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Use of an Analogy to Demonstrate the Origin and Nature of Steady-State Errors in Control Systems

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Abstract

An introductory control systems course can be challenging to undergraduate students due to its fairly sophisticated mathematical nature. For example, it can be difficult to comprehend how even a system composed of perfect components could have a steady-state error. To help students understand such non-intuitive concepts, it is beneficial to offer them a visual example that involves a familiar scenario. This paper describes a car race analogy which leverages these two complementary techniques in a one semester course for juniors and seniors in automatic control systems.

The analogy consists of two competing cars of differing masses and air drags with various inputs via the gas pedal. Equations of motion are presented for the displacement, velocity, and acceleration for step, ramp, and parabolic inputs. MATLAB® software is used to solve the equations and plot the results for analysis and comparison. This familiar illustrative scenario allows students to discover easily and quickly how steady-state differences (analogous to errors) depend on the nature of the system and its type of input. It also demonstrates the effects of some easily understood corrective actions to reduce or eliminate the differences and reinforces understanding of the derivative-integral relationships between the displacement, velocity, and acceleration responses.

The graphical nature of this illustration fits well with the visual learning style of many students. Through this multi-faceted investigative analogy, they gain an intuitive understanding of steady-state errors as a complement to the traditional mathematical treatment. Results of a voluntary survey completed by the students indicated that they found the car race analogy helpful in understanding the origin and nature of steady-state errors in control systems.

I. Introduction

Steady-state errors are an important consideration of control systems in a multitude of applications, such as the use of machine tools and robotics in manufacturing. As such they form a key aspect of the theory of control systems that students must understand. The origin and nature of steady-state errors can be easily discovered by students fairly early in their study of control systems. This preparation is helpful as background for the more mathematically challenging aspects of corrective design measures, such as the use of proportional-integral-derivative (PID) compensators, which typically follow later in their study of control systems.

Through a simple graphical example introduced in a lecture, the instructor can guide students to understand how steady-state errors can occur in a system. If time allows, the students can investigate the example in more depth as a homework assignment or on their own initiative. That such errors can occur in a system, <u>even if</u> it could be composed of ideal components, is not intuitive to most students. The MATLAB® numeric computation software package, as used in this paper, can readily perform the simple calculations and construct plots of the results to demonstrate the origin and nature of steady-state errors for various values of system parameters and types of input signals.

The graphical example of a car race analogy presented in Section III of this paper is designed to match the preferred learning style of most engineering students which is visual, sensing, inductive, and active; and it provides balance to the traditional lecture presentation which is usually auditory, intuitive, deductive, and passive.[1] As preparatory background to the car race analogy, the author's pedagogy in engineering courses makes extensive use of analogies and demonstrations to illustrate concepts, as described in the following section.

II. Pedagogy--Extensive Use of Analogies and Demonstrations

To address and enhance the varied learning styles of students in a typical engineering class, the author uses an array of supplemental teaching approaches, including the use of a numerical exercise to demonstrate the effects of feedback, to complement the primarily lecture style in this introductory course on automatic control systems.[2] While most of the course with its prerequisite of ordinary differential equations is devoted to developing and applying the theory in this mathematically-based course, several complementary analogies and demonstrations are used to help the students understand the underlying concepts and performances. The focus is primarily on first and second order systems, and especially ones dealing with motion control since the class typically includes both electrical and mechanical engineering students.

After extensive coverage in several lectures of the underlying mathematical theory of the position and speed of a motion control system, the response of the system to a step input is demonstrated in the laboratory using a configurable MS150 Modular Servo System from Feedback, Inc. Important aspects of this hardware demonstration include the effects of gain, inertia, and damping on the response of the system. Students often express appreciation and state that this complementary demonstration helps to make the theory more meaningful to them and, hence, the motion responses less mysterious and more understandable.

Damping is an important factor in motion performance as graphically illustrated for the car race analogy in the next section of this paper. To demonstrate to students the effect of damping on the responses of systems using readily available materials, clear glasses of liquids with different viscosities are placed on a table and then a leg of the table is given a swift, unannounced kick, often startling the students! This physical action (without breaking a toe as yet!) approximates the application of an impulse excitation to the system and the resulting impulse responses for different levels of damping. As another demonstration of damping, the classroom door with its damping cylinder is opened some and then slowly pulled shut. For comparison of the required pulling forces, the door is opened again, but then quickly pulled shut. (So far, the door has not been pulled off its hinges during this enthusiastic demonstration of the effect of damping!)

The subject of this paper is a car race analogy, refined over several years, used to help students understand the potentially perplexing concept of steady-state errors in systems. Coverage of this topic occurs about mid-semester following a review of the Laplace Transform, developing mathematical models for components and systems of components using transfer functions, and analyzing the responses of feedback control systems by hand and with MATLAB® software. As a close follow-up to the introduction of steady-state errors, the use of PID compensators is investigated to improve system performance, including eliminating steady-state errors.

