AC 2010-1675: USING CONSTRUCTIVIST TEACHING STRATEGIES IN PROBABILITY AND STATISTICS

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Using Constructivist Teaching Strategies in Probability and Statistics

Abstract

This paper discusses the early results of an NSF EEC project that focuses on the impact of using constructivist approaches to teaching probability and statistics for engineers. Twelve exercises were developed and used in a modified version of the course to promote student learning. For example, one exercise enables students to build upon previously acquired knowledge related to counting techniques and the concept of statistical independence and, through self-discovery, derive the probability mass function for the binomial distribution. Data were collected from both a control group and a treatment group. Preliminary results regarding the efficacy of such an approach are presented.

1. Introduction

The need for a strong engineering workforce in the United States has been affirmed by several national studies including some sponsored by both the National Science Foundation and the National Academy of Engineering[1-3]. Existing research on learning and teaching offers pedagogical approaches that have proven to be effective in teaching mathematics instruction, namely Realistic Mathematics Education (RME) and inquiry-oriented (IO) teaching and learning. However, while these pedagogical approaches have been applied in pure mathematics courses, there is limited evidence that the pedagogies have been employed in a more applied context like the engineering curriculum.

The RME and IO constructivist pedagogies are significantly different from traditional engineering instructional methods. Traditional engineering pedagogy dictates that instructors should present students with a series of known facts or procedures and then demonstrate their use to solve sample problems. Students are then given homework problems that require the repetitive use of the same facts or procedures. It is assumed that such repetition fosters student learning. However, what this method oftentimes creates are students who are capable of successfully solving problems without truly thinking or understanding. When assessment of student outcomes is measured by having students solve similar problems on an exam, it is entirely possible for students to perform well on such assessments without really having an understanding of the underlying concepts. Constructivist learning and teaching pedagogies are a response to this deficiency.

The objectives of this research are to: (1) develop educational materials based on the RME and IO constructivist learning pedagogies to more effectively teach probability and statistics in engineering; (2) evaluate the impact of the RME and IO learning environment on student learning outcomes; and (3) disseminate results on campus, in local K-12 education, and throughout the engineering education community to facilitate further implementation of constructivist pedagogies in the engineering curriculum.

2. Literature Review

The traditional pedagogy within engineering education is built upon the premise that it is the job of the engineering educator to explain existing truths to students who will then "learn" these truths by applying them as they solve problems presented by the educator. While the engineering curriculum at most schools has expanded to include topics such as teamwork, ethics, and the benefits of diversity, and capstone courses that seek to integrate work through team projects, many of the mathematics based courses still teach in a passive manner. Formulas are presented to students, a few example problems are solved, and students practice by doing homework. An assessment of student learning is to solve similar problems on an exam. However, what is generally not assessed is a student's understanding of the very formulas that are employed. In fact, students can perform quite well on such exams with very little understanding at all.

At its core, engineering is the application of mathematics and science to solve practical problems of the human race. That is, at its core, engineering is not just problem solving, but practical problem solving. And it is when engineers encounter interesting problems and grapple with them that discovery and real learning occurs. Many of the formulas that we simply present as fact were discovered as a result of individuals grappling with problems until these formulaic relationships emerged. This process of self discovery is a very important aspect of engineering education that is now missing.

The constructivist learning classroom presents the learner with opportunities to build on prior knowledge and understanding to construct new knowledge and understanding from authentic experience. Students are allowed to confront problems full of meaning because of their real-life context. In solving these problems, students are encouraged to explore possibilities, invent alternative solutions, collaborate with other student (or external experts), try out ideas and hypotheses, revise their thinking, and finally present the best solution they can derive. This is in contrast to the abovementioned traditional engineering approach to education.

There are three streams of research that are related to this constructivist approach. They are: 1) Realistic Mathematics Education[4-10], 2) inquiry-oriented education[4-5, 10], and 3) cognitive science research on teaching and learning[11-13]. Each of these lends support to the efficacy of the proposed approach.

There is evidence that learning can be aided by contextualizing problems[4-6]. This is the thought underlying problem-based learning (PBL) that has been most often associated with medical education. One reason that PBL may have taken hold in medical education is that the problem-based scenarios used so closely parallel the real life experiences of practicing clinicians. Realistic Mathematics Education is an analogous pedagogy associated with mathematics education.

