute value of x a differentiable function?
S: Yes, I think.
P 1: Is sin(x) differentiable function?
S: Yes.
P 1: … what is the meaning of differentiable? Do you remember?
S: Differentiable is when you do the derivative and able to plug in any number from domain to get like a correct answer, like a real number.
P 1: Can you think an example of a function that is not differentiable?
S: One over the ln of x.
P 1: One over the ln of x, where is it not differentiable?
S: No I don't think so.

Participant 6 with the written response displayed in Figure 2 was able to explain the concept image and justify that the function is differentiable from 0 to 2π initially. During the oral interview the participant mentioned the differentiability of the composition function to be the entire domain.

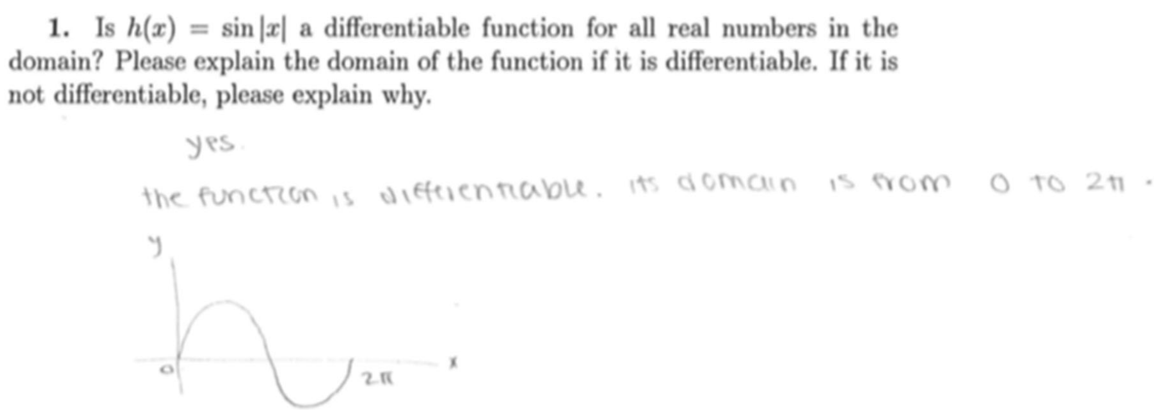
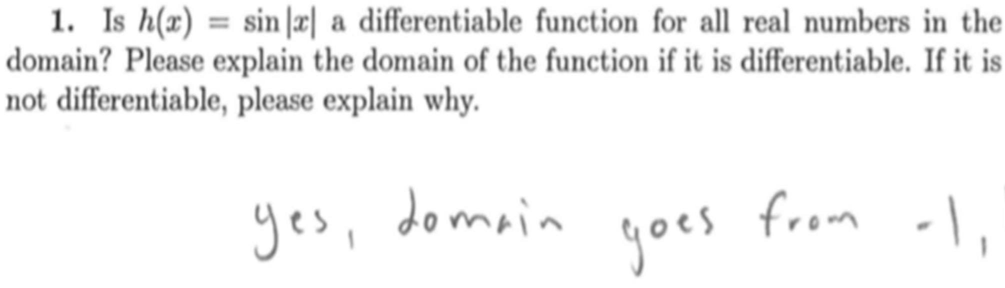


Figure 2. Response of participant 6 with partially correct response.

P 6: Can you please explain your answer to question 1?
S: …so I said the function is differentiable because there are no discontinuities in it like there is no jumps or breaks so like yeah that was my though process behind it.
P 6: Okay
S: And then the domain, I really wasn't sure but usually like sine and cosine curves it goes from like 0 to, oh actually now, as I am looking at it, because it is like absolute value, so maybe that changes it.
P 6: How would it change?
S: It can't be negative.
P 6: Okay, is absolute value of x differentiable everywhere in its domain?
S: I don't know
P 6: Okay, is the function differentiable everywhere?
S: Yes

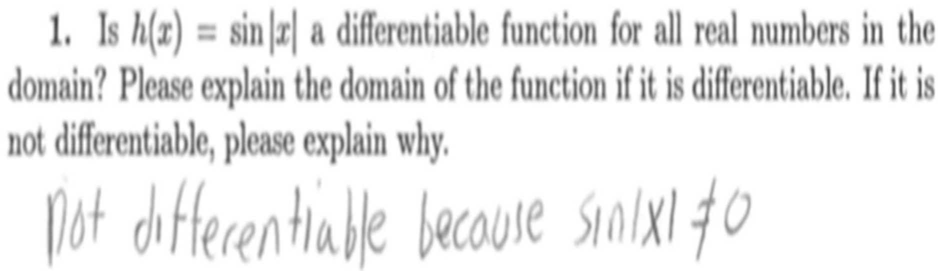
Participant 9 had a misconception of the domain of the function and stated the range of the function instead of the domain itself.


