

2006-1702: USING EQUATION SOLVER PROGRAMS TO TEACH DESIGN OF PIPE NETWORKS

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Using Equation Solver Programs to Teach Design of Pipe Networks

Introduction

Pipe networks are one of the fundamental components in the design of thermal systems. Many universities teach the design of these systems in courses such as Fluid Mechanics or Thermo-Fluid Component Design. Traditionally, the analysis of pipe networks was done using iterative methods, such as the Hardy-Cross method¹. However, modern mathematical software such as Mathcad² and EES³, which include equation solvers, can be used to analyze these systems more efficiently.

The advantage of using software like Mathcad to teach students the design of pipe networks is that students focus in the fundamental concepts rather than in the mathematical solution. To set up the solution to the network problem, they need to enter the correct set of governing equations. Once this is done, the software solves the system of equations and provides them the solution. As a consequence, students spend most of their time analyzing their system rather than computing. This is very useful in design because many changes can be done in a short time to optimize the final design.

The goal of this paper is to describe how the design of pipe networks is taught using the software Mathcad. Three examples that are used in class are discussed. The first is a very simple pipe network that introduces the technique to students. The second example also describes a simple network that shows students how to modify pipe sizes and other parameters to design a stable and controllable network of pipes. The third example discusses the design of a more complex pipe network system, similar to the ones used in hydronic cooling or heating of buildings. The goal of this example is to show students how to design and optimize a complex network of pipes.

Example 1: Analysis of a simple pipe network

This simple pipe network is shown in figure 1. The goal of this problem is to calculate pipe flows Q_1 , Q_2 , and Q_3 in pipes 1, 2, and 3 when the following information is provided:

- All pipe sizes and materials
- Flow rates coming in or out at points A and B, as shown in the figure

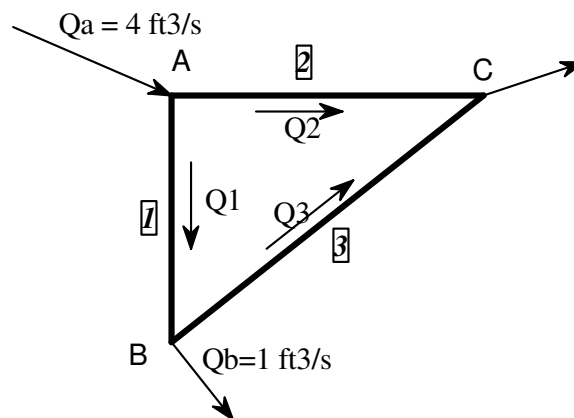


Figure 1. Analysis of a Simple Network

The problem has three unknowns, and to solve for them, three independent equations are needed. The conservation of mass applied to two nodes such as A and B provides two equations.

$$Q_a = Q_1 + Q_2 \quad (1)$$

$$Q_1 = Q_b + Q_3 \quad (2)$$

The conservation of energy applied to the loop A, B, and C provides the third one.

$$h_1 + h_3 = h_2 \quad (3)$$

In this equation, h_1 , h_2 , and h_3 are the head losses in each pipe.

This system of equations cannot be easily solved since the expression for the head losses h as function of Q is very complicated. The head loss is given by

$$h = f \frac{L V^2}{D 2g} \quad (4)$$

In this expression, V is the velocity, L the length, D the diameter of the pipe, and f is the friction factor. The velocity may be written as function of the volumetric flow rate Q . The friction factor f for turbulent flow may be approximated by one of the explicit equations such as the one given by Swamee and Jain⁴