In particular, the primary aim of this car race analogy is to graphically show students how steady-state errors can occur even if one could build a system using perfect components. An error can occur in the steady-state response following a transient depending on the nature of the system (such as its mass and damping) and the type of input (such as a step, ramp, or parabolic change). As an added benefit of graphically displaying the displacements, velocities, and accelerations of the two cars in a race, the plots reinforce how these variables are related though derivatives and integrals that the students first encountered in calculus and applied in physics. To relate to as many students as possible, a race between runners is mentioned to them as a similar analogy.

Details of the illustrative car race analogy with the equations of motion and the comparative responses of two racing cars for various mass and damping conditions and types of inputs are presented in the next section. The equations, along with the sample MATLAB® program in the Appendix, will enable interested readers to implement the graphical demonstration for their students.

III. Illustrative Car Race Analogy

Imagine that two cars, labeled as Car #1 and Car #2, are in a car performance race at an automobile test track. Car #1, with its lighter mass M and more streamlined design yielding less air drag as represented through a viscous damping coefficient B, will be considered as the reference car. The following analyses will determine the differences in car performance of displacements, velocities, and accelerations as a function of time, including the steady-state differences which can be considered as analogous to steady-state errors in control systems. The race scenario is represented in the block diagram of Figure 1.

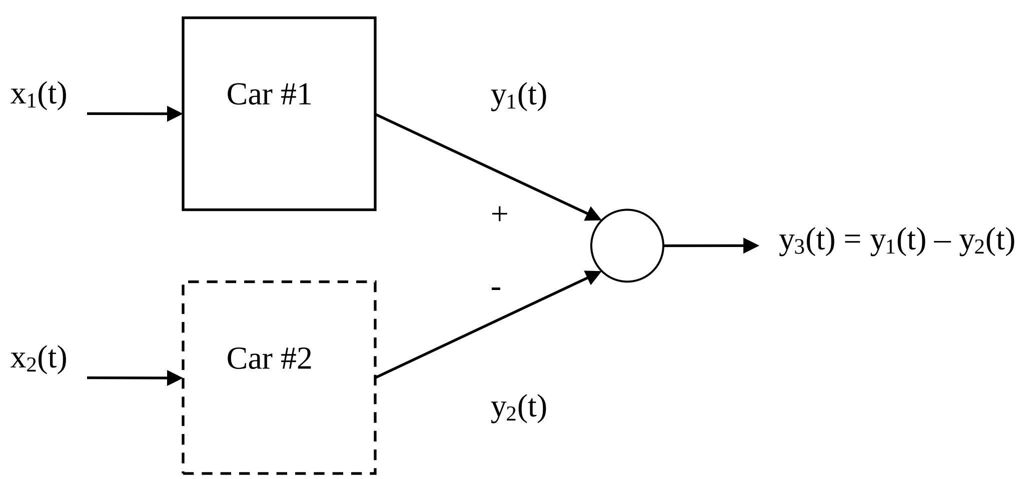

Fig. 1. Block diagram.

The motion of each car can be represented in the time domain by the following general differential equation of Newton's Second Law:

$$Md^2y(t)/dt^2 + Bdy(t)/dt + Ky(t) = x(t) \qquad (1)$$

where M is the mass of a car, B is the viscous damping coefficient representing air drag, and K is the "spring" constant taken as zero in this simple modeling of car motion, and where x(t) is the input (force excitation resulting from pressing the gas pedal) and y(t) is the output (displacement response of a car).

The Laplace transform of this differential equation is:

$$Ms^2Y(s) + BsY(s) + KY(s) = X(s) \qquad (2)$$

where X(s) and Y(s) are the Laplace Transforms of x(t) and y(t), respectively.[3]

The displacement, velocity, and acceleration responses of the two cars for various car and input parameters are illustrated in the following sections. For simplicity, units for the parameters and variables are not included in the equations of motion of the two cars.

A. Displacement, Velocity, and Acceleration Responses of Each Car to a Step Input.

Select the following parameter values for the two cars, with Car #2 heavier (larger M value) and less streamlined (greater B value) than Car #1:

Car #1:	Car #2:
$M_1 = 1$; $B_1 = 1$; $K_1 = 0$	$M_2 = 1.25$; $B_2 = 1.25$; $K_2 = 0$

1) Displacement for a Step Input: The transfer function for displacement is

$$T_D(s) = Y(s)/X(s) = \frac{1}{Ms^2 + Bs + K} \qquad (3)$$

where, for a step input, X(s) = A/s corresponding to x(t) = Au(t) where A is the amplitude of the step with u(t) being the unit step function, giving the displacement, Y(s), as

$$D_S(s) = \frac{1}{Ms^2 + Bs + K}\,(A/s) \qquad (4)$$

Note: The MATLAB® program used in this paper to generate the displacement responses of the two cars to unit step inputs is listed as an example in the Appendix.

The displacements of the cars for identical step inputs with $A_1 = 1$ and $A_2 = 1$ are shown in Figure 2.

