Use of Excel and MATLAB to Design General Linkage Systems for Orthopedic Devices

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Abstract

This paper describes a generalized method for calculating the static and dynamic behavior of orthopedic systems. The orthopedic system under consideration is a flexor hinge hand splint, an assistive device for individuals who have limited use of their hands due to spinal cord injury. Typically, these systems take force from wrist movement and translate this force to the fingertips for gripping. However, for those individuals who cannot supply enough force from the wrist, a small motor-based device can be added. In designing such a device, concerns arose over the force applied in the linkages and joints of the system. It became necessary to analyze how the forces on the linkage system varied throughout its range of motion. To accomplish this analysis, a MATLAB code[1] was written and was later translated into Excel (using Visual Basic Macros).

The MATLAB code borrowed numerical techniques from a variety of sources. The numerical techniques used include root finding methods, Boolean or connectivity matrices, and cross products for use in moment balance equations. A general outline of the overall approach is as follows. Each pin joint is labeled numerically and a 3-D vector to each pin is constructed. Each link in the overall linkage system is numbered as well. A connectivity matrix is constructed describing which pin joints are members of which link. This numerical configuration is exactly that which exists within the finite element method. When the orthopedic device begins its movement the connectivity matrix never changes. However, the position of each pin joint will change relative to other points on the linkage. During the movement of the system rigid links must remain rigid. Rigid links yield constraint equations in that all pins on that link remain the same distance from each other. For well-constructed linkage systems, specification of one link's motion will in turn describe the motion of all links on the system. Throughout the range of motion, the force in each link and pin joint is calculated based on the applied torque of a motor system.

1. Introduction

In the course of seeking an engineering degree all students at Arkansas Tech University must complete a senior design course[2]. The final product of this course is usually a finished or paper design. In spring 2000 an orthopedic company[3] residing in Hot Spring, Arkansas approached the author about motorizing an orthopedic device. This paper is a result of the methods used in the design of the system. The original design has been in production for years, shown in figure 1. This design consists of a four bar linkage system connected in such a way that a reverse wrist movement results in the finger and thumb closing. This results in the patient being able to grip objects through this wrist movement.

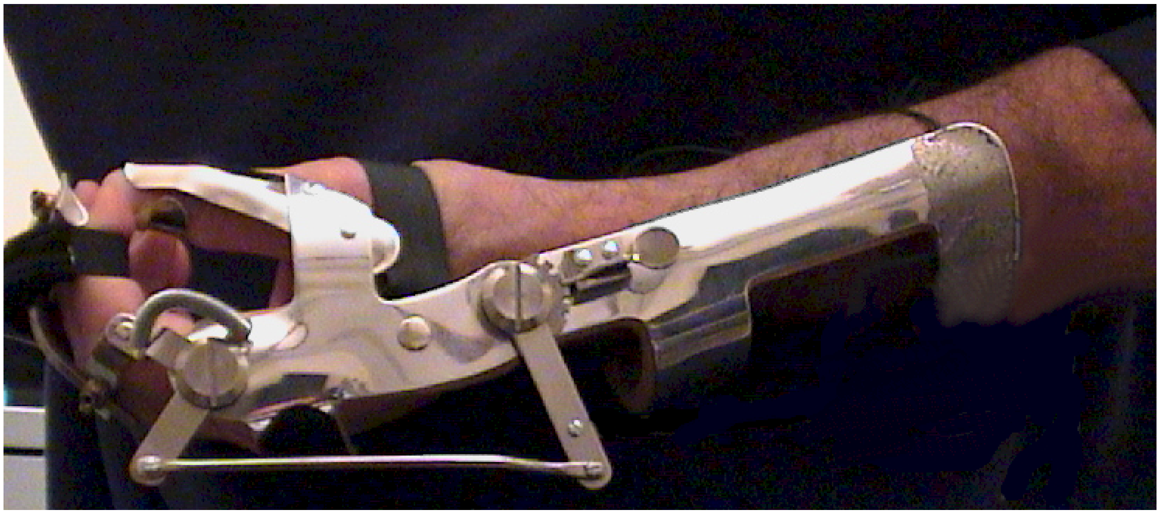

Figure 1

The client wished to motorize this system while maintaining its same basic structure. Installing a motor on this system may increase the forces on the linkages and the force applied by the patient on the object. Therefore, in this design process it is imperative to determine these forces so the patient does not break the flexor hinge or even worse, hurt their fingers by placing too much force in their grip. The major focus of this paper is a methodology of determining the kinematics and force in each member in the linkage.

