Using concept inventories to gauge preparedness and assess learning objectives in engineering economy classes

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Abstract

A ten-question, multiple-choice Concept Inventory was developed and administered to students at two universities: (1) students enrolled in a “Cost Analysis” course at an English-language, ABET-accredited Civil Engineering program outside of the United States, and (2) students enrolled in an “Engineering Economy” course utilized by students in civil, mechanical, and electrical engineering, along with computer science and safety technology, at a university in the United States. This paper describes the process that was utilized to identify which problem types and underlying concepts would be included in the inventory, includes an analysis and comparison of student performance on beginning-of-semester administrations of the inventory, and describes how administration of the inventory at the end-of-semester can be utilized for purposes of direct assessment of course objectives.

When administered at the beginning of a course, Concept Inventories can be used by instructors to gain insight about which ideas are already understood by how many students. By focusing on foundational concepts, such as compound interest, cash-flow diagrams, the time value of money, breakeven, cost indexing, and others, a Concept Inventory can identify what ideas students bring with them into the course, and which topics may need additional emphasis during the semester. Additionally, Concept Inventories allow for direct assessment of course objectives. When a Concept Inventory is administered at the beginning and then again at the end of a semester, it is possible to measure the improvement over time, and identify which subject areas may need additional emphasis or adjustment in presentation strategy in subsequent semesters.

Introduction

Among undergraduate engineering courses, Engineering Economy is unique in that it is often taught with a wide range of academic backgrounds among the students who enroll in the course. A given section may have second-semester freshman as well as final-semester seniors. This difference in student backgrounds is largely determined by prerequisites, and in some instances only a single college-level math course is required prior to enrollment in an Engineering Economy course. Which courses follow after Engineering Economy in the curriculum also is a contributing factor. In some programs, Engineering Economy is a terminal course which does not lead to follow-on classes in the same way that there is often a linear progression in other courses (e.g., physics → statics → dynamics→ structural analysis → reinforced concrete design), meaning that it can be pushed to a student’s final semester. Further contributing to a wide spectrum in student types and preparation is also the fact that Engineering Economy
courses are sometimes shared among different engineering disciplines (i.e., mechanical, civil, electrical, etc.), and in some cases also with students from fields beyond engineering (e.g., computer science, safety technology, etc.). This diversity in academic field further contributes to the wide range of student background and experience in the classroom, and presents instructors with challenges related to what students may already know at the outset of a course. An additional complication is that this experience distribution can be variable from semester to semester, meaning that what one student population knew at the beginning of one semester does not necessarily ensure that the same things are already known by a different student population in a subsequent semester. All of these factors complicate the teaching of Engineering Economy, and make it especially beneficial to make some effort to assess which course concepts students may already be acquainted with at the beginning of the semester. And since broad conceptual information may be retained by students for a longer duration than specific course details (e.g., equations used to solve certain problems), assessing conceptual understanding at the end of a course may be an especially valuable indicator of what students have learned. This is particularly true when a beginning-of-semester assessment is combined with an end-of-semester assessment, in which case it becomes possible to infer learning gains that have arisen as a result of student learning during a course of study.

Concept inventories have been developed to assess student understanding in a variety of subjects, including physics1, statics2, biology3, genetics4, thermodynamics5, fluid mechanics6, light and spectroscopy7, dynamics8, chemistry9, digital logic10, thermal and transport science11, geoscience12, statistics13, and engineering hydrology14. Perhaps reflecting the wide range of subjects they seek to assess, there is substantial variation in the format and length of existing concept inventories. Some utilize a multiple-choice structure, assigning an all-or-nothing outcome to student responses, while others incorporate more detailed assessment of student responses in an attempt to develop a more nuanced understanding of the factors confounding whether or not a student is able to answer a certain question correctly. Likewise, there is considerable variation in the length of a concept inventory instrument, and the amount of time required to administer it to students. Since this concept inventory was envisioned as being used for both pre- and post-semester assessment, it was developed to be relatively quick to administer (i.e., students were given approximately 30 minutes to answer the 10 multiple choice questions).