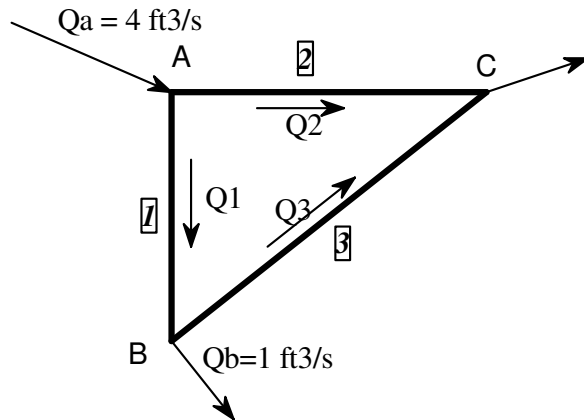
$$f = \frac{0.25}{\left[\log \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}} \right) \right]^2} \quad (5)$$

In this equation ε is the roughness and Re is the Reynolds number which can be expressed as a function of the volumetric flow rate Q , as well. Then, equations (4) and (5) can be written as a function of Q and combined with (3) to express equations (1), (2), and (3) as functions of Q . Since the resulting system of equations is very complicated, an iterative process is required to obtain its solution. In more complex networks with over 10 pipes the problem is even more difficult.

Using the software Mathcad, the solution is very simple, as long as all three equations can be written as functions of Q_1 , Q_2 and Q_3 . The solution using Mathcad is shown in Figure 2.

This solution has three main sections. In the first, all known values, such as lengths and diameters, are entered. In the second, all required functions are defined. Some highlights of these functions are that the Reynolds number should be always positive since the function for friction factor “ f ” requires a positive Reynolds number. Also, the head loss function “ h ” may be positive or negative depending on the flow direction, which is not known in advance. The third section is where the system of equations is solved. To solve the system of equations, Mathcad requires guess values for the unknowns Q_1 , Q_2 , and Q_3 , and the keyword *Given* and *Find*. As it can be observed in this figure, the system of equations is clear and meaningful. All three equations display the conservation of mass and conservation of energy principles.

EXAMPLE NETWORK OF PIPES



Given dimensions find volumetric flow rates Q1, Q2 and Q3

Volumetric flow rates coming in and out

$$Q_A := 4 \cdot \frac{\text{ft}^3}{\text{s}}$$

$$Q_B := 1 \cdot \frac{\text{ft}^3}{\text{s}}$$

Properties of water at 50 °F

$$\gamma := 62.4 \frac{\text{lb}_f}{\text{ft}^3}$$

$$\nu := 1.4 \cdot 10^{-5} \cdot \frac{\text{ft}^2}{\text{s}}$$

Pipe Dimensions

$$\varepsilon := 0.00015 \text{ft} \quad (\text{roughness of commercial steel})$$

$$L := (200 \ 300 \ 450)^T \cdot \text{ft} \quad (\text{lengths of each pipe})$$

$$D := (0.5 \ 2.0 \ 2.0)^T \cdot \text{ft} \quad (\text{diameters of each pipe})$$

Function Definitions

$$\text{Re}(q, d) := \frac{4 \cdot |q|}{d \cdot \pi \cdot \nu} \quad \text{Reynolds number}$$

$$f(q, e, d) := \begin{cases} \frac{0.25}{\log\left(\frac{e}{3.7 \cdot d} + \frac{5.74}{\text{Re}(q, d)^{0.9}}\right)^2} & \text{if } \text{Re}(q, d) > 2300 \\ \frac{64}{\text{Re}(q, d)} & \text{otherwise} \end{cases} \quad (\text{friction factor for turbulent flow as given by Swamee and Jain})$$

Figure2. Mathcad Solution of Example 1

$$h(q, d, e, l) := \frac{8f(q, e, d) \cdot l \cdot q \cdot |q|}{d^5 \cdot g \cdot \pi^2} \quad (\text{head loss in pipes as given by } h = f \cdot L / D \cdot V^2 / 2g)$$