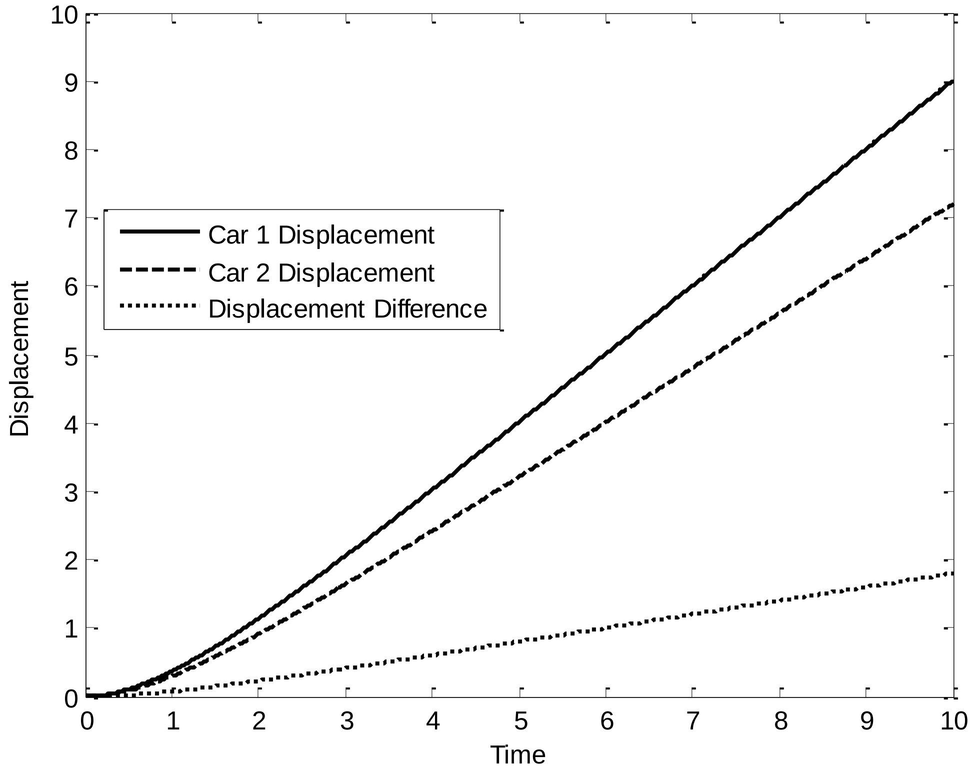

Fig. 2. Displacements of the cars for step inputs.

The difference of the car displacements for step inputs tends toward infinity as demonstrated in Figure 2.

2) Velocity for a Step Input: The transfer function for velocity is

$$T_V(s) = [sY(s)]/X(s) = \frac{s}{Ms^2 + Bs + K} \qquad (5)$$

where, for a step input of amplitude A, the velocity, sY(s), is

$$V_S(s) = \frac{s}{Ms^2 + Bs + K}(A/s) = \frac{1}{Ms^2 + Bs + K}(A) \qquad (6)$$

The velocities of the cars for identical step inputs with $A_1 = 1$ and $A_2 = 1$ are shown in Figure 3.

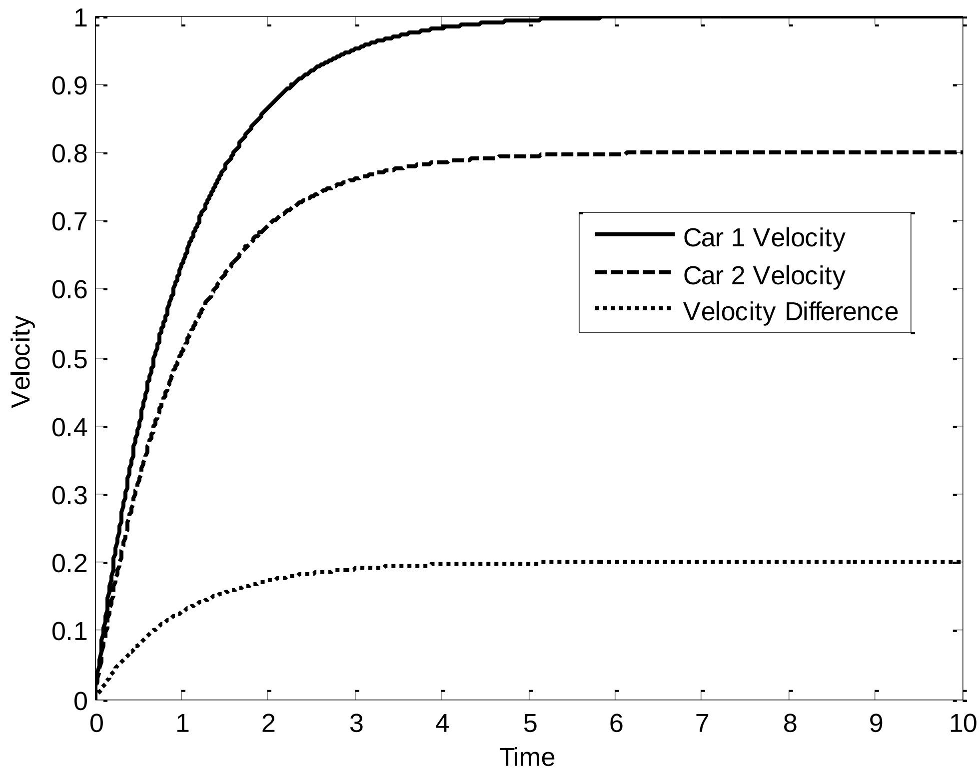

Fig. 3. Velocities of the cars for step inputs.