1. Is $h(x) = \sin|x|$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

yes, domain goes from -1,1

Figure 3. Response of Participant 9 mixed up the domain and the range of the function.

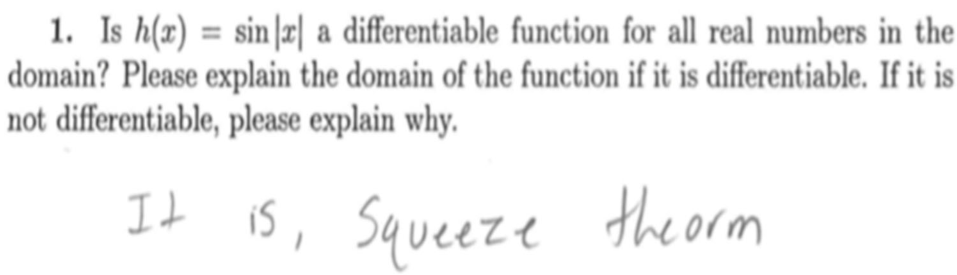
One of the most interesting responses was provided by Participant 14 in Figure 4 below. The participant tried to justify that the function is not differentiable because the given composition function cannot be equal to zero.


1. Is $h(x) = \sin|x|$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

not differentiable because $\sin|x| \neq 0$

Figure 4. Response of Participant 14 with differentiation depending on function's equality to zero.

Another unexpected response was by Participant 16 displayed in Figure 5 below. This participant stated that the function is differentiable because of the squeeze theorem.

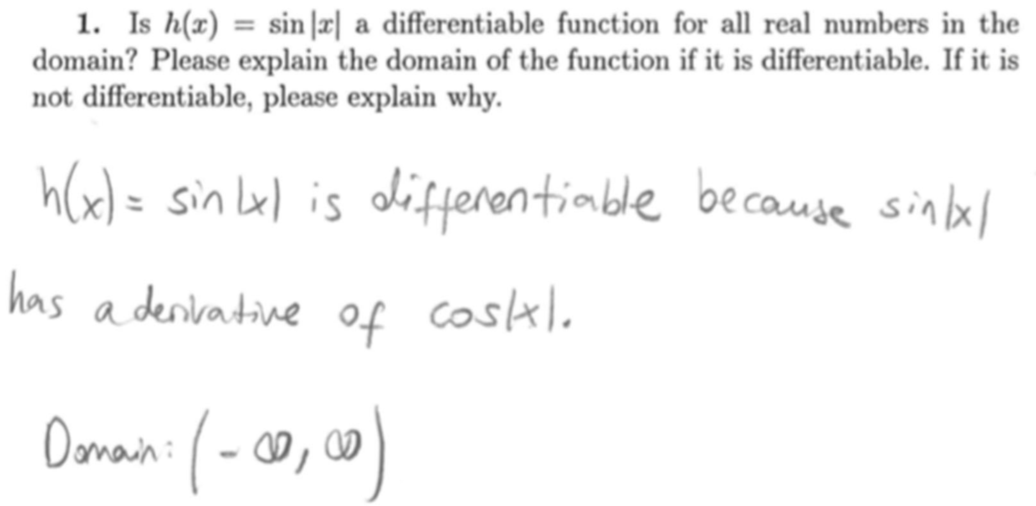

1. Is $h(x) = \sin|x|$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

It is, Squeeze theorm

Figure 5. A participant showing squeeze theorem as the reason for differentiation of the function.

One of the participants, Participant 17, knew that sinus function is differentiable and stated that |x| is not a differentiable function without justification (in Figure 6):

P 17: Okay, now we are recording. Can you please explain your answer to the first question?
S: So for the first question asking if h(x) equals to sin(x) is a differentiable function, and I said that it is differentiable because differential means that if it has a derivative of the function, it can be differentiated. And in this example, in this question sin(x) has derivative of cos(x). That's why I believe that it is a differentiable function.
P 17: Okay, is absolute value of x differentiable?
S: The absolute value is not but sine is, so I kept the absolute value same and I changed into cos.
P 17: Okay, so is sine and absolute value of x are two different functions? If it is the composition of these two functions how would be derivative of two compositions functions work? Do you recall?
S: I'm not sure but I only know sin has a derivative of cos that's why I switched it like that.