Solution

Guesses for volumetric flow rates to start the iteration process in Mathcad

$$Q := (2 \ 2 \ 1)^T \cdot \frac{\text{ft}^3}{\text{s}}$$

Given

$$h(Q_1, D_1, \varepsilon, L_1) + h(Q_3, D_3, \varepsilon, L_3) = h(Q_2, D_2, \varepsilon, L_2)$$

$$Q_A = Q_1 + Q_2$$

$$Q_1 = Q_B + Q_3$$

$$Q := \text{Find}(Q)$$

$$Q^T = (0.125 \ 3.875 \ -0.875) \frac{\text{ft}^3}{\text{s}}$$

$$h(Q_1, D_1, \varepsilon, L_1) = 0.065 \text{ft}$$

$$h(Q_2, D_2, \varepsilon, L_2) = 0.059 \text{ft}$$

$$h(Q_3, D_3, \varepsilon, L_3) = -6.01 \times 10^{-3} \text{ft}$$

(These values of head loss satisfy the energy equation. The values of volumetric flow rates satisfy the conservation of mass. The negative value of the volumetric flow rate Q_3 indicates that this flow is in the opposite direction of the arrow)

Figure 2. Continuation

Example 2: Simple Hydronic Network

The second example, shown in Figure 3, describes a simple hydronic network. Hydronic networks of pipes deliver hot or cold water to different zones in a building. The desire flow rate of water into a zone depends on the thermal load of that zone. In general, the flow rate is controlled by a valve that opens and closes according to a thermostatic reading. However, when the system is not properly designed, the flow rate into a zone may change also when the flow rates in other zones change. This will force the valve controlling this zone to correct the problem by adjusting its position. This adjustment may also affect the valves in other branches and create an unstable system.

To illustrate the problem, the simple hydronic closed loop system of Figure 3 is discussed. This example shows two branches that include two air handlers or heat exchangers. As will be shown, when pipe sizes are too small a change in the flow rate in one branch affects the flow rate in the other. The goal of this problem is to calculate the change in flow rate ΔQ_2 in branch 2 when a valve in branch 1 changes from fully opened to fully closed. This change in flow rate is calculated for different pipe sizes of pipes 3, 4 and 5. Five pipe sizes were evaluated 0.5, 0.75,

1.0, 1.25, and 1.5 inches. The system of equations and the Mathcad solution for an initial diameter of 0.5 inches is displayed in Figure 4. The complete solution includes all dimensions and functions definitions not displayed in this figure. The final plot of ΔQ_2 vs. the diameter of pipes 3, 4, and 5 is shown in Figure 5.

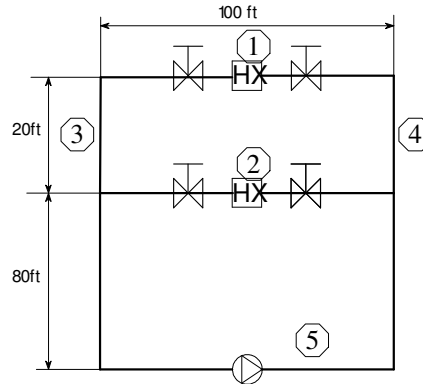


Figure 3. Simple Hydronic Network

There are 6 unknowns, 5 Q s and the head of the pump. Guess values for Q_i and h_p are:

$$Q := (6 \ 6 \ 6 \ 6 \ 12)^T \cdot \frac{\text{gal}}{\text{min}} \quad h_p := 60 \cdot \text{ft}$$

The system of equations and their solution are given by:

Given

The mass conservation equations are

$$Q_4 = Q_1$$

$$Q_3 = Q_1$$

$$Q_5 = Q_2 + Q_4$$

$$Q_5 = Q_p$$

head losses for loops 1 and 2 are

$$h_p = h(Q_5, D_5, \varepsilon, L_5) + h(Q_2, D_2, \varepsilon, L_2) + 2 \cdot hf(Q_5, D_5, K_e) + 2 \cdot hf(Q_5, D_5, K_{tb}) + 2 \cdot hf(Q_2, D_2, K_v) \dots + hf(Q_2, D_2, K_{hx})$$

$$h_p = h(Q_5, D_5, \varepsilon, L_5) + h(Q_4, D_4, \varepsilon, L_4) + h(Q_1, D_1, \varepsilon, L_1) + h(Q_3, D_3, \varepsilon, L_3) + 2 \cdot hf(Q_5, D_5, K_e) \dots + 2 \cdot hf(Q_1, D_1, K_e) + 2 \cdot hf(Q_5, D_5, K_{ts}) + 2 \cdot hf(Q_1, D_1, K_v) + hf(Q_1, D_1, K_{hx})$$

$$\begin{pmatrix} \text{Res}_1 \\ \text{Res}_2 \end{pmatrix} := \text{Find}(Q, h_p) \quad Q := \text{Res}_1 \quad h_p := \text{Res}_2$$

$$Q^T = (2.763 \ 3.237 \ 2.763 \ 2.763 \ 6) \frac{\text{gal}}{\text{min}} \quad (\text{Volumetric flow rates in pipes 1, 2, 3, 4, and 5})$$

$$h_p = 335.085 \text{ ft} \quad (\text{Required head of the pump})$$

Figure 4. Solution of a Simple Hydronic Network

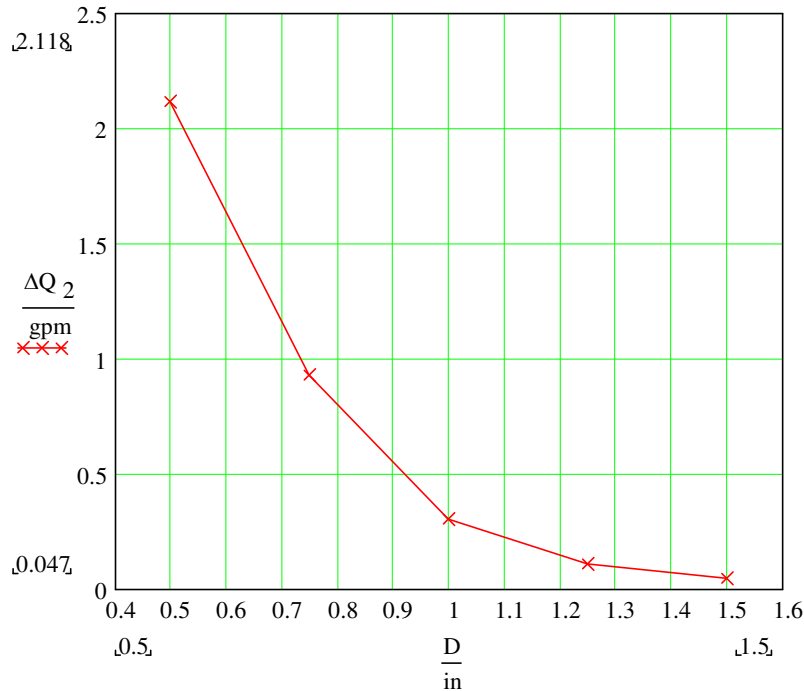


Figure 5. Change in Volumetric Flow Rate in Branch 2 for Different Pipe Sizes

Figure 5 shows that when pipe sizes are too small, the change in the flow rate in one branch, in this case branch 1, affects the flow in other branches such as branch 2. When pipes 3, 4 and 5 are of ½ inch and the valve in branch 1 changes from fully opened to fully closed, the flow rate in branch 2 increases from 3.237 gpm to 5.355 gpm (or 2.118 gpm). However, when these pipes are 1.5 inches in diameter, the change in flow rate is very small (0.05 gpm). This example illustrates the value of equation solver programs, including Mathcad, in the design of hydronic systems that are stable and easy to control. Changes can be done quickly and many different options can be analyzed in a short time.