During development of questions for the concept inventory used in this project, attention was given to identifying ideas that were both suitable for determining student pre-exposure to course concepts and that could be linked to course outcomes. The following course outcomes illustrate the range of ideas that students are expected to gain proficiency in during the semester:

1. Compute economic equivalency values for cash flows, including uniform and non-uniform expenses and revenues, deferred annuities, gradients, lump sum payments or receipts, and salvage values.
2. Use software tools and standard interest tables for determining present, annual, and future worth values of cash flows, in addition to calculating rates of returns and determination of the number of payment periods.
3. Perform alternative evaluation utilizing incremental rate of return analysis, replacement/retention analysis, and economic service life analysis.
4. Evaluate projects according to benefit/cost analysis, cost effectiveness analysis, and alternative selection with budget limitations.
5. Incorporate uncertainty analysis into simulations that address random project variabilities, including cost, inflation, and revenue.
6. Calculate the effect of taxes on project rates of return, and incorporate various depreciation models into project planning models.

Of these outcomes, some have elements that are not suitable for assessment through a pen and paper instrument at the beginning of the semester (e.g., the “software tools” mentioned in Outcome 2). Similarly, there are other elements that assessment of which would simply require more detailed effort (and thus time) than can reasonably be made available, such as problems relating to economic service life analysis (Outcome 3) or uncertainty analysis in a simulation that includes random project variables (Outcome 5). For these reasons, and others, simplified concept inventories are not suitable replacements for more exhaustive and detailed assessment, such as the typical midterm and final exams that might be administered in such a course. However, compared to the uncertainty about student preparation and understanding that instructors generally face at the beginning of the semester, even a relatively simple concept inventory can provide meaningful insight and a baseline for judging improvements.

Methods

The concept inventory was utilized at two universities: the American University of Sharjah (AUS), a private not-for-profit institution with a total student population of approximately 5,000 students, following the American model of higher education, located in a country outside of North America; and Marshall University, a public institution with a total student population of approximately 14,000 with a Carnegie classification of “Master’s Colleges and Universities (larger program). At the American University of Sharjah the students enrolling in the Engineering Economy course were exclusively from a civil engineering program, while at Marshall University the students in the course included engineering students majoring in civil, electrical, and mechanical engineering, as well as students from a safety technology program and computer science degree.

The questions and multiple-choice answer options that are included in the concept inventory are provided at the end of this paper as Appendix A. Question topics and related course outcomes are summarized in Table 1.
Table 1 – Mapping of concept inventory question number, topic, and related course outcomes.

<table>
<thead>
<tr>
<th>Question</th>
<th>Topic</th>
<th>Marshall University, Related Course Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve for i given P and F</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>Find F given P and i</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Cash flow diagram interpretation</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Find P given F and i</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Find P given A and i</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Find A given P and i.</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>Breakeven</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>Inflation</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Cost estimation: cost index</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>Depreciation</td>
<td>6</td>
</tr>
</tbody>
</table>

The number of students answering each question on the concept inventory correctly at the beginning and end of the semester was analyzed using a one-tailed Z-test to determine whether the difference is statistically significant. This was done to differentiate between differences in performance that might have been merely a result of random variation over time from those where the difference in number of students answering correctly can confidently (\(\alpha = 0.05\)) be attributed to an actual effect. The Z-test statistic is calculated by first determining the Pooled Sample Proportion, \(P\) (Equation 1), then calculating the Standard Error, \(SE\) (Equation 2), and finally the Z-test Statistic, \(Z\) (Equation 3).

\[
P = \frac{P_1 n_1 + P_2 n_2}{n_1 + n_2} \quad \text{(Equation 1)}
\]

\[
SE = \sqrt{P(1-P)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \quad \text{(Equation 2)}
\]

\[
Z = \frac{P_1 - P_2}{SE} \quad \text{(Equation 3)}
\]