The difference of the car velocities for step inputs tends toward a finite value as demonstrated in Figure 3. In addition, the velocity plots in Figure 3 correspond, as expected, to the rate of change (slope or 1st derivative) of the displacement plots of Figure 2.

3) Acceleration for a Step Input: The transfer function for acceleration is

$$T_A(s) = [s^2Y(s)]/X(s) = \frac{s^2}{Ms^2 + Bs + K} \tag{7}$$

where, for a step input of amplitude A, the acceleration, $s^2Y(s)$, is

$$A_S(s) = \frac{s^2}{Ms^2 + Bs + K}(A/s) = \frac{1}{Ms^2 + Bs + K}(As) \tag{8}$$

The accelerations of the cars for identical step inputs with $A_1 = 1$ and $A_2 = 1$ are shown in Figure 4.

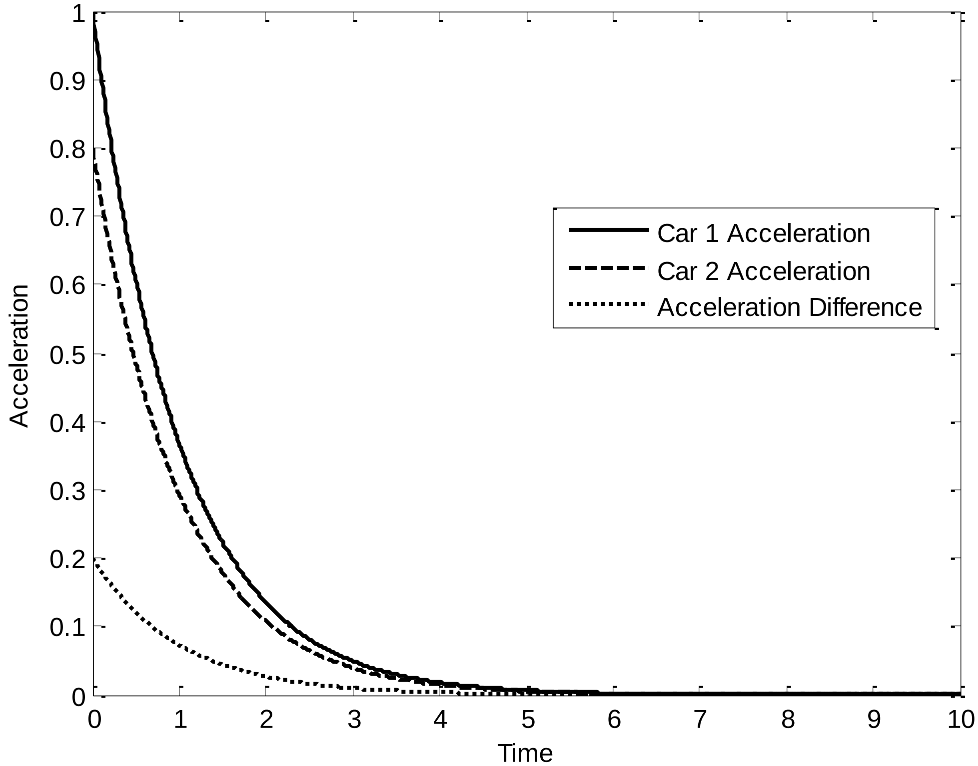

Fig. 4. Accelerations of the cars for step inputs.

The difference of the car accelerations for step inputs tends toward zero as demonstrated in Figure 4. In addition, the acceleration plots in Figure 4 correspond, as expected, to the rate of change (slope or 1st derivative) of the velocity plots of Figure 3.

B. Displacement, Velocity, and Acceleration Responses of Each Car to a Ramp Input.

Select the same parameter values as applied in Section III.A for the two cars.

1) Displacement for a Ramp Input: For a ramp input, $X(s) = A/s^2$ corresponding to $x(t) = Atu(t)$ where A is the rate of change (slope) of the ramp, giving the displacement, Y(s), as

$$D_R(s) = \frac{1}{Ms^2 + Bs + K}\,(A/s^2) \qquad (9)$$

The displacements of the cars for identical ramp inputs with $A_1 = 1$ and $A_2 = 1$ are shown in Figure 5.