2. Methodology

The present method is general with regards to any geometry under consideration. This flexibility is incorporated in the method considering the possibility of altering the design away from a four bar linkage system. The present design is shown in figure 2 with angles α, β, θ, δ, and Δ. During the movement of the system all angles vary except Δ and α. The angles are all interdependent such that the length of all linkages in the system remains fixed. This fact is used in the calculations as discussed below. Figure 3 shows the same drawing including pin joint numbers and linkage (Rod) numbers. The pin numbers are shown in circles and the linkage numbers are shown without circles. In this figure a connectivity matrix is shown. This type of matrix is often used in finite element method. Each row of the matrix represents a linkage (Rod) and the two entries in that row represent the two pin joints connected on each end. Pin 9 represents a ghost or false pin. This pin is only listed as a convenience for calculating the location of pin 8, the thumb. This data structure represents the entire configuration of the system. If the coordinate of each pin joint is known as well, the entire system is defined.

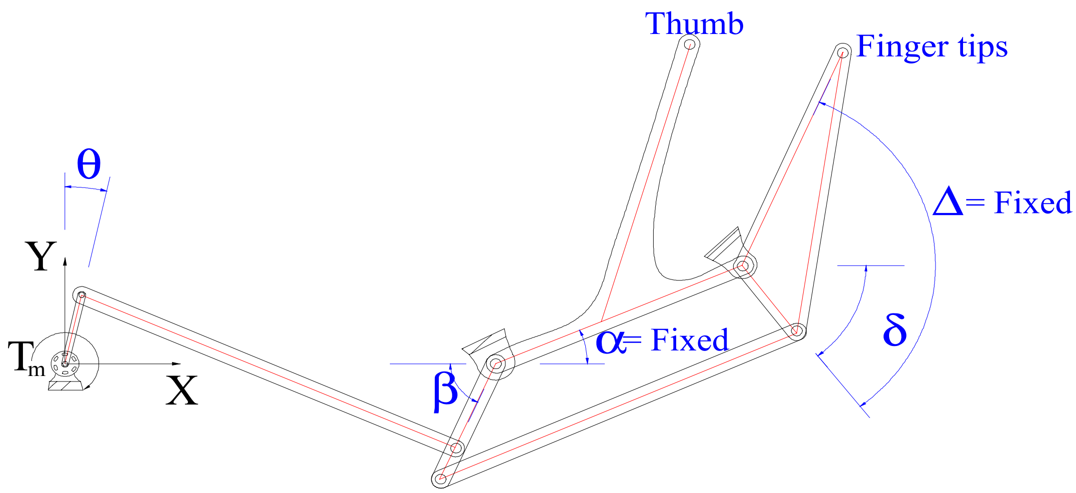

Figure 1

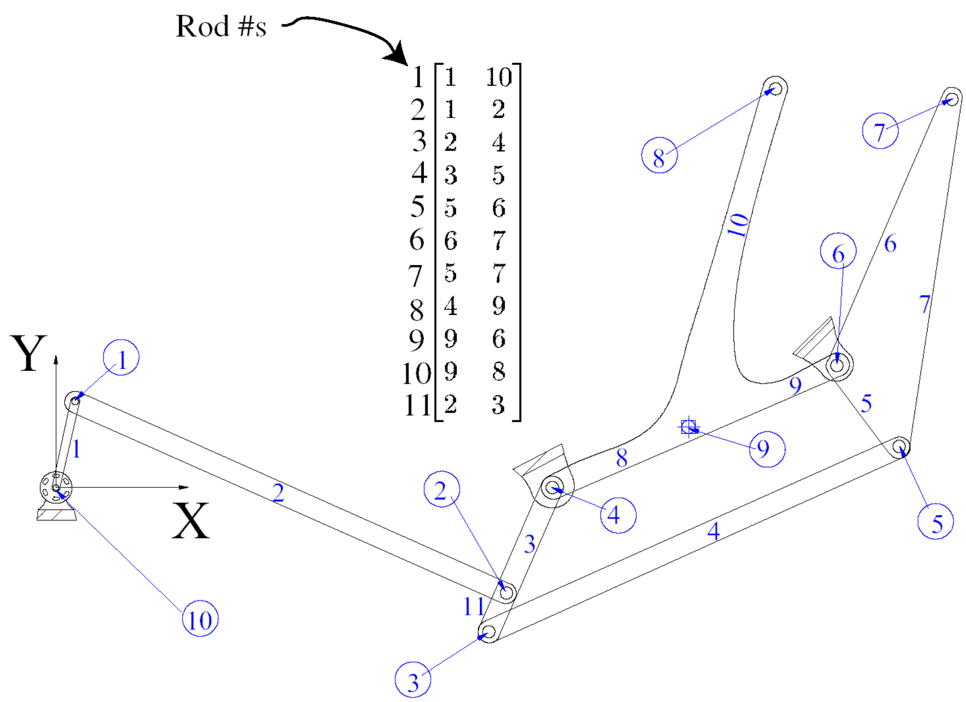

Figure 2

With a coordinate system defined at the center of the motor the location of each pin may be expressed using the angles in figure 2. These nine equations are shown below.

$$\bar{r}_1 = L_1 \sin\theta \ \hat{i} + L_1 \cos\theta \ \hat{j}$$
$$\bar{r}_2 = [5.62 - L_3 \cos\beta] \ \hat{i} - L_3 \sin\beta \ \hat{j}$$
$$\bar{r}_3 = [5.62 - (L_3 + L_{11}) \cos\beta] \ \hat{i} - (L_3 + L_{11}) \sin\beta \ \hat{j}$$
$$\bar{r}_4 = 5.62 \ \hat{i}$$