Example 3. More Complex Hydronic Network

Figure 6 shows a more complex pipe network that includes three pumps. In this example, flows through air handlers a, b, and c are provided. The goal is to calculate the heads of each pump Hp1, Hp2, and Hp3. All pipe sizes and all loss coefficients Ks are known for all fittings and components. Since flow rates are known in each pipe, head losses in them can be calculated directly. The heads of each pump were obtained applying the three loop equations as shown in Figure 7.

After some examples similar to the ones presented here, students are assigned a design project that requires generation of concepts, analysis using Mathcad, selection of components and drawing of a final solid model. The next example shows some pictures of the design of a group of students.

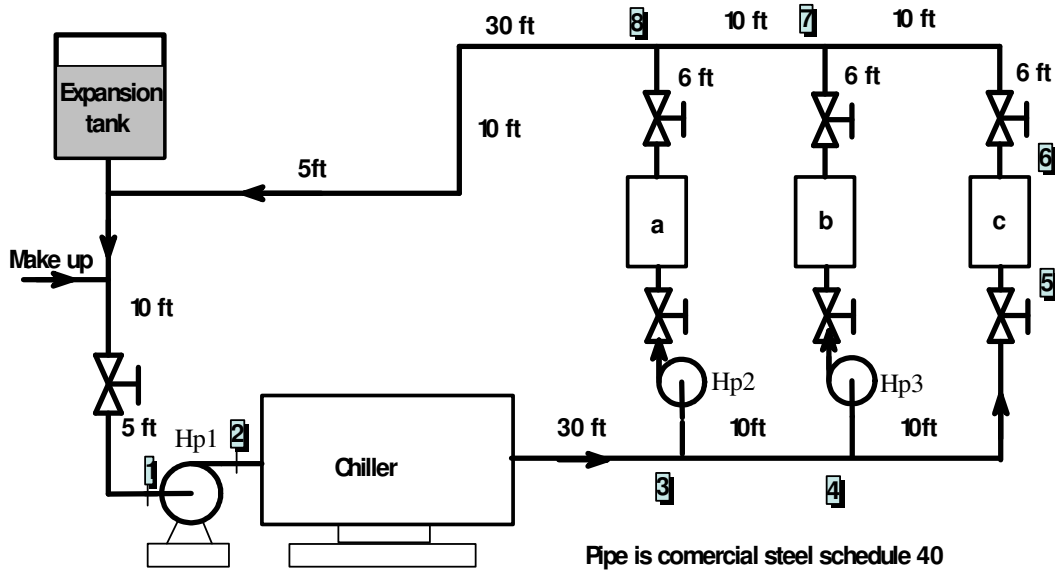


Figure 6. More Complex Hydronic Network

Computation of heads of pumps using energy equations in three paths			
Path 1-2-3-4-5-6-7-8-1			
$H_{p1} := h_{23} + h_{34} + h_{457} + h_{78} + h_{81}$	$H_{p1} = 28.312 \text{ ft}$	$Q_{p1} := Q$	$Q_{p1} = 60 \frac{\text{gal}}{\text{min}}$
Path 1-2-3-8-1			
$H_{p2} := h_{23} + h_{38} + h_{81} - H_{p1}$	$H_{p2} = 4.4 \text{ ft}$	$Q_{p2} := Q_a$	$Q_{p2} = 30 \frac{\text{gal}}{\text{min}}$
Path 1-2-3-4-7-8-1			
$H_{p3} := h_{23} + h_{34} + h_{47} + h_{78} + h_{81} - H_{p1}$	$H_{p3} = 0.024 \text{ ft}$	$Q_{p3} := Q_b$	$Q_{p3} = 20 \frac{\text{gal}}{\text{min}}$

Figure 7. Loop Equations to Determine Heads of the Pumps

Example 4. Student Design Problem

Students in this part of the class are assigned to design a pipe network. One of the design problems is represented in Figure 8. The objective was to design a pipe network including selection of components for a five-story building, as represented in this figure. The required cooling load for each floor was given to them. One of the conceptual networks developed by a

group of students is represented in Figure 9. Using this sketch and Mathcad, the dimensions of every component were obtained. Different sizes and configurations are tested and compared with supplier specifications during this stage. Finally, a solid model using Pro/Engineer was drawn, as shown in Figures 10 and 11.