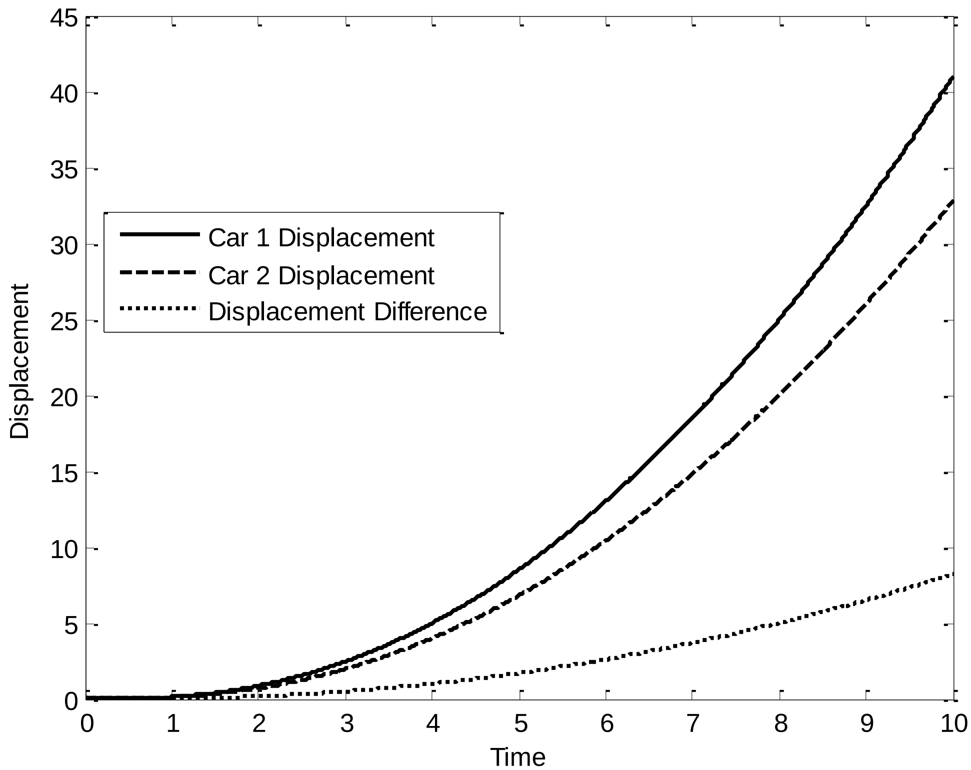

Fig. 5. Displacements of the cars for ramp inputs.

The difference of the car displacements for ramp inputs tends toward infinity as demonstrated in Figure 5.

2) Velocity for a Ramp Input: The corresponding ramp velocity, sY(s), is

$$V_R(s) = \frac{s}{Ms^2 + Bs + K} (A/s^2) = \frac{1}{Ms^2 + Bs + K} (A/s) \qquad (10)$$

This is the same result as the function for displacement with a step input (Equation 4). Therefore, the velocity responses of the cars to ramp inputs are the same as the displacement responses of the cars to step inputs as shown in Figure 2.

3) Acceleration for a Ramp Input: The corresponding ramp acceleration, $s^2Y(s)$, is

$$A_R(s) = \frac{s^2}{Ms^2 + Bs + K} (A/s^2) = \frac{1}{Ms^2 + Bs + K} (A) \qquad (11)$$

This is the same result as the function for velocity with a step input (Equation 6). Therefore, the acceleration responses of the cars to ramp inputs are the same as the velocity responses of the cars to step inputs as shown in Figure 3.

C. Displacement, Velocity, and Acceleration Responses of Each Car to a Parabolic Input.

Select the same parameter values as applied in Sections III.A and III.B for the two cars.

1) Displacement for a Parabolic Input: For a parabolic input, $X(s) = A/s^3$ corresponding to $x(t) = (A/2)t^2u(t)$ where A is the rate of change (slope) of the parabola, giving the displacement, Y(s), as

$$D_P(s) = \frac{1}{Ms^2 + Bs + K} (A/s^3) \tag{12}$$

The displacements of the cars for identical parabolic inputs with $A_1 = 1$ and $A_2 = 1$ are shown in Figure 6.