Inquiry-oriented teaching and learning environments emphasize presenting students with problems that build upon current knowledge. Instead of lecturing, the instructor facilitates students working in small groups to solve problems themselves by asking questions and posing possible solutions. Through interactive inquiry, students develop their own approaches to solving the problems. They also develop a deeper understanding of how various concepts are connected[4-5,10].

Cognitive science offers new models of the learning process that are consistent with the RME pedagogy such as that introduce by Reyes and Zarama[11]. The work of Parra and Yano[12] builds upon this work and posits that the model is recursive in nature and involves interaction between both the teacher and student who are both involved in the learning cycle. Similarly, the work of Dreyfus and Dreyfus[13] is consistent with the learning model outlined above. The interested reader is encouraged to explore this rich literature by reading the aforementioned references.

3. Description of Constructivist Exercises

The Probability and Statistics for Engineers course was redesigned to incorporate twelve exercises that were designed to promote a constructivist approach to learning. A brief description of the twelve exercises appears in Table 1 below. Traditionally, the approach taken in the course was to introduce a new topic via instructor-led teaching of textbook material via an exchange of written notes between the instructor and the students. Students were then assigned relevant homework to provide an opportunity for them to *master* the material via repeated application and practice. In contrast, the constructivist approach presents students with carefully constructed problems or contexts that encourage students to create their own knowledge and develop their own understanding. The study of the binomial distribution provides a nice example of the contrasting approaches.

Exercise Title	Course Topic	Brief Description/Objective
#1: Projectile Motion	Introduction to uncertainty	Student teams develop the deterministic formula for the distance a projectile travels based on their knowledge of physics and then perform the experiment with an actual catapult and measure the distance traveled with each launch.
#2: The Problem of the Points	Introduction to probability	Student teams are introduced to the original Problem of the Points as discussed by Fermat and Pascal and asked to develop an equitable solution and explain its strengths and weaknesses.
#3: USF BULL RUNNER	Counting techniques including permutations and combinations	Students work in teams to count the number of ways simple words can be rearranged using the same letters. The questions begin very easy and become increasingly difficult and abstract until the students are led to derive some very common counting formulas.
#4: Exit Polling	Conditional probability and Bayes' Theorem	Using real exit polling data presented to two ways, student teams are asked simple questions regarding voting patterns. The questions begin very easy and become increasingly difficult ultimately unknowingly requiring the understanding of a conditional probability and the use of Bayes' Theorem.
#5: Rolling Dice	Introduction to the probability mass function for discrete random variables	Student teams roll two dice and record the sum. After completing this experiment 30 times, the students graph the result. The teams are then instructed to develop the theoretical outcome in both a tabular and graphical form.
#6: Counting Revisited	The binomial distribution	Student teams are asked a series of leading questions that build upon their knowledge of the binomial distribution. The initial question is easy, but they come increasing difficult and abstract leading to the derivation of the formula for the probability mass function of the binomial distribution.
#7: Chugalug	Introduction to the probability density function for continuous random variables	Student teams are unknowingly introduced to the concept of a probability density function. Initial questions only require simple counting of squares on a graph to determine probabilities. The more advanced questions require students to determine the equation for a straight line and to integrate over specified limits to determine a probability.

Table 1a: Summary description of the implemented constructivist exercises.

Exercise Title	Course Topic	Brief Description/Objective
#8: The Poisson Distribution Revisited	The link between the Poisson and exponential distributions	Student teams are reminded of the probability mass function of the Poisson distribution previously studied. A series of leading questions that build upon previous knowledge are asked ultimately leading to the derivation of the probability density function for the exponential distribution.
#9: Online Distribution Quiz	Developing the ability to discriminate between various probability distributions	Individual students access the online quiz and can practice analyzing probability problems. The objective is for students to improve their ability to discern the underlying probability distribution of a random variable based on information provided in the context of a problem.
#10: Soldier Fatalities	Using graphical techniques to analyze data for decision making	Student teams are introduced to the use of statistics as a *macroscope* and are provided with a large database of numerical data representing the cause of soldier fatalities. With limited funds, the student teams are to make a single recommendation to reduce the number of soldier fatalities. Because the numerical data is so numerous, students must use statistics or graphical methods to make a defendable recommendation.
#11: Sampling	The sampling distribution	Leveraging previous knowledge from the course, student teams are asked a series of leading questions culminating in the development of the sampling distribution.
#12: 2009 0-60 mph Time	Confidence intervals and hypothesis testing	Students are presented with sample data and a marketing claim issued by a vehicle manufacturer. Students must determine if they believe the claim or not. Questions are posed that encourage students to independently develop a confidence interval and a procedure akin to a statistical hypothesis test.