$$\overline{r}_5 = [(L_8 + L_9)\cos\alpha + 5.62 + L_5\cos\delta]\ \hat{\imath} + [(L_8 + L_9)\sin\alpha - L_5\sin\delta]\ \hat{\jmath}$$
$$\overline{r}_6 = [5.62 + (L_8 + L_9)\cos\alpha]\ \hat{\imath} + (L_8 + L_9)\sin\alpha\ \hat{\jmath}$$
$$\overline{r}_7 = [5.62 + (L_8 + L_9)\cos\alpha + L_6\cos(\Delta - \delta)]\ \hat{\imath} + [(L_8 + L_9)\sin\alpha + L_6\sin(\Delta - \delta)]\ \hat{\jmath}$$
$$\overline{r}_8 = [5.62 + L_8\cos\alpha + L_{10}\cos(\alpha + \gamma)]\ \hat{\imath} + [L_8\sin\alpha + L_{10}\sin(\alpha + \gamma)]\ \hat{\jmath}$$
$$\overline{r}_9 = [5.62 + L_8\cos\alpha]\ \hat{\imath} + L_8\sin\alpha\ \hat{\jmath}$$

Equations 1

In addition to the position of each pin joint, the relative position must be such that the linkages 2 and 4 do not change in length. A constraint equation can be written for each of these linkages. For link 2 the equation is

$$(L_2)^2 = [L_1\sin\theta - 5.62 + L_3\cos\beta]^2 + [L_3\sin\beta + L_3\sin\beta + L_1\cos\theta]^2.$$

Given the rotation angle of the motor θ, solving this constraint equation determines the angle β. Since a closed form solution to the angle β is too complicated and not general in scope, an approximate solution will be obtained. A bisection type method is used to search for an angle β such that this equation is satisfied. Once the angle β is determined a constraint equation for link 4 is solved. This equation is shown below.

$$(L_4)^2 = [(L_8 + L_9)\cos\alpha + L_5\cos\delta + (L_3 + L_{11})\cos\beta]^2 + [(L_8 + L_9)\sin\alpha - L_5\sin\delta + (L_3 + L_{11})\sin\beta]^2$$

This equation is solved using a bisection method for the angle δ. Once all the angles α, β, θ, δ, and Δ have been determined, the location of the new pin points can be determined from the group of equations 1. Using these new positions and the connectivity a new diagram of the motion can be plotted.