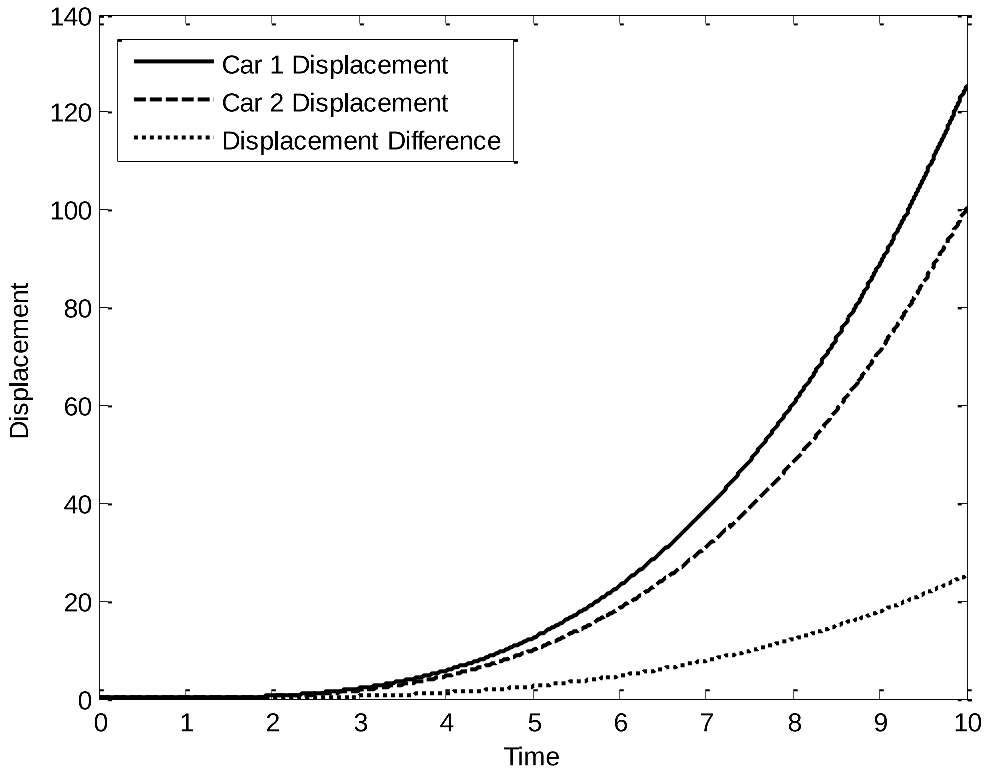

Fig. 6. Displacements of the cars for parabolic inputs.

The difference of the car displacements for parabolic inputs tends toward infinity as demonstrated in Figure 6.

2) Velocity for a Parabolic Input: The corresponding parabolic velocity, $sY(s)$, is

$$V_P(s) = \frac{s}{Ms^2 + Bs + K} (A/s^3) = \frac{1}{Ms^2 + Bs + K} (A/s^2) \qquad (13)$$

This is the same result as the function for displacement with a ramp input (Equation 9). Therefore, the velocity responses of the cars to parabolic inputs are the same as the displacement responses of the cars to ramp inputs as shown in Figure 5.

3) Acceleration for a Parabolic Input: The corresponding parabolic acceleration, $s^2Y(s)$, is

$$A_P(s) = \frac{s^2}{Ms^2 + Bs + K} (A/s^3) = \frac{1}{Ms^2 + Bs + K} (A/s) \qquad (14)$$

This is the same result as the function for velocity with a ramp input (Equation 10), or for displacement with a step input (Equation 4). Therefore, the acceleration responses of the cars to parabolic inputs are the same as the displacement responses of the cars to step inputs as shown in Figure 2.

D. Summary of Steady-State Differences.

The results of the steady-state differences (analogous for illustrative purposes to traditional steady-state errors in control systems) of the displacement, velocity, and acceleration responses for step, ramp, and parabolic inputs for this illustrative car example are summarized in Table I.

Table I. Summary of Steady-State Differences

	Step	Ramp	Parabolic
Displacement	Infinite	Infinite	Infinite
Velocity	Finite	Infinite	Infinite
Acceleration	Zero	Finite	Infinite

The results in Table I agree whether obtained by examining the responses in the time-domain as t approaches infinity, or obtained in the s-domain by applying the Final-Value Theorem of Laplace Transform Theory.[3-5] As expected, rapidly changing inputs make it difficult for Car #2 with its larger mass and greater air drag to match the performance of Car #1.

E. Implementation of Changes to Impact the Car Performances.

The performance of control systems depends on the nature of both the system and its input, as illustrated by varying the car and input parameters of this illustrative example, such as follows:

1) Impact of Car Parameters on Performance: As observed in the above figures, the displacement, velocity, and acceleration of Car #2 with its larger mass and greater damping (air drag) system characteristics lag those of Car #1 for identical inputs. One way to enable Car #2 to catch up with Car #1 in performance is to redesign it to use lighter materials to reduce its mass and to make it more streamlined to reduce its air drag.

To model this scenario, select the following identical parameter values for the two cars, with the parameters of Car #2 equal to those of Car #1:

Car #1:	Car #2:
$M_1 = 1$; $B_1 = 1$; $K_1 = 0$	$M_2 = 1$; $B_2 = 1$; $K_2 = 0$

To illustrate the car performances for this scenario, the drivers apply step inputs of equal amplitudes to the gas pedals, specifically $A_1 = 1$ for Car #1 and $A_2 = 1$ for Car #2.

As expected with identical parameters values for both cars, the displacements, velocities, and accelerations of the cars for step inputs of equal amplitudes are identical to those of Car 1 in Figures 2, 3, and 4, respectively.

2) Impact of Input Parameters on Performance:

a) Displacements of the Cars for Different Step Inputs:

As observed in Figures 2-6, the displacement, velocity, and acceleration of Car #2 with its larger mass and greater damping (air drag) system characteristics lag those of Car #1 for identical inputs. Another way to enable Car #2 to catch up with Car #1 in performance is for the driver of Car #2 to apply more input to the gas pedal than by the driver of Car #1.

To model this scenario, select the same parameter values as applied in Sections III.A and III.B for the two cars:

Car #1:	Car #2:
$M_1 = 1$; $B_1 = 1$; $K_1 = 0$	$M_2 = 1.25$; $B_2 = 1.25$; $K_2 = 0$

To illustrate the car performances for this scenario, apply a step input of amplitude $A_1 = 1$ for Car #1 and a step input of larger amplitude $A_2 = 1.25$ for Car #2.