1. Is $h(x) = \sin|x|$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable, please explain why.

h(x) = sin|x| is differentiable because sin|x| has a derivative of cos|x|.

Domain: (-∞, ∞)

Figure 6. Response of participant 17 to the research question.

Another participant, Participant 18 (with the response displayed in Figure 7), was able to sketch the graph of the absolute function and showed geometric realization of it but yet this participant ignored the definition of differentiation by stating "absolute function doesn't matter" during the interview.

P 18: Can you please explain your answer to the first question?
S: So in the first question it was asking that h(x) equal to sin(x) differentiable function for all real numbers. I think it is and differentiable function and the derivative h(x) equals to cos(x) and the domain of the x is negative infinity to infinity because it says all real numbers.
P 18: Is absolute value of x differentiable function?
S: Yes. I think yes because it doesn't matter if I put negative infinity in the absolute value, It's gonna be positive infinity and if I can have sin positive infinity as a derivative, I think absolute function doesn't matter.
P 18: So, can you sketch the graph of absolute value of x?
S: …so you want me to sketch the sin|x|.
S: Okay, so it is gonna be
P 18: Do you remember?
S: The thing is if I give 1 here it is going to be 1, if I give 2 here it is going to be 2. Right? So this will be like that and if I give negative 1 here it is going to be 1 again, if I give negative 2 here it is going to be 2 again so.
P 18: Okay, is this a differentiable function?
S: Yes

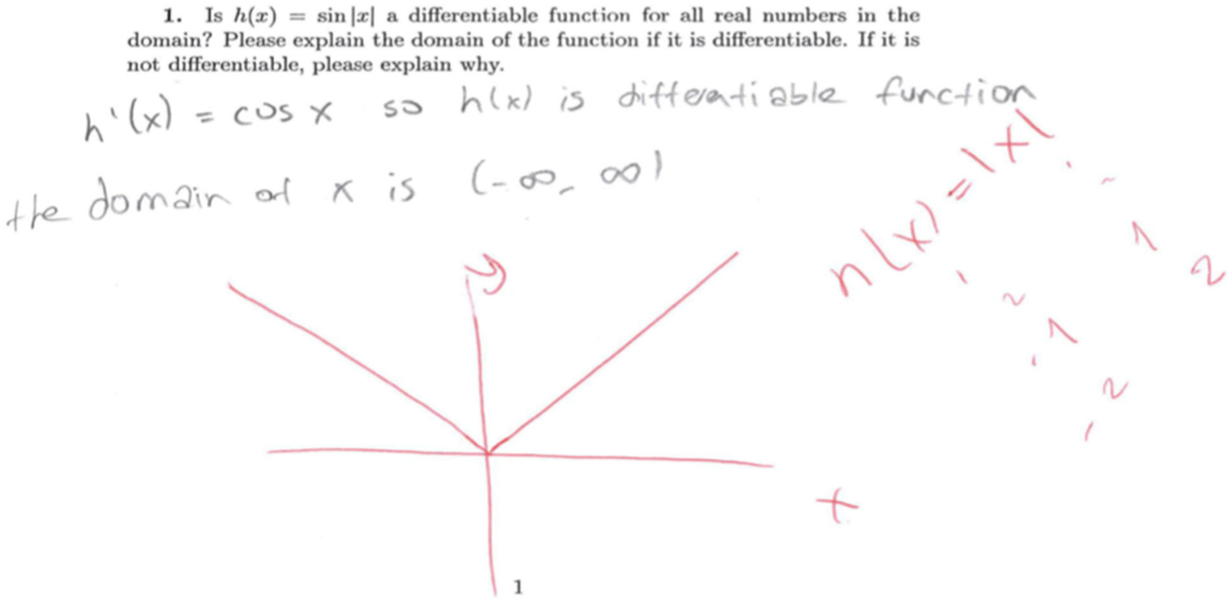


Figure 7. Response of Participant 18 to the research question.

Participant 20 with the response displayed in Figure 8 below tried to justify the response by stating "sin(2) does not exist". This response was also unexpected, and the participant doesn't further explain the response later.

