

Use of FEA in an Introductory Strength of Materials Course

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Abstract

The new generation of PC-based computer aided engineering software is relatively easy to learn and use. Most have robust graphics user interfaces and adequate CAD modules that greatly enhance the teaching and learning process. These facts coupled with product availability, relatively low cost, and reliability make them a convenient tool to be included within the curriculum of many introductory engineering technology courses.

This paper describes how one such computer program in the area of structural finite element modeling and analysis may be included within an introductory course in strength of materials. The software is used to solve one and two-dimensional structural problems. Students enrolled in this course acquire sufficient knowledge within the field of finite element related stress analysis to proceed on to more challenging experiences in other courses or industry.

Nomenclature

A_0 - Cross-section area of tension member
 d - Nodal deflection
 E - Modulus of elasticity
 F - Global nodal force
 f - local nodal force
 L_0 - Original length of tension member
 P - External tension axial force load
 u - deflection in the local x direction
 v - deflection in the local y direction
 δ - Axial deformation of tension member
 σ - Normal stress
 θ - Orientation angle of truss element measured w/r to the x-axis

Introduction

The method of finite element analysis may be introduced into the curriculum of an associate degree mechanical engineering technology program. The use of a commercially available, windows-based, FEA computer package could complement other computer software typically used in most engineering technology programs. The NINTENDO factor would thus be reinforced and perhaps additional interest generated. An existing academic program was used as a model to help describe the pedagogical approach taken within this paper. It contains a sequence of courses that are vital to the success of this approach by providing students with the necessary background to gain basic working knowledge of the analysis method. These courses should introduce the following general topics: engineering materials, statics, strength of materials, and machine design. Materials testing such as tension, impact, shear, and fatigue are introduced within the engineering materials course. The

strength of materials course provides instruction in the various mathematical relationships and procedures that may be used to perform stress analysis on structural parts of simple geometry. Instead of the usual test laboratory a computer laboratory module is provided. To help prepare students for use of the computer analysis program the concept of finite element structural analysis is introduced through simple examples of two-dimensional truss structures. Two-dimensional matrix truss analysis is introduced early in the course by the definition of a two-dimensional truss element. The necessary computer program commands are introduced through scripted examples that are designed to complement manufacturer supplied documentation. The academic model used allows students to gain additional finite element analysis experience within a first course in machine design. An outline describing the basic sequence of events and the general intent of each is provide.

Overview

The following outline describes the fundamental pedagogical steps that may be taken to help prepare students to deal with structural analysis problems using the method of finite elements. Clearly the general topics listed below are contained within the curriculum of most associate degree mechanical engineering technology programs.

1. Ample instruction and hands-on experience with materials testing should be provided, especially in the area of tension tests.¹
2. A firm foundation in the area of equilibrium force analysis of static structures should be provided. The concepts of structural system and structural component equilibrium should be emphasized.²
3. A thorough understanding and comprehension of material mechanical properties and Hooke's law as related to loads and displacements of structural components is necessary.³
4. A basic understanding of index notation and matrix algebra operations is necessary.⁴
5. Derivation of the local truss element equations using Hooke's law and matrix representation of transformation relationships should be presented.⁵
6. Truss element equilibrium equations should be derived in matrix form.
7. The global equilibrium nodal force/displacement equations should be developed in matrix form.
8. The matrix analysis procedure should be illustrated through the use of examples.
9. Hand computed matrix analysis examples should be reinforced with scripted examples of the computer commands necessary to solve the same problems.⁶
10. The finite element analysis concept may be extended by the introduction of the triangular plane stress element. The introduction of this element should be timed to coincide with lecture discussions of plane stress. To help students gain additional appreciation for the increased complexity of the triangular plane stresses element the element stiffness matrix may be introduced. Emphasis should be placed on the necessity of computer program application.
11. Tutorial examples of the computer software may be provided to the students so plane stress structural models and analyses may be performed.
12. Students within the academic model program gain further use of the computer FEA software within a capstone design project. Typically, a simple machine component is designed by hand computation and verified by use of the computer.⁷

While each of the items of the above outline are of equal importance only items 3 through 9 are emphasized within this paper. Only enough detail is provided so a general understanding of the proposed pedagogical procedure may be gained. If presented with continuity, the beginnings of a firm foundation in FEA should result.