With different parameters values for the cars, the displacements, velocities, and accelerations of the cars for step inputs of the selected compensating amplitudes are identical to those of Car 1 in Figures 2, 3, and 4, respectively.

b) Displacements of the Cars for Step versus Ramp Inputs:

Another way to enable Car #2 to catch up with Car #1 in performance is for the driver of Car #2 to increase the input to the gas pedal over time, eventually exceeding the input by the driver of Car #1.

To model this scenario, select the same parameter values for the two cars as used in Section III.E.2.a.

To illustrate the car performances for this scenario, apply a step input of amplitude $A_1 = 1$ for Car #1 and, for example, a ramp input of slope $A_2 = 0.5$ for Car #2. The resulting displacements of the two cars are shown in Figure 7.

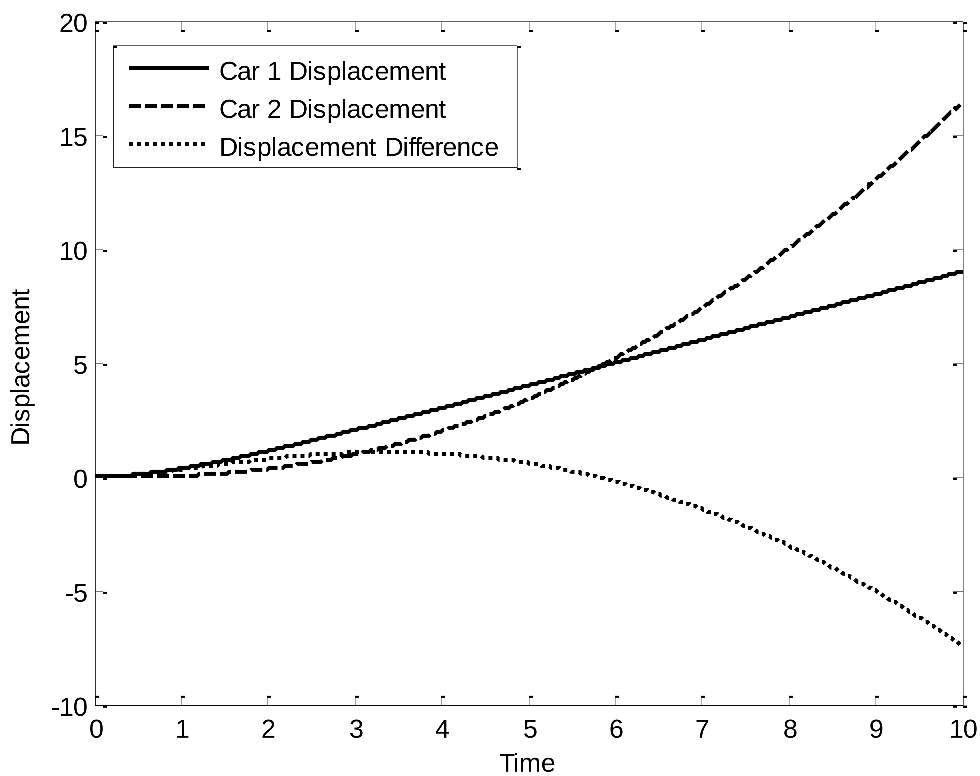

Fig. 7. Displacements of the cars for step versus ramp inputs.

As observed with different parameters values for the cars, the displacements of the cars for step versus ramp inputs are different as shown in Figure 7. Car #2 eventually catches, and passes, Car #1. Similarly, the velocities and accelerations of the cars for step versus ramp or parabolic inputs will be different.

IV. Assessment and Evaluation of the Car Race Analogy

A copy of the manuscript of this car race analogy paper was distributed to the students prior to analytical coverage of steady-state errors. The nature and differences of the transient responses and their steady-state errors for various system characteristics and types of inputs were then explained to them as a tutorial using the graphical results in this paper.

To assess and evaluate the effectiveness of the car race analogy to explain steady-state errors in automatic control systems, the students were asked to provide their opinions using a survey form. Space was available on the form for them to also provide comments and suggestions.

The course enrolled 14 students, consisting of 6 women and 8 men in 2010, and 20 students, with 7 women and 13 men, in 2011. Nineteen (56%) of the 34 combined number of students enrolled in this fall semester, 3-credit hour, course responded to this voluntary survey shown in Table II.

Table II. Survey to Assess and Evaluate the Effectiveness of the Steady-State Error Analogy

EGR 404 Automatic Control Systems (3 credit hours)
(Combined Results for Fall Semesters 2010 and 2011)

Questions:	Very Much	Acceptably	Very Little	Not at All
1. Did you have any previous knowledge of steady-state errors?		7	8	4
2. Did the analogy motivate you to learn more about steady-state errors?	6	12	1	
3. Did it increase your understanding of the <u>sources</u> of steady-state errors in automatic control systems?	9	8	2	
4. Did it increase your understanding of the <u>effects</u> of steady-state errors on the performance of automatic control systems?	11	8		
5. Do you recommend the use of this analogy in future offerings of the course?	10	9		

Comments about the Effectiveness of the Analogy:

- This analogy was very understandable and related to real life examples. This made it easy to comprehend. The analogy was well researched and made a clear relationship between negative feedback and stability. It was also amazing how it applied to different order systems.
- Was very helpful and useful for me in my senior project class.
- It is indeed useful. I still visualize it when working with control/feedback systems in other classes.
- The analogy was very easy to imagine in the case of the car race because I drive a car and have first-hand knowledge of how driving a car works. By taking an analogy that was easily translated and related to everyday life, I believe I was more accurately able to understand steady-state errors.
- The analogy helped me to understand what would happen due to errors, but not what caused them.