Where:

- \(P_1\) = Sample proportion from population 1 (i.e., beginning of semester)
- \(P_2\) = Sample proportion from population 2 (i.e., end of semester)
- \(n_1\) = Size of sample 1 (i.e., beginning-of-semester; 19)
- \(n_2\) = Size of sample 2 (i.e., end-of-semester; 21)
The confidence level of 95% was selected such that when a significant difference of “yes” is indicated, there is a 95% probability that it is correct to reject the null hypothesis that the two groups are the same – that the proportion of students answering the question correctly is the same between the pre- and post-test. When a significant difference of “no” is indicated, this means that there is less than a 95% probability that the two groups are actually different.

The significance of differences in overall student score for the questions of the pre- and post-test were determined using a one-tailed t-test (Equation 4).

\[
t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
\]

(Equation 4)

Where:

\( \bar{x} \) = Mean score on concept inventory, end-of-semester
\( \mu_0 \) = Mean score on concept inventory, beginning-of-semester
\( s \) = Standard deviation, end-of-semester
\( n \) = Number of students, end-of-semester

The internal consistency of the concept inventory was evaluated with the Kuder-Richardson Formula 20 approach (Equation 5).

\[
\rho_{KR20} = \frac{k}{k - 1} \left( 1 - \frac{\sum_{j=1}^{k} p_j q_j}{\sigma^2} \right)
\]

(Equation 5)

Where:

\( \rho_{KR20} \) = Kuder and Richardson Formula 20 test statistic
\( k \) = number of questions in concept inventory
\( \sigma^2 \) = variance of the total scores of all people taking the test
\( p_j \) = number of people who answered question j correctly
\( q_j \) = number of people who did not answer question j correctly

In cases where examinees were randomly guessing on questions, or where performance on certain questions did not tend to align with performance of the instrument as a whole, the \( \rho_{KR20} \) tends to be low due to the outliers. By contrast, when performance individual questions on an instrument matches the overall results, this indicates internal consistency and the \( \rho_{KR20} \) will be relatively high. In other words, this statistic can be used to quantify whether examinee
performance is consistent across the entirety of a test instrument, or whether there are outliers that may confound results.

Results and Discussion

In general, and as one might hope, student performance on the concept inventory improved from the beginning of semester pre-test to the end-of-semester post-test. At the American University of Sharjah there was a statistically significant increase in percentage of students answering correctly on 8 of the 10 questions. At Marshall University, the statistically significant increase was on 5 of the 10 questions, along with a somewhat more perplexing and difficult to explain significant decrease in student performance on one of the questions (i.e., question 10). A comprehensive summary of student performance at both institutions is provided in Table 2. At the American University of Sharjah, the average performance on the beginning-of-semester concept inventory was 4.0 questions correct out of 10, compared to 8.5 out of 10 at the end-of-semester. At Marshall University, the average improvement was from 5.1 to 8.0 questions being answered correctly.

Table 2 shows that there were several questions where there were increases in the number of students answering correctly, but for which the improvement was not statistically significant. In most cases this was because students had a relatively strong performance at the beginning of semester administration of the concept inventory. For example, on Question 1 (which is related to calculating an interest rate from a given present value and given future value), the percentage of students answering correctly at the American University of Sharjah started at 83% at the beginning of the semester, and finished at 92% at the end of the semester. While the increase in performance is welcome, the magnitude of the improvement, relative to the baseline condition, is not statistically significant. This same situation, of relatively high pre- and post- performance, also explains the lack of significant difference on Questions 3 at the American University of Sharjah, and Questions 1, 3, and 8 at Marshall University. One notable exception to this trend is for Marshall University, where performance on Question 9 appears to show that the majority of students did not know the underlying concept (i.e., how to apply cost index data) at the beginning of the semester, and likewise did not correctly know how to apply the concept at the end of the semester. This observation is notable in that it identifies a lack of student achievement (related to course outcomes) that may require corresponding adjustment in lecture materials and/or the types of supporting learning activities (e.g., in-class exercises, homework assignments, etc.).