Item 3 - Tension Tests

The most natural and perhaps the most revealing test of material mechanical properties is the tension test. A strip or cylinder of the material, having length L and cross-section area A is constrained at one end and subjected

to an axial pull-load, P . A gradual increase in the load is accompanied by a corresponding increase in the axial deflection, δ . At some value of P the material begins to behave in a way that would be very detrimental to a structure. The value of P for which this happens is referred to as the ultimate load. This value is dependent not only on the material but also on the size and shape (cross-sectional area) of the test specimen. Thus when reporting the strength of materials loaded in tension it is customary and convenient to account for the area effect by dividing the ultimate load P_u by the original cross-sectional area A_o , i.e. $\sigma_u = P_u / A_o$

The difference between stiffness, which is a measure of the load needed to induce a given deformation in the material, and strength, which usually refers to the material resistance to failure by fracture or excessive deformation, is very important. The stiffness is measured experimentally, by applying known axial loads well short of fracture and measuring the resulting axial deformation. To good approximation, the load P and the resulting deformation δ are related linearly. This relationship is generally known as Hooke's law and may be written as:

$$P = k\delta \quad 1.$$

k is a constant of proportionality called stiffness. The stiffness as defined by k is not a function of the material alone but is also influenced by the specimen shape. A useful way to adjust the stiffness so as to be a purely materials property is to define a parameter that is area independent. This may be accomplished by dividing the load by the stressed area. Further, the deformation δ may be made independent of the original length, L_o , by defining a deformation per unit length $\epsilon = \delta / L_o$. ϵ is called strain. Using these more general measures of load per unit area and displacement per unit length Hooke's law becomes:

$$P = (A_o E / L_o) \delta \quad 2.$$

E is a function of material alone and may be determined from tension tests on various materials. Typically E is called the modulus of elasticity.

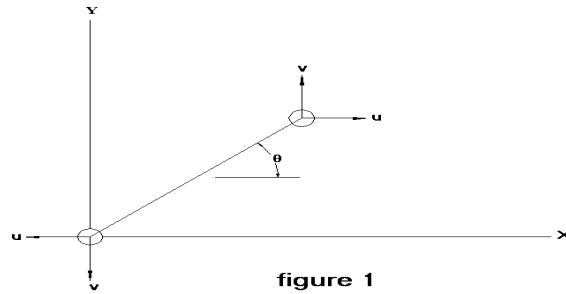
Items 4 - Matrix Algebra

At the associate degree level, little information regarding index notation and matrix algebraic operations is provided. The rules that apply to multiplication of matrices are very useful in the representation of large numbers of equations in a compact form and in a form that lends itself to index looping within a computer program.

The general rule for matrix multiplication may be stated as follows: the multiplication of an $M \times N$ matrix, \mathbf{A} , containing elements a_{ij} by an $R \times P$ matrix, \mathbf{B} , containing elements b_{ij} is defined only if $R = N$ and always results in a new $M \times P$ matrix, \mathbf{C} , whose elements, c_{ij} , are given by the equation $c_{ij} = \sum a_{ik} b_{kj}$ for $k = 1$ to n and $n = P$. In general the matrix product AB is not equal to BA . Several examples of matrix multiplication should be provided and explained in detail. Students should be required to hand-compute several practice examples.

Items 5-7 - Matrix Truss Analysis

In matrix truss analysis the preceding rules and conventions may be used to evaluate the forces that accompany known imposed displacements and to evaluate the displacements at the joints or nodes where known external forces are applied. The key to this method is the precise definition of the single truss element such as the one shown in figure 1. Hooke's law in the form of equation 2 is used to relate nodal forces and displacements. Index notation and matrix multiplication are used to represent the truss element equilibrium equations in terms of the nodal forces, element stiffness matrix, and nodal displacements as in equation 3.



The nodal force components for a single element may be represented by equation 3 below.

$$\begin{bmatrix} f_x \\ f_y \\ f_x \\ f_y \end{bmatrix} = (AE/L) \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{bmatrix} \quad 3.$$

The product of (AE/L) and the 4 x 4 coordinate transformation matrix is called the element stiffness matrix. The assembly of elements into a system that represents the complete structure will result in a set of force/displacement, linearly independent, equilibrium equations that may be solved by any one of several methods.

These global equations will be indexed with a global numbering system. The number of degrees of freedom of the complete structure will determine the range of the indices. Thus if the truss contains 3 nodes it will have six degrees of freedom and the indices will range between 1 and 6. The global stiffness matrix will be square with dimensions M x N where M = 6 and N = 6.