3. Force Balance

In addition to the kinematics a moment/force balance can be calculated. Since the movement of the device is slow and the basic components are lite, the motion is considered quasi-static. In addition, the torque on the motor is a function of the force applied to the finger by an object the user wishes to grasp. Therefore, the worst case in operation is during a static grasp of an object bringing the motor to its stalling torque. These fact all accentuate the validity of a quasi-static assumption.

Using the definition of the moment equation each link is broken down and each pin joint is analyzed according to the following moment equation.

$$\sum \overline{M}_{pin} = \sum \overline{r}_i \times \overline{F}_j = 0$$

Appling this equation to each pin joint we get at pin 10,

$$\sum \overline{M}_{10} = 0 = -T_m + \overline{r}_1 \times \overline{F}_2.$$

Resulting in the determination of the force in linkage 2 as a function of the torque in the motor.

$$F_2 = -\frac{T_m L_2}{\overline{r}_1 \times \overline{r}_2}$$

At pin 4 the moment equation gives the following.

$$\sum \overline{M}_4 = 0 = \overline{r}_{3/4} \times \overline{F}_4 + \overline{r}_{2/4} \times \overline{F}_2$$

Hence, the force in link 4 can be determined as a function of the force in link 2,

$$F_4 = -\frac{F_2 L_4}{L_2} \left[\frac{\overline{r}_4 \times \overline{r}_1 - \overline{r}_4 \times \overline{r}_2 - \overline{r}_2 \times \overline{r}_1}{\overline{r}_4 \times \overline{r}_5 - \overline{r}_3 \times \overline{r}_5 - \overline{r}_4 \times \overline{r}_3} \right].$$

Finally, summing moments about pin 6 gives

$$\sum \overline{M}_6 = 0 = \overline{F}_f L_6 + \overline{r}_{5/6} \times \overline{F}_4,$$

Resulting in the force at the finger tips (F_f) being a function of the force in linkage 4 as shown in the equation below.

$$F_f = \frac{F_4}{L_2 L_6} \left[\overline{r}_6 \times \overline{r}_3 - \overline{r}_5 \times \overline{r}_3 - \overline{r}_6 \times \overline{r}_5 \right]$$

From this set of equations the force at the fingers can be determined as a direct function of the torque of the motor. Using these equations, the force can be determined at any point in the movement of the system. One particular geometric configuration was run and gave the plots in figure 4 and 5. These forces were determined based on a motor applying a 120 oz-in torque. These two plots show the compressive load in each rod as a function of the motor angle (in degrees). Although these plots are useful in design, they do not describe the patient's experiences. The patient experiences a force related to the opening of the finger-thumb combination. A plot of this relationship is shown in figure 6. This figure shows a very large force in the system at the finger-thumb combination for an opening greater than 3 inches. This plot alone shows the need for a computational tool described in this paper. A force this large may not be acceptable for an actual working system. Also the force in the finger-thumb combination for smaller distances may be too small thereby forcing a possible redesign of the system.