Table 1b: Summary description of the implemented constructivist exercises.

Traditionally, students were introduced to the concept of a discrete random variable and the associated concept of a probability mass function. Subsequently, students were taught several common discrete random variable distributions such as the binomial distribution, geometric, hypergeometric, negative binomial, etc. For each of these distributions, the name of the distribution was introduced followed by a quick definition of the random variable and a presentation of the probability mass function associated with the distribution. This was typically followed by sample problems. This process was repeated for each of the covered discrete distributions in rapid succession. In contrast, the constructivist approach to teaching this same material begins with the following questions:

> A multiple-choice test contains 5 questions, each with four answers. Assume a student just guesses on each question.
>
> a) What is the range of the random variable X, the number of questions the student answers correctly?
>
> b) Construct the probability mass function for the random variable X, the number of questions that the student answers correctly.

Note that the question allows students to build upon material that they have already studied and mastered. Namely, the students build upon their knowledge of statistical independence, the counting technique known as a combination, and the concept of a probability mass function. All of these topics were covered prior to introducing this new topic. After dealing with this familiar type of question, the next question in the sequence becomes increasing complex and abstract:

> A multiple-choice test contains n questions. If answers are randomly selected, the correct answer will be selected with probability p. What is the probability that X, the number of questions that the student answers correctly, equals x?

In the answering the above question, the students unknowingly derive the probability mass function for the binomial distribution. Once the students construct this function, building upon knowledge they had previously acquired, the students then use the function to answer a specific question:

> A multiple-choice test contains 25 questions, each with four answers. Assume a student just guesses on each question.
>
> a) What is the probability that the student answers 5 questions correctly?
>
> b) What is the probability that the student answers more than 2 questions correctly?

Only after having completed this exercise are students then formally introduced to the binomial distribution by the instructor. The hypothesis is that the level of comprehension the students gain is greatly enhanced by this process of self-discovery. As one might expect, however, this hopeful gain in comprehension is not without cost for the use of such exercises involves a great deal of time. This trade-off is one that every instructor must make in a manner that best supports his/her stated objectives.

4. Preliminary Results

The Motivated Strategies for Learning Questionnaire (MSLQ) has been used to collect data to facilitate a comparison of the control and treatment groups and these data are currently being analyzed. In conjunction with the MSLQ data, empirical data has been collected from students in the treatment group to gauge the usefulness of the new exercises. This paper presents these data.

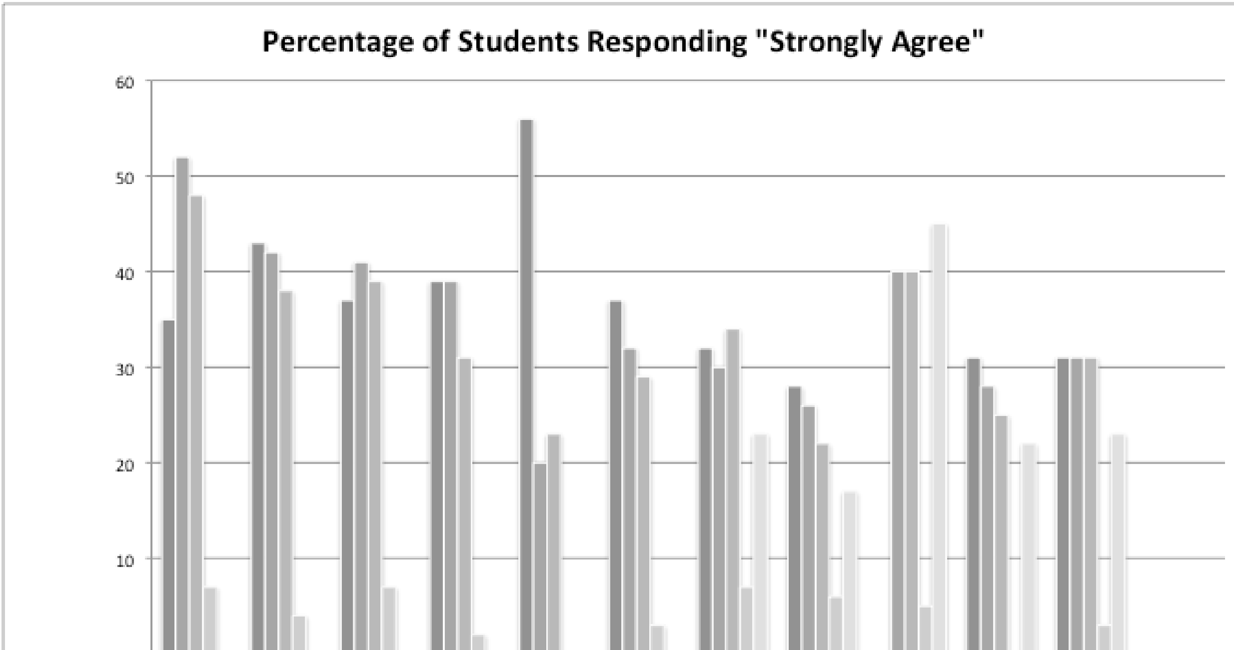
A survey was administered to the treatment group following each of the twelve exercises. A six-point opinion scale was used for the survey statements with the following possible responses: Strongly Agree, Agree, Neither Agree nor Disagree, Disagree, Strongly Disagree, and Not Applicable. Table 2 shows the questions that were posed to the students. (Note that no data are available for Exercise 12 or for survey item number one for Exercise 9.)