Item 8 - Matrix Truss Analysis

Examples of two dimensional, three-element, truss structure like the one shown in figure 2 may be used to illustrate the assembly and solution procedure. It is required that the vertical and horizontal displacements of

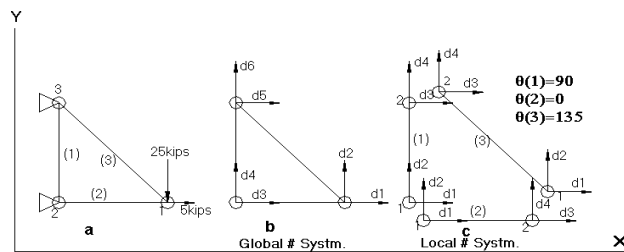


figure 2

node 1 and the force reactions at nodes 2 and 3 be computed. Figure 2a shows the labeled elements and nodes. Figure 2b shows the global number system to be used for the nodal displacements while figure 2c shows the corresponding local numbering system for nodal displacements and defines the element direction angles (θ). A nodal degree of freedom transformation table may be used to help students track the matrix assemble process. For the example problem shown in figure 2 the following data apply:

element (1) $A = 5 \text{ in}^2$, $E = 10\text{e}6 \text{ psi}$, $L = 100 \text{ in}$
 $\theta^{(1)} = 90^0$, $c = 0$, $s = 1$

element (2) $A = 6 \text{ in}^2$, $E = 30\text{e}6 \text{ psi}$, $L = 100 \text{ in}$
 $\theta^{(2)} = 0^0$, $c = 1$, $s = 0$

element (3) $A = 4 \text{ in}^2$, $E = 30\text{e}6 \text{ psi}$, $L = 141 \text{ in}$
 $\theta^{(3)} = 135^0$, $c = -0.707$, $s = 0.707$

The stiffness matrices may be evaluated for all three elements by applying equation 3. These local stiffness matrices may be assembled to form the global stiffness matrix. Multiplication of the global stiffness matrix and the global displacement vector will produce six equilibrium equations to which boundary conditions may be applied. The boundary conditions are: $d_3 = d_4 = d_5 = d_6 = 0$ and $f_1 = 5 \text{ kips}$, $f_2 = -25 \text{ kips}$. The required nodal displacements, d_1 and d_2 , may be computed. The nodal reaction forces at the supports may then be computed by “back” substitution of d_1 and d_2 into the remaining equilibrium equations. As may be inferred from the above discussion a significant amount of time may be devoted to the computational procedure as well as the physical interpretation of the results.

Item 9 - Computer Software

The computer software used was ANSYS Multi-Physicstm release 5.3. Windows NTtm version 4.0 was the operating system. The computer laboratory used within the course is equipped with computers configured with Intel-586, single processor, 233MHz chips, 96 megabytes RAM, and 2 gigabyte hard drives. Each machine is equipped with a 17-inch monitor and 3-D graphics card. Students were introduced to the use of the software through scripted examples. Once sufficient knowledge of the basic pre-processing and post-processing phases had been gained they were assigned additional problems to solve on their own.

The following outline summarizes the basic steps that should be taken to model and analyze engineering problems using the ANSYStm software. This outline was distributed to the students and used to help define the software modeling and analysis procedures.

1. Specify a job name
2. Set Preferences
3. Define element type(s) and options
4. Define real constants
5. Define material properties
6. Create geometry
7. Mesh geometry
8. Apply constraints
9. Apply loads
10. Activate the solver engine
11. Display results in various ways

Tables 1 and 2 show a summary of the numerical nodal displacement and reaction force results obtained from a computer analysis of the example outlined in item 8.

Nodal Displacement Results (in.)

NODE	UX	UY
1	-.11111E-01	-.70037E-01
2	.00000	.00000
3	.00000	.00000

Table 1

Nodal Reaction Results (lb)

NODE	FX	FY
2	20000.	.00000
3	-25000.	25000.

Table 2

Conclusion

A method has been presented for the introduction of computer aided engineering into an associate degree course in strength of materials. This method may be used to present difficult-to-understand concepts related to structural finite element analysis. A commercially available computer program and a method for its inclusion within the curriculum of the course were suggested.

It has been suggested that the use of computer analysis software within the laboratory portion of the course may result in an increase of student interest. This method is currently being used within the academic model program in an introductory strength of materials course. There are fourteen students enrolled in the course. To date only informal student comments have been obtained related to the acceptance and success of the use of analysis software within the laboratory. It appears, at this time, that increased interest and enthusiasm has been generated. A formal class/instructor evaluation is planned at the end of the course. Student comments regarding the use of FEA software will be solicited at that time.

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