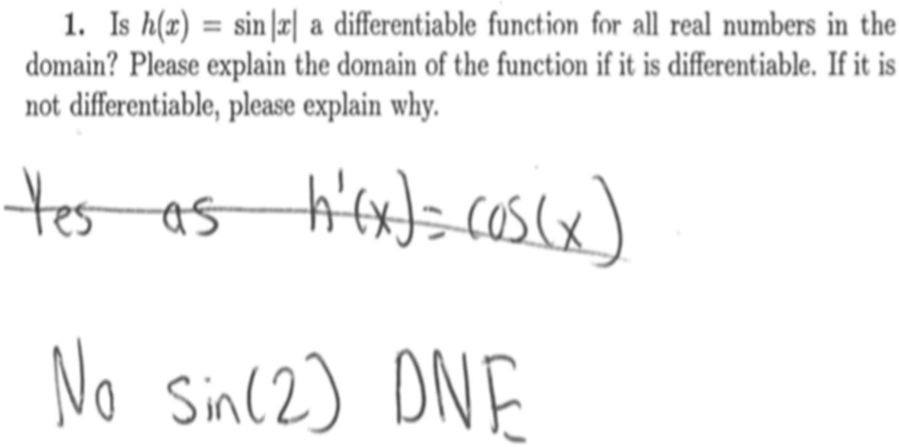


Figure 8. Response of Participant 20 with the justification depending on a value of the function.

The APOS classification of the participants appeared to be heavily distributed into Action stage with 60% of the participants grouped in this stage. The main challenges faced by the participants included basic misconceptions on the domain of sine function, graphs of sine and absolute value functions, and differentiation of absolute value function. Process stage classification consisted of 15% in which case the corresponding participants were able to graph and differentiate the sine and absolute value functions. 25% of the participants were not classified in any one of the APOS stages due to their responses not fulfilling any of the group specifications.

The concept image and concept definition were used for observing participants' understanding of the connections between the geometric view and definition of the sine, absolute value function, composition function, and differentiation. The participants were mainly capable of sketching the graph of the absolute value function but could not justify the derivative of the absolute value function. The participants who were able to respond to the research question at the Action stage mentioned that the derivative of the sine function is cosine function and had the ability to sketch the graph of the sine function. The domain knowledge was one of the misconceptions of the participants; 10% of the participants mixed up the definition of the domain and range of a function.

5. Conclusion

In this work we used APOS theory and concept image and concept definition to analyze and evaluate engineering undergraduate students' comprehension of the derivative of a composition function. The empirical data is collected from 20 undergraduate STEM students who majored in a STEM field at a mid-sized university located on the Northeast side of the United States. The research procedure followed for data collection received IRB approval. The research participants are compensated for both their written responses to the following research question as well as the follow up video recorded oral interviews.

Is $h(x)=\sin(|x|)$ a differentiable function for all real numbers in the domain? Please explain the domain of the function if it is differentiable. If it is not differentiable please explain why.

The calculus sub-concepts used for evaluation of the research included the following:

- Differentiability
- Function domain
- Composition of functions
- Graph of a trigonometric function
- Absolute value function

There were two types of data collected during the research period: The first type, quantitative data, consisted of pre- and post-interview responses of the participants. The pre-interview written responses consisted of the written responses of the participants to the research question prior to the video recorded interviews. The post-interview transcription of the data was based on the responses collected during the oral interview; the oral interviews were conducted to have a better understanding of participants' conceptual knowledge. The oral interviews were video recorded and transcribed for analysis of the data that are displayed in this work. The collected data for the research question is evaluated by using the following APOS classification:

- **Action:** The participants classified in this group were able to act on their knowledge to determine the derivative and graph of the sine function. These persons respond to the research questions during the pre- or post-interviews based on each sub-calculus concept.
- **Process:** The Action group members are classified in this group due to their ability to act on their knowledge to determine the derivative and graph of the absolute value function. The participants in this

group respond to the research question during the pre- or post-interview period based on each sub-calculus concept.

- **Object:** The individuals classified in this group are members of the Process group that could compose the sine and absolute value functions by graphing the composition function.
- **Schema:** Object group members are classified to be in this group if they could justify how absolute value function impacts the differentiation and graph of the composition function. The participants are expected to reflect their Action, Process and Object knowledge in a comprehensive manner to be able to outline all the details on the composition function's graph and its derivative.

APOS classification indicated 60% of the participants to be in the Action group while 15% of the participants are classified to be in the Process stage and 25% of the participants were not classified in any one of the APOS stages due to their responses not fulfilling any of the group specifications. The results shared in this work suggest that educators need to find ways for undergraduate students to build mental structures to advance their calculus knowledge and improve their conceptual definition and conceptual image understanding. The main challenges faced by the participants included basic misconceptions on the domain of sine function, graphs of sine and absolute value functions, and differentiation of absolute value function. We invite other researchers to expand on the research conducted in this work.

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