Using an Integrated Engineering Curriculum to Improve Freshman Calculus

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ABSTRACT

This paper addresses the following question: What are some of the ways that the beginning calculus course for engineers can be improved, if it is part of an integrated curriculum that also includes physics, engineering, and chemistry courses? The authors have had the opportunity to participate in such an integrated curriculum at Texas A&M for the past two to four years. Several major changes were made in the first-year calculus sequence in order to present various topics at the times they were applied in other courses. We have found that these changes not only serve the needs of the partner disciplines, but also provide a more unified and coherent treatment of some topics from the point of view of mathematics itself. Vectors, parametric curves, line integrals, and especially centers of mass and moments of inertia are topics that students traditionally find difficult, unmotivated, or confusing because of inconsistent notation or terminology in different courses; covering them "early" actually improves their presentation. Other topics, such as multiple integrals, orthonormal bases, ordinary differential equations, and numerical approximation of derivatives and integrals, can be introduced in a motivated way in preparation for their more in-depth treatment in later years. Following "learning cycle" and "learning style" ideas, we have made an effort to provide more motivation and practice within the mathematics course; but the most effective and efficient motivators and practice fields are coordinated courses in other disciplines where the mathematics is actually used.

INTRODUCTION

We were recently presented with the challenge of rethinking how to teach calculus to freshman engineering students as part of an integrated curriculum. We immediately saw this as an opportunity to motivate calculus concepts by linking them to topics being covered in other courses. What educator wouldn't jump at the opportunity to use other courses to provide motivation, reinforcement, and credibility for one's own course? Upon closer examination of the idea, however, it became obvious that compromises were required by all of the disciplines involved, primarily in the order and depth in which topics are covered. Since presumably there are very good, time-tested reasons for the existing, traditional course syllabi, we wanted to make only changes that were, from the viewpoint of the overall curriculum, clearly improvements.

The purpose of this paper is to describe some of the major changes that were made to the freshman calculus courses, and our experiences using them in a pilot program (the Foundation Coalition) at Texas A&M over the past four years. The thrusts of this program are curriculum integration, classroom technology, active and team learning, and continuous assessment. We hope that the assessment efforts will soon provide validation of the conclusions we argue for here.

OVERVIEW

The Foundation Coalition integrated curriculum for the freshman year contains mathematics, physics, engineering, and English courses both semesters, and a chemistry course in the spring. The main impetus for rearranging topics in calculus came from physics, but the engineering and chemistry courses also influenced our choices. The most radical change was to cautiously introduce, in the first semester, vectors and multidimensional calculus concepts, most of which are traditionally not covered until third semester. We have the following reasons for considering this to be feasible: Most important, the students are seeing these topics in their physics, engineering, and chemistry courses anyway, so our efforts should be seen as providing them with extra help with their other courses, rather than overloading them with advanced material. Also, we (almost entirely) restrict to two dimensions (2D); and we don't require the same mastery and depth of understanding as in third-semester calculus, where all of the concepts will be revisited. In fact, we believe that this two-stage approach to vectors and multidimensional calculus (easy and motivated 2D in the freshman year, 3D in the sophomore year) is one of the unanticipated strengths of our revised course. It gives students early exposure and additional practice with traditionally difficult material that requires the development of geometrical intuition more than learning of facts.

The next broad area of significant change is in “approximation techniques”, including estimation of derivatives from numerical data, numerical methods of integration, and finite Taylor expansions. For example, we cover the Midpoint, Trapezoidal, and Simpson's Rules for approximating definite integrals in the first semester, in conjunction with the definition of the Riemann integral and before the Fundamental Theorem of Calculus turns the students' attention to analytical methods. The increased emphasis on approximation is mainly due to the influence of the engineering course — particularly our participation in “integrated examinations” based on semirealistic engineering problems, which often lead to mathematical problems that can't be solved exactly by methods covered in first-semester calculus. (We also admit to some influence from the Calculus Reform movement's “Rule of Three”: Concepts should be presented geometrically, numerically, and algebraically.)

The current textbook situation is not ideal, since our rearranged syllabus requires that we skip around in the book a lot. We now use *Calculus* by James Stewart, 3rd Ed., Brooks/Cole Publ., one of the best of the current generation of “traditional” texts. (An independent effort within our department is devoted to revising Stewart's text to include some vectorial material in the first semester.) Pending the availability of a text that matches our syllabus, we smooth over the rough spots with supplementary notes provided to the students as Web pages and handouts. (The Web site in question is under continual development, and its URLs are subject to change; it is most easily found from our home pages at http://www.math.tamu.edu/.)

The remainder of the paper will discuss in more detail some of the specific ways we have changed, and improved, the freshman calculus course. The year is conveniently divided into 30 weeks, 15 each semester.

FIRST SEMESTER

1. The transcendental functions e^x, $\ln x$, a^x, and $\log_a x$ briefly appear early in the semester, because engineering is using them to fit data. We return to them early in the second

semester for the standard, thorough calculus treatment. In the meantime, it is sometimes convenient to have available these extra examples of nonpolynomial functions.

2. We cover antiderivatives and one-dimensional (1D) motion in week 4, to coincide with their use in physics. This compares with about week 10 in the traditional course. The students get a head start on integration by keeping antidifferentiation in mind while learning differentiation rules.