Many students will have taken a course related to engineering economy (e.g., a conventional economics social science course, as a part of university general education requirements) prior to the semester that they take engineering economy. These students may, for example, have already had exposure to principles such as inflation, how to apply interest rates, and related terms such as “book value”. To ascertain the degree to which this may have been the case at the American University of Sharjah, discussions were held with several students, after completing the Cost Analysis course at that university. Each of the students who participated in this discussion
identified previous university courses related to the principles covered in the Cost Analysis course. A “Principles of Microeconomics” course was most frequently mentioned, and “Fundamentals of Financial Accounting” was identified as well. When asked about what may have inhibited their performance on the concept inventory at the end of the semester, some students indicated a general sense of being overworked and overwhelmed at that point in the semester, which may mean some students under-achieved on the concept inventory relative to how they could have scored in the absence of the last minute projects, assignments, and cramming that often accompanies that time of the semester.

Table 2 – Summary of student performance on concept inventory.

<table>
<thead>
<tr>
<th>Question</th>
<th>Spring 2016 - INST 1</th>
<th>Fall 2016 - INST 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% of Students Answering Correctly</td>
<td>Z</td>
</tr>
<tr>
<td></td>
<td>Pre-</td>
<td>Post-</td>
</tr>
<tr>
<td>1</td>
<td>83</td>
<td>92</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>92</td>
</tr>
<tr>
<td>3</td>
<td>67</td>
<td>75</td>
</tr>
<tr>
<td>4</td>
<td>42</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>92</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>83</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>58</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>10</td>
<td>42</td>
<td>75</td>
</tr>
<tr>
<td>Avg:</td>
<td>40</td>
<td>85</td>
</tr>
</tbody>
</table>

Performance on the end of semester concept inventory was compared relative to two other measures of student achievement: overall average course grade at the end of the semester, and students’ score on the comprehensive final exam at the end of the course. Figure 1, which depicts the relationship between end of semester concept inventory performance along with average course grade, shows a trend of increasing course grade with increasing performance on the concept inventory. This is as one might anticipate: those who did well on the concept inventory also did well in the course overall. There is, however, significant variance in the data – the inclusion of students who, for example, did particularly well on the concept inventory but less well in the course, or those whose achievement in the course as a whole outperformed their percentage correct on the concept inventory. This variance can be partly explained by the fact
that the weighted end of semester course grade is comprised of several different item categories, such as homework assignments, quizzes, project assignments, midterm exams, and a comprehensive final examination. Students who are prone to underperforming in higher-stakes scenarios, such as the concept inventory (which is a multiple-choice instrument with no possibility of partial credit for incorrect answers), may have their weighted course average increase through high achievement on low-stakes assignments, or on grade items that allow for teamwork and student collaboration.

Figure 1 – Relationship between end-of-semester concept inventory performance and final grade.

Figure 2 – Relationship between end-of-semester concept inventory performance and final exam score.
Variance is likewise high on the comparison of concept inventory performance with grade on the final exam (see Figure 2). While both the concept inventory and final exam were higher-stakes instruments, the final exam had a broader variety of item types (e.g., short-answer questions, problem solving where partial credit was available in case of student mistakes, and questions where students had to define course concepts in their own words). Thus, while the general trend of increasing performance on the concept inventory does correlate with increasing grade on the final exam, it seems clear that attempting to assess student achievement solely with this multiple-choice concept inventory would be an unjustified simplification of the more comprehensive view that a more lengthy and intricate final exam can provide. So, while a concept inventory administered twice in a semester can be a valuable tool for assessment, it clearly does have limits beyond which it may not be the best tool.