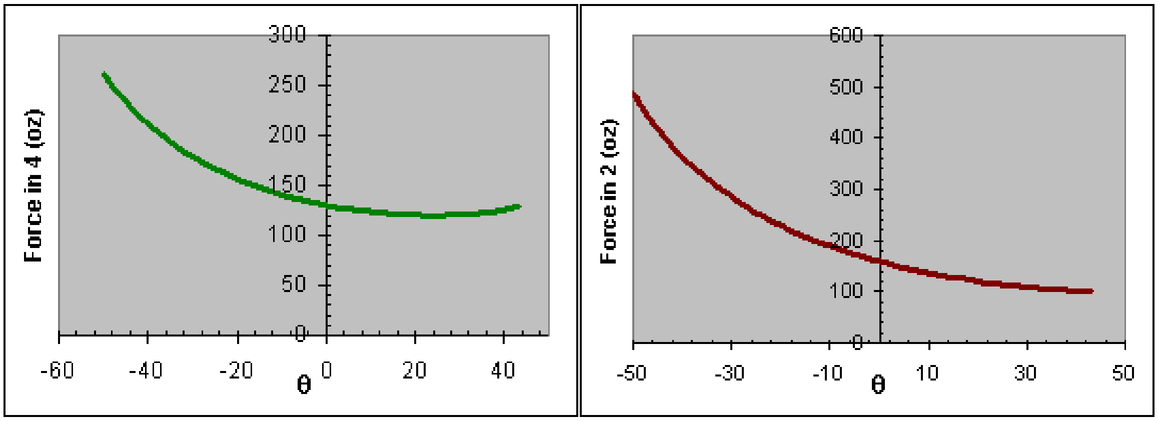

Figure 4 & 5

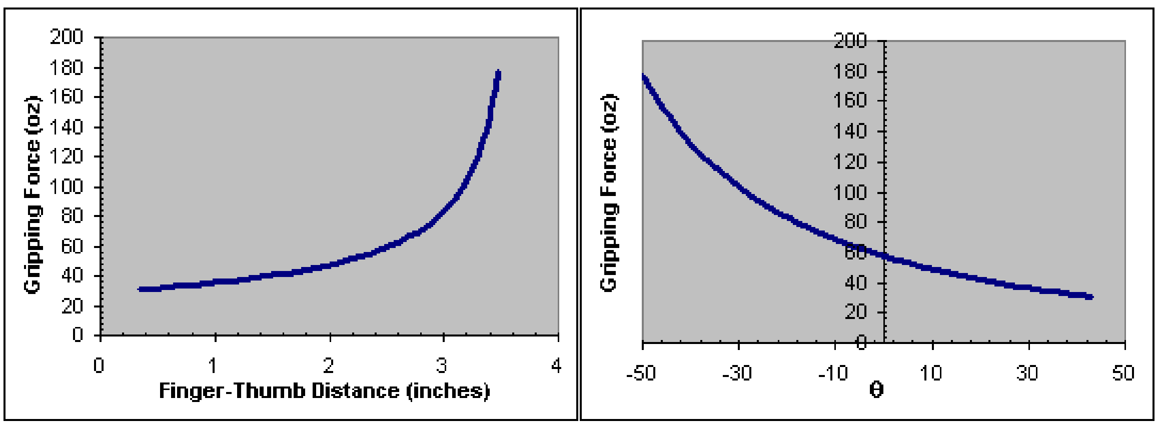

Figure 6 & 7

4. Conclusions

A generalized method for determining the behavior of a linkage system has been presented. This method involves the use of constraint equations for linkage lengths. These equations depend on a vector description for each pin joint in the system. Each linkage and node is numbered and these numbers are placed into a connectivity matrix. This matrix has the same structure as a connectivity matrix commonly used in the finite element method. A particular configuration of the system is simulated and plots of the force in each member as well as the gripping force are shown. This method should be easily adaptable for arbitrary linkage systems.

Bibliography

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3. Jaeco Orthopedic, “Products Catalog” 214 Drexel, Hot Spring, AR 71901

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Bob Fithen is an assistant professor at Arkansas Tech University. He received his B.S. in Mechanical Engineering from Louisiana Tech University, M.S. in Mechanical Engineering from Texas A&M University, and his PhD in Engineering Mechanics from Virginia Tech University. He spent four years working at General Dynamics, Fort Worth and a total of five years working in the research division of Wright Laboratories in Dayton, Ohio. Further information may be obtained at http://mengr.atu.edu