Survey Statements
1. This exercise was an appropriate use of class time.
2. This exercise had a clear learning objective.
3. This exercise was successful in meeting its associated learning objective(s).
4. I recommend DISCONTINUING the use of this exercise in future semesters.*
5. Please provide any additional comments you may have regarding this exercise:
6. This exercise was an effective learning tool.**

* "Discontinuing" was listed in all capital letters for exercises 3 through 11.
** This survey item was only posed for exercises 7 through 11.

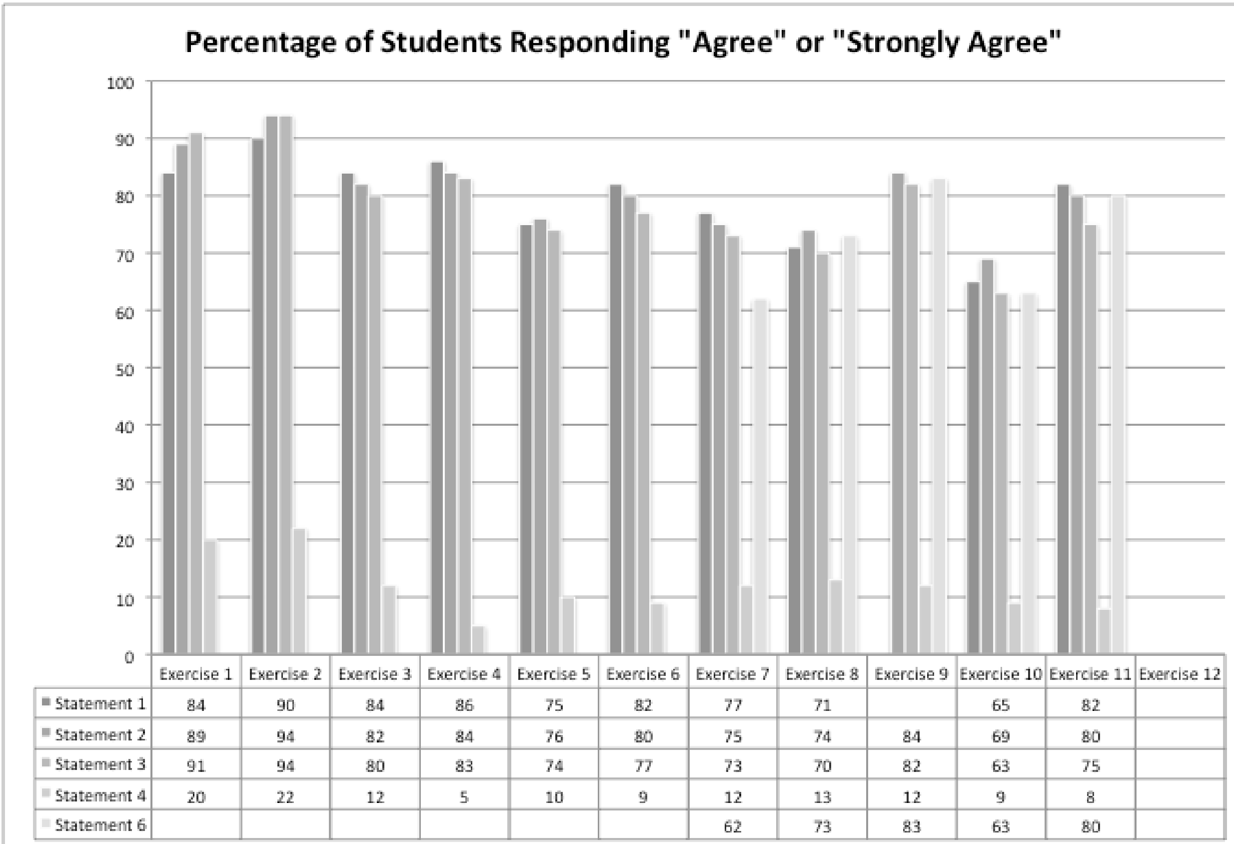
Table 2: Statements for surveys administered to students following completion of each exercise.