The Kuder-Richardson Formula 20 test statistic computed for the Fall 2016 end-of-semester administration of the concept inventory at the American University of Sharjah is $\rho_{KR20} = 0.51$, indicating acceptable reliability and internal consistency. Since this concept inventory only contains ten questions, it should be noted that the Kuder-Richardson Formula 20 test statistic is biased in favor of instruments with more items. Since students can randomly guess and get some questions correct without knowing the related concepts, instruments with relatively few questions are resultantly weaker at distinguishing between random guessers and those who correctly answer a question due to working it correctly. Therefore, increasing the number of questions on this concept inventory would further increase the test statistic (and the underlying reliability of the concept inventory to discriminate between students who understand course concepts and those who do not), though it would come at the cost of increased time spent taking the assessment in the classroom. One means of avoiding lost instructional time that arises out of administering the concept inventory should be to shift it to an online administration route. This would also yield additional benefits, including the ability to assess the amount of time that students spend solving each problem\textsuperscript{16}, the capacity to introduce innovative question types (such as matching or on-screen hot-spots, and an ease of assessing short-answer problems.

Conclusion

The application of this engineering economy concept inventory at two different universities demonstrated that many students arrive in such courses already possessing a workable knowledge of key course concepts, and able to solve related problems. This presents challenges to instructors, who are charged with covering a comprehensive curriculum in a given subject, in that some high-achieving students will inevitably feel that the course material is proceeding too slowly, since they already have a basic understanding of how to solve problems. Application of a concept inventory, however, does also provide a unique opportunity to identify such high-
performing, already-knowledgeable students at the beginning of the semester, and could allow for innovative learning activities such as paired problem solving with lower-performing classmates. In this way partnered learning and collaborative student mentoring could be facilitated, providing benefits to students at both ends of the spectrum. Another significant benefit of application of concept inventories relates to their value as a direct assessment tool. Rather than relying on subjective interpretation of student submissions such as homework assignments or project reports, a concept inventory (particularly one that is multiple-choice in nature) is considerably more consistent between students and from semester to semester. This ability to track achievement over time enables for the effect of changes in lecture materials, assignments, and other course content to be assessed, in hopes of closing the loop and achieving continuous improvement.

Refinement and expansion to the existing engineering economy concept inventory is planned over the coming offerings of the related courses, including the possible consolidation of questions related to time value of money calculations (for which performance on, say, a P/A type question is closely related to performance on a F/A type question), and shifting of the instrument from a pen-and-paper format to an online format. Additionally, new questions could be developed to address topics not currently included in the inventory, such as incremental analysis, benefit/cost, and taxes. Instructors who are interested in using the engineering economy concept inventory in their courses are welcome to utilize the version that is included as an appendix to this paper, or to contact the paper authors for any revised versions that may be developed over time.

Bibliography


Appendix – Concept Inventory Questions.

Note: For a copy of the solution, please email one of the authors.

1. You make a single deposit of $8735 into an interest-bearing savings account, and one year later the account balance is $9794. The interest rate for the savings account is most nearly:
   a) 4.5%  
   b) 10.6%  
   c) 10.8%  
   d) 12.1%  
   e) 14.7%

2. You make a single deposit of $8000 into a savings account that pays 7.5% interest, compounded annually. The account balance at the end of 9 years is most nearly:
   a) $5,400  
   b) $8,600  
   c) $13,400  
   d) $15,300  
   e) $72,000

3. As shown in the figure below, your company is starting a project that will require and investment of $150,000 now and again in one year. Which of the following statements is true?
   a) Ten years from now your company will need to invest another $200,000.  
   b) Eight years from now your company will have revenues totaling $200,000.  
   c) The net cash flow four years from now is $180,000 in revenue.  
   d) Your company will receive $100,000 at the end of year 7.  
   e) There are no years in which the revenue equals $130,000.