Conclusion

Mathcad, and most equation solver programs, provide students with good tools to analyze a design problem, as shown in the previous examples. These programs help students concentrate in the alternative solutions rather than in the mathematics behind them. An additional benefit is that these programs retain all governing equations, making them useful for teaching purposes.

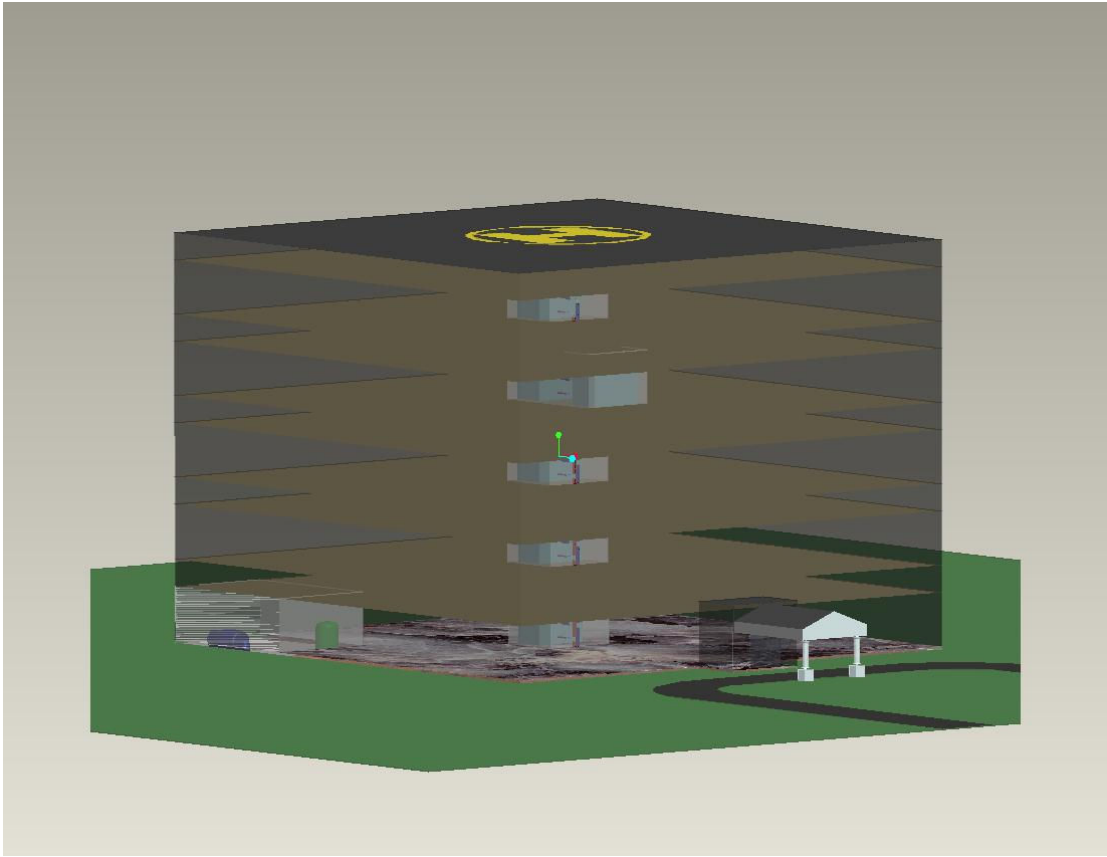


Figure 8. Design of a Hydronic System for a Building

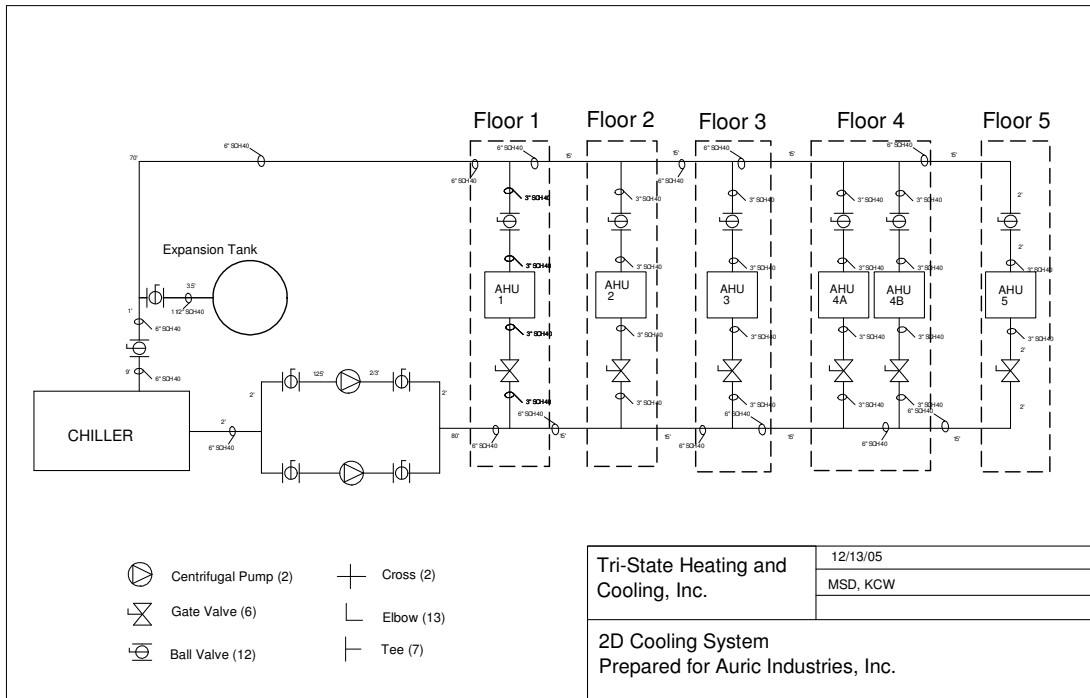


Figure 9. Two-Dimensional Sketch of Design Solution

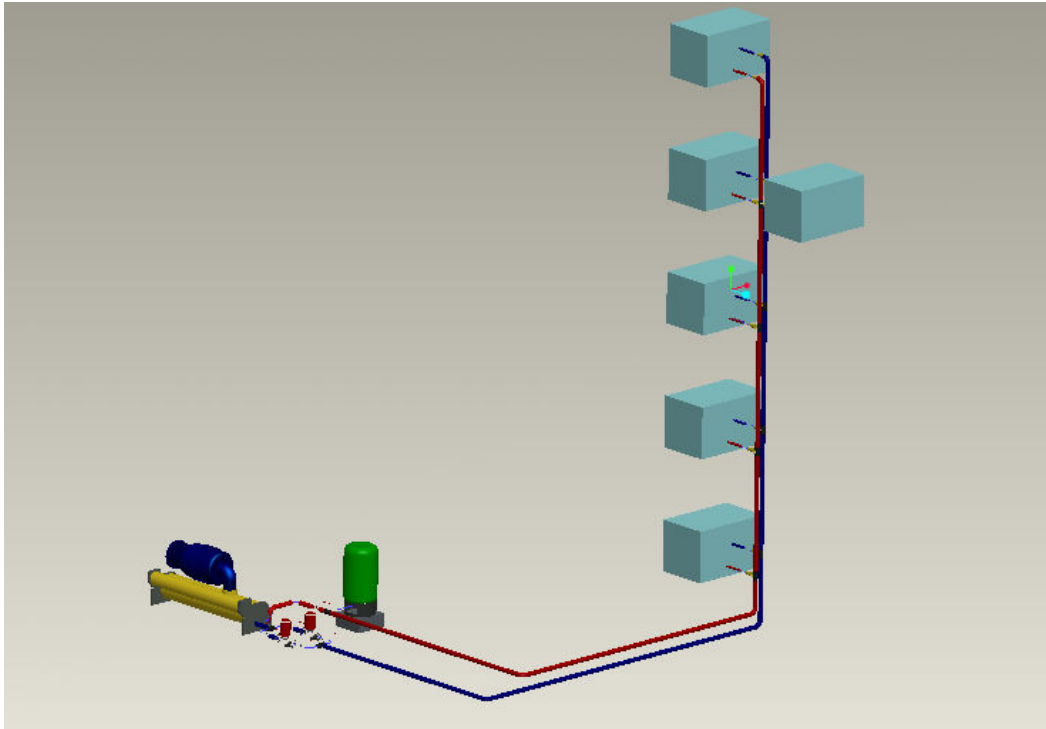


Figure 10. Three-Dimensional Representation of Design Solution

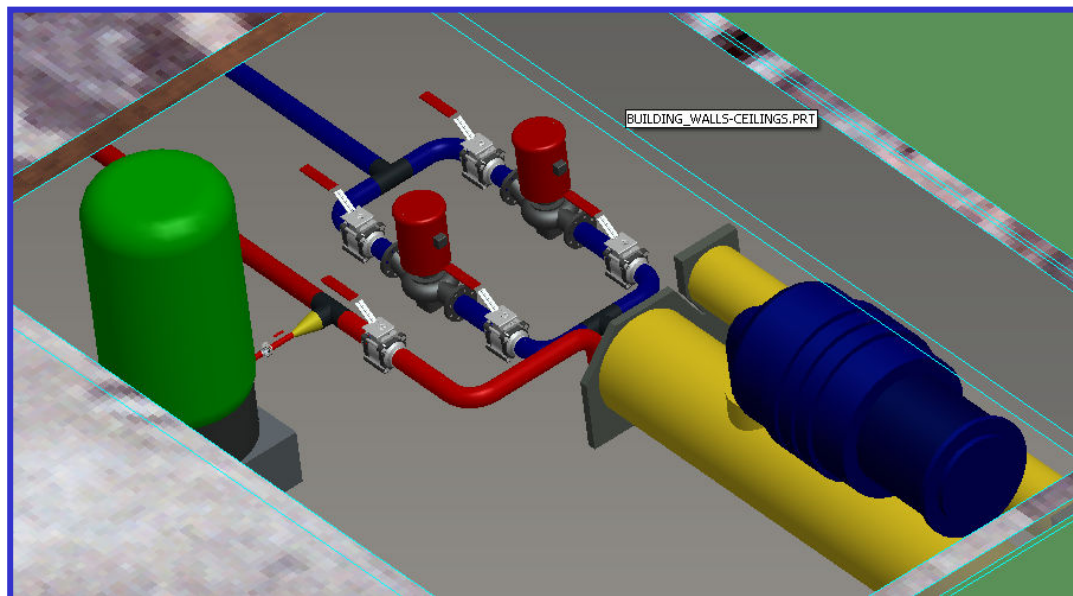


Figure 11. Solid Model of Final Design

References

- ¹ Hodge and Taylor, *Analysis and Design of Energy Systems, 3rd Edition*, Prentice Hall, 1999.
- ² Mathcad, MathSoft Engineering & Education, Inc.
- ³ Engineering Equation Solver, F-Chart Software, Box 44042, Madison, WI 53744.
- ⁴ Crowe, Elger, and Roberson, *Engineering Fluid Mechanics, 5th Edition*, Wiley, 2001, pg 419.