3. We interrupt the usual coverage of derivatives to introduce 2D vectors, parametric curves, and polar coordinates in week 6. Physics has already begun 2D motion and vectors in week 5, so the two courses are able to reinforce each other on difficult material. (This is the first time many of the well-prepared students with good high school backgrounds are seeing something in math class that they haven't seen before. Some experience a little panic, but most are reassured somewhat by the fact that they are seeing it applied.)

4. We postpone most of the traditional applications of derivatives, so that we can begin definite integrals in week 8. This is done because physics begins the concept of work in week 9, requiring integrals and eventually 2D line integrals. Immediately after defining the definite integral, we show how to evaluate it using a computer algebra system (Maple) and how to approximate it numerically. The early introduction of numerical integration both consolidates the conceptual understanding of the definite integral and directs the student's attention immediately from Riemann sums to approximations of more practical value.

5. In week 10 we begin two or three weeks of material traditionally not covered until third-semester calculus, such as line integrals in the plane and 2D definite integrals. To deal with the general case of a line integral along a curve, one must first represent the curve parametrically. We have been preparing the students for this since week 6, and we take a slow approach with easy problems. The treatment ends with a statement of Green's theorem, for which we need to introduce both partial derivatives and 2D integrals. The level of understanding of partial derivatives needed at this stage takes less than 15 minutes to convey. One or two days of simple examples of 2D integrals provide preparation for their application in the second semester to moment and centroid problems.

SECOND SEMESTER

1. In week 20 we begin two weeks on applications of (easy) 2D and 3D definite integrals to problems of finding volumes, centroids and centers of mass, and moments of inertia. It happens (and not by accident) that some or all of these topics are being used at approximately the same time in the physics, engineering, and chemistry courses. Looking at how they were presented in these other courses (partly in response to complaints of student confusion), we decided to change the way we would present them in calculus. In the traditional calculus course, for example, the usual approach to centroids and moments of inertia is to first treat planar regions using 1D definite integrals, leaving 3D regions to third semester. Unfortunately, the contortions that are necessary to obtain these 1D integrals are never clear to a large percentage of the students, and they simply memorize some formulas of dubious long-term value. We, in contrast, start with 3D systems of point particles (as encountered in physics and chemistry) and hence build the natural conceptual foundation for these topics. It is then easy to pass to very

simple 3D continuum problems and slightly more ambitious 2D problems, taking advantage of the students' exposure to double integrals near the end of the previous semester. The coordination with the other disciplines has revealed some conflicts in terminology and notation of which we were previously unaware; we are now able to warn our students about these. Improvement of the treatment of centroid and moment problems was not originally a central agenda item of the integrated curriculum, but it has turned out to be one of its most valuable byproducts.

2. We begin almost three weeks on introductory differential equations in week 22. (Our traditional calculus sequence already includes some introductory material on differential equations, but less.) By making connections with physics, we are able to bring a great deal of motivation to our coverage of differential equations, and prepare students for later courses in this important subject.

3. Near the end of the second semester we develop finite Taylor series (Taylor's theorem with remainder, and calculations with the series of particular functions) before getting into the matter of convergence of infinite series, which is really conceptually quite separate. At present there is not much direct tie-in of this to material being covered in the other courses, but the subject is highly relevant to later science and engineering courses.

4. Before turning to infinite series and the related topics of sequences and improper integrals, we return to the rigorous definition of a limit, which was glossed over very rapidly at the start of the first semester. We now have the leisure to discuss this matter at a pedagogically realistic pace, even taking the opportunity to introduce some notation of symbolic logic. With almost a year of college math behind them, students are much better able to understand and appreciate this traditional September bugbear. They are shown how functional limits, asymptotes, sequences, series, and improper integrals are all different manifestations of the same basic idea. The apparent disruptions of the conceptual structure of calculus forced by curricular integration in the first semester are now largely healed; the students, now more mature, are exposed to the logical underpinnings and conceptual unity of mathematics. Our task of handmaiden to technology having been performed, we can now stress that mathematics is an intellectual structure in its own right, independent of its applications.

WHAT IS LEFT OUT?

It is evident from the foregoing that this yearly syllabus is rather crowded. What has been omitted to make room for multidimensional math and other nontraditional emphases?

Much of the change is simply a matter of rearrangement. As remarked, the discussion of limits in depth is postponed to the end of the year. The mean value theorem also is postponed to the second semester, where it naturally combines with Taylor's theorem with remainder. The proofs of consequences of the mean value theorem are omitted, and “curve sketching” is taught quickly and mostly by means of examples; this is in accordance with the general deemphasis of that topic under the pressure of new graphing technology. Advanced techniques of integration (trigonometric substitution and partial fractions) are postponed to the third semester. Some traditional applications (such as hydrostatic pressure, area of a surface of revolution, and areas in polar coordinates) are omitted. Various preliminary nonvectorial treatments of such things as arc length and parametric equations are redundant in our treatment, and we believe that conceptual clarity is improved by (more efficiently) working in the better conceptual framework from the beginning.

CONCLUSION

One of us recalls from high school a driver education lesson whose main slogan was "Aim high in steering." That is the central principle of the sort of curricular reform exemplified here. An integrated curriculum necessarily sacrifices, in its first year, some of the depth, logical organization, and elementary drill traditionally associated with mathematics courses. This is the price of giving engineering majors a better understanding of why they are in the math class and where their education is heading. Somewhat unexpectedly, we have found that an "integrated" calculus syllabus also promotes such a heads-up treatment of some of the mathematical concepts themselves, as we have documented above. More generally, the frequent connections with applications encourage the student to focus on what calculus means, rather than on memorization of calculational techniques.

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Biographical Information

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