Analysis indicates that students in the treatment group largely received these exercises positively. The percentage of students who "agreed" or "strongly agreed" that the exercises were an appropriate use of class time ranged from 65 percent for Exercise 10 to 90 percent on Exercise 2. The percentage of students who "agreed" or "strongly agreed" that the exercises had clear learning objectives ranged from 69 percent to 94 percent. Moreover, a majority of the students felt that the exercises were successful in meeting their respective objectives. Finally, only a minority of students suggested discontinuing use of the exercises. Refer to Figures 1 and 2 for these data.



	Exercise 1	Exercise 2	Exercise 3	Exercise 4	Exercise 5	Exercise 6	Exercise 7	Exercise 8	Exercise 9	Exercise 10	Exercise 11	Exercise 12
Statement 1	35	43	37	39	56	37	32	28		31	31	
Statement 2	52	42	41	39	20	32	30	26	40	28	31	
Statement 3	48	38	39	31	23	29	34	22	40	25	31	
Statement 4	7	4	7	2	0	3	7	6	5	0	3	
Statement 6							23	17	45	22	23	

Figure 1. Results from student evaluations after constructivist exercises were completed.



	Exercise 1	Exercise 2	Exercise 3	Exercise 4	Exercise 5	Exercise 6	Exercise 7	Exercise 8	Exercise 9	Exercise 10	Exercise 11	Exercise 12
Statement 1	84	90	84	86	75	82	77	71		65	82	
Statement 2	89	94	82	84	76	80	75	74	84	69	80	
Statement 3	91	94	80	83	74	77	73	70	82	63	75	
Statement 4	20	22	12	5	10	9	12	13	12	9	8	
Statement 6							62	73	83	63	80	

Figure 2. Results from student evaluations after constructivist exercises were completed.

Though in the minority, there were some negative reactions to the exercises as well. For example, some students did not like working in groups and felt that the exercises would have been more beneficial if done individually. Also, some students got confused during the exercises and felt that some of the exercises did not have clear objectives while being performed. Such students wanted more direction from the instructor and felt frustrated by a lack of it. Moreover, some students felt that class time would have been better spent had the instructor simply explained the material "from the get go" and foregone the exercises.

5. Summary

This paper presented the ongoing work of a NSF EEC project for analyzing the impact of using constructivist approaches to teaching probability and statistics for engineers. Twelve exercises were developed and used in a modified version of a Probability and Statistics for Engineers course to promote student learning. Preliminary results show that a majority of students in the treatment group "agreed" or "strongly agreed" that the exercises had clear learning objectives and were successful in meeting these learning objectives. Furthermore, a majority of respondents in the treatment group "agreed" or "strongly agreed" that the exercise for which these data were collected were effective learning tools. Lastly, a majority of the responding students "agreed" or "strongly agreed" that the exercises were an appropriate use of time, while only a minority of students "agree" or "strongly agreed" that the exercises should be discontinued in future semesters.

Additional data from the control and treatment groups have been collected using the Motivated Strategies for Learning Questionnaire (MSLQ) and are currently being analyzed to determine the impact of these constructive learning approaches on students' learning.

Acknowledgements

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