- I could relate to it just fine. It made sense and I saw the connection easily.
- I think if effectively illustrates the problem of steady-state errors. It helps to picture and bring physical meaning to steady-state error which is a hard problem to understand.
- Applying the concept of steady-state errors to something that everyone has a general understanding of, racing cars, is very helpful.
- Very simple analogy; makes the concept easy to understand.
- I liked the analogy because it is easy to understand. Most people can grasp the idea of how weight and engine performance can affect the acceleration and velocity of a car.
- The analogy offers a better understanding of steady-state errors because the analogy is based on the well known concept of cars with displacement, velocity, and acceleration.
- Easy to relate to and understand.
- The analogy really described the importance and different effects of steady-state errors. The different examples showed the variation of the errors.
- The analogy was very clear; I liked how one car was used as a baseline to compare with car 2.
- Good description of each scenario before analysis was shown. Helpful introduction to get us started in understanding steady-state errors. <u>Very, very</u> helpful comments about what we should notice in each graph, even in comparison to others.
- I thought it was great to relate topics such as position, velocity, and acceleration to a system familiar to everyone – cars.
- It was easy to understand.

Suggestions for Improvements of the Analogy:

- Though the analogy was very easy to understand and imagine, I think it could be bettered in the way you present the analogy. Try and find a way to keep it fun and engaging.
- Perhaps use an analogy that would help the students to understand both what causes errors and what would happen to the system due to errors.
- I have none, sorry, it's a great analogy.
- Make a lab demonstration.
- I found it a little repetitive; however, it was very clear.
- Make a MATLAB® code we can play with to graph it on a computer and have the students interact with it more and see how the graphs change in that sense.
  (Author's Note: The sample MATLAB® program that is included in the Appendix of this paper was distributed to the students as part of the manuscript of this paper.)

As expected, and as a good starting point for learning a new topic, the majority (12 of 19 or 63%) of the students had combined "very little" or "not at all" prior knowledge of steady-state errors as indicated by the results for question #1 in Table II. And as the results for questions #2, 3, 4, and 5 indicate, the vast majority of the students found the car analogy to be helpful in illustrating how an automatic control system can have a steady-state error, even if the system could be built using perfect components, based solely on the nature of the system and its input. Table II shows that the combined results for the "very much" and "acceptably" categories are

18/19 or 95%, 17/19 or 89%, 19/19 or 100%, and 19/19 or 100%, respectively, for questions #2, 3, 4, and 5.

Based on student feedback, such as above, the author continues to refine and enhance the nature and presentation of this car race analogy with each offering of the course. Since the students have experience driving, or at least riding in cars, none of them have ever expressed any written or verbal misgivings about the use of the car race analogy based on their gender, minority status, nationality, etc. The goal is to provide a valuable conceptual understanding prior to detailed analytical coverage of steady-state errors in control systems.

V. Summary

Through an illustrative car race example (or, if preferred, runners in track or cross country events) that is familiar to students through life experiences, the author has presented a graphical example that allows students to discover the origin and nature of steady-state errors that vary with the system and input characteristics of control systems. This flexible example also enables students to explore various compensation measures to reduce or eliminate such errors. As an additional benefit, students can view the derivative and integral relationships between the sets of graphs that relate displacement, velocity, and acceleration to complement and reinforce their learning in calculus, physics, and engineering. This multi-faceted example is designed to appeal to the preferred learning style of most engineering students, namely visual, sensing, inductive, and active.

Acknowledgment

The author acknowledges the participation of his students in automatic control systems during the past several years in helping to refine the graphical illustration of steady-state errors demonstrated in this paper.

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Appendix

A sample MATLAB® program, used in this paper to generate the displacement responses of the two cars to unit step inputs in Figure 2, is:

```
M1 = 1;
B1 = 1;
K1 = 0;
num1 = [0 0 1];
den1 = [M1 B1 K1];
den1 = conv(den1,[1]);

M2 = 1.25;
B2 = 1.25;
K2 = 0;
num2 = [0 0 1];
den2 = [M2 B2 K2];
den2 = conv(den2,[1]);

t = 0:0.01:10;

step(num1,den1,t);
step(num2,den2,t);

y1 = step(num1,den1,t);
y2 = step(num2,den2,t);

y3 = y1 - y2;

plot(t,y1,'k-',t,y2,'k--',t,y3,'k:','LineWidth',1.5)

x = t;

xlabel('Time')
ylabel('Displacement')

legend('Car 1 Displacement','Car 2 Displacement','Displacement Difference','Location','Best')
```