4. You are making a loan to someone who has agreed to repay you with a one-time payment of $15,000 in 13 years. For 4% interest that compounds annually, the amount you should lend to the person today is most nearly: (note: this problem can be solved with the provided equations or the 4% table that is given)
   a) $7800  
   b) $9000  
   c) $11,000  
   d) $14,400  
   e) $25,000
5. A certain project will earn your company $400,000 per year for the next 11 years. Your company will borrow money at an interest rate of 5% per year to finance the project. Based on this, the maximum amount that your company should be willing to spend today to begin the project is most nearly: (note: this problem can be solved with the provided equations or the 5% table that is given)
   a) $680,000  
   b) $3,320,000  
   c) $4,400,000  
   d) $4,620,000  
   e) $7,530,000

6. As shown in the figure below, today you deposit $35,000 into a savings account that pays 6.8% interest, compounded annually. The maximum amount that can be withdrawn from the account at the end of each year for four years is most nearly:
   a) $2,400  
   b) $6,700  
   c) $8,800  
   d) $9,400  
   e) $10,300

   ![Diagram](image1.png)

7. You spend $5000 on an investment that will return $750 per year. You wish to receive a 10% return on this investment. Based on this, the amount of time required for the investment to break even is most nearly: (note: this problem can be solved with the provided equations or the 10% table that is given)
   a) 6 years  
   b) 7 years  
   c) 10 years  
   d) 12 years  
   e) 14 years

8. Which of the following statements about inflation is true?
   a) Inflation corresponds to a decrease in the value of money, and increase in the cost of goods and services.
   b) Inflation corresponds to an increase in the value of money, and increase in the cost of goods and services.
   c) Inflation corresponds to a decrease in the value of money, and decrease in the cost of goods and services.
   d) Inflation corresponds to an increase in the value of money, and decrease in the cost of goods and services.
9. You are planning a highway rest area that will be constructed in 2016, and it will have a planned capacity of 500 visitors per hour. In 2010, when a certain construction cost index was 158, a similar highway rest area was constructed with a capacity of 380 visitors per hour, at a cost of $2.85 million. Assuming a linear relationship between capacity and cost, and a 2016 construction cost index of 214, the anticipated construction cost for the planned highway rest area will be most nearly:
   a) $1.60 million  
   b) $2.76 million  
   c) $3.42 million  
   d) $5.08 million  
   e) $9.59 million

10. Today your company has purchased a drill rig for $550,000 that will have a salvage value of $100,000 in eight years. The book value for the drill rig can be found according to a linear, “Straight Line” depreciation in value. The book value of the drill rig in three years is most nearly:
   a) $56,250  
   b) $100,000  
   c) $381,250  
   d) $493,750  
   e) $650,000
NOTE: Those taking the concept inventory are provided with compound interest factor tables for $i = 4\%$, 5\%, and 10\%, along with the following equations:

<table>
<thead>
<tr>
<th>Conversion</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>To $F$ from $P$</td>
<td>$\frac{F}{P} = (1 + i)^N$</td>
</tr>
<tr>
<td>To $P$ from $F$</td>
<td>$\frac{P}{F} = (1 + i)^{-N}$</td>
</tr>
<tr>
<td>To $A$ from $F$</td>
<td>$\frac{A}{F} = \frac{i}{(1 + i)^N - 1}$</td>
</tr>
<tr>
<td>To $A$ from $P$</td>
<td>$\frac{A}{P} = \frac{i(1 + i)^N}{(1 + i)^N - 1}$</td>
</tr>
<tr>
<td>To $F$ from $A$</td>
<td>$\frac{F}{A} = \frac{(1 + i)^N - 1}{i}$</td>
</tr>
<tr>
<td>To $P$ from $A$</td>
<td>$\frac{P}{A} = \frac{(1 + i)^N - 1}{i(1 + i)^N}$</td>
</tr>
</tbody>
</table>