

Using Excel's Goal Seek and Solver Functions as Effective Computational Tools in Solving Heat Transfer Problems

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Abstract

An introductory course in heat transfer covers basic modes of heat transfer, namely conduction, convection and radiation. The analytical solutions to heat transfer problems are typically limited to steady-state one-dimensional heat conduction, simple cases of one dimensional transient conduction, two-dimensional conduction, and calculation of radiation view factors for objects displaying simple geometries. Chart or graph are provided in textbooks to aid students in solving more complex transient or multi-dimensional heat conduction problems. Empirical formulas are usually used for solving heat convection problems. The heat transfer application includes fin design and heat exchanger analysis involving simple flow patterns. Integration of a computational tool in the course is an effective way to aid students in solving more complex problems. Excel which is usually available on almost all desktop or laptop computers, can serve as an effective and inexpensive computational tool in a heat transfer course. In recent publication of papers in the proceeding of the conferences we described how Excel can be used in solving fin and heat exchanger problems. Others have demonstrated the use of Excel in solving boundary layer equations and transient heat conduction problems. This paper focuses on the application of "Solver" and "Goal Seek" functions of Excel to solve heat transfer problems requiring iteration solution process. It presents a summary of several papers previously presented on this topic and provides several examples demonstrating the application of Excel in solving heat transfer problems.

Introduction

The topic coverage in an undergraduate course in heat transfer typically includes an introduction to basic modes of heat transfer, solutions of steady state and transient conduction problems, introduction to free and forced convection, and an exposure to radiation heat transfer. Introductions to condensation and boiling heat transfer processes may also be included in the coverage. Fin design and heat exchanger analysis are also included in the course introducing to examples of practical heat transfer applications.

The focus of analytical solutions to heat transfer problems in an introductory heat transfer course is usually limited to one-dimensional conduction problems, including fins. The coverage of analytical solutions to transient and multi-dimensional heat transfer problems is very narrow. Empirical equations are mainly used for solving convective heat transfer problems. Many of the more complex analytic solutions to heat transfer problems presented in most textbooks¹⁻¹⁵ are in forms of graphs or charts. A few examples include graphs for fin efficiencies, transient temperature distribution charts for heat transfer in slabs, cylinders, or spheres (Heisler Charts),

heat exchanger correction factors, NTU-effectiveness charts, and radiation shape (view) factor charts.

Even though the topic coverage of fundamentals of heat transfer concepts has not changed very much in the last 40 years, there has been a tremendous evolution in the computational tools that can be employed in solving heat transfer problems. For example calculators replaced slide rules in the early 1970's as the basic computational tool for solving engineering problems. A few years later programmable calculators became available and modules containing basic solutions to heat transfer problems were developed for these calculators. In addition, authors began to include sections in their textbooks, introducing students to numerical techniques for solving heat transfer problems.

Prior to the introduction of personal computers (PCs) in the early 1980's, complex computer codes were needed for numerical solution of heat transfer problems. Access to mainframe computers and proficiency in such programming languages as FORTRAN and PASCAL were necessary for solving complex heat transfer problems. As the personal computers became more available and affordable, and as the operating systems became more user friendly, their applications were gradually integrated into introductory heat transfer courses. BASIC programming language was used for solving simple heat transfer problems. Similarly, spreadsheets became useful tools for solving engineering problems.

Traditionally in the past, all mechanical engineering programs required a course in one of the structured computer programming languages. The knowledge of a computer programming language was essential to solve engineering problems numerically. However, in more recent years, many mechanical engineering degree programs no longer require a course in computer programming. The trend is now shifted toward using software packages to solve problems numerically. Currently, many publishing companies provide computer software with heat transfer textbooks¹⁰⁻¹⁵. There are also other software packages available in the market that could be used to solve heat transfer problems. Many of the software packages available today are extremely useful tools which could be utilized for analysis and design in undergraduate or graduate introductory heat transfer courses. The most significant advantage of these software programs is that no prior knowledge of programming language is necessary in their applications.

Currently, the most commonly used software package accompanying heat transfer text books are Interactive Heat Transfer (IHT)¹⁶ and Engineering Equation Solver (EES)¹⁷. These programs are general purpose, non-linear equation solvers with built-in property functions. They are capable of exploring and graphing the effects of change in variables on the solution to a given problem. There are also software packages available in the market that could be integrated into a heat transfer course and used in the analysis and design of heat exchangers. These include Microsoft Excel spreadsheet, Mathcad, MATLAB, and Maple. All these software programs can be used as useful tools in solving open-ended problems or parametric studies of heat transfer problems. Excel, which is available on almost all desktop or laptop computers, is an example. This paper focuses on the application of Microsoft Excel in solving heat transfer problems.

Excel Spreadsheet

It has been shown¹⁸⁻²⁷ that Excel is an effective computational tool in solving heat transfer problems. Excel operates with data and formulas entered by the user into a spreadsheet. This software recognizes 39 engineering functions, as well as various math and trigonometry functions. The engineering functions include Bessel functions, error functions, and other functions necessary for the analysis of heat transfer problems.

To use engineering functions in the formulas entered into cells of a worksheet, the insert button on the Excel spreadsheet could be used. Then clicking on function, a window appears on the screen, as shown on Fig. 1. One can search for the desired function by typing a description of the function (financial, engineering, etc.) in the search box or using the “select category” box by scrolling through options for the desired function.

For problems requiring iterative calculations, the “Goal Seek” or “Solver” tools can be employed. By using the tool menu and selecting the solver option a dialog box appears, as shown in Fig 1. By selecting the target cell and fixing the desired value for that cell, values in the selected cells automatically change to correspond to the solution given for the target cell. This will be demonstrated later in several examples.

The following sections demonstrate how “Goal Seek” or “Solver” functions of Excel can be used as a tool to solve heat transfer problems requiring trial and error process. Several examples requiring solutions for steady heat conduction in fins, transient conduction in a semi-infinite region, boundary layer similarity transform equations, and heat exchanger problems are presented to demonstrate the effectiveness of Goal Seek and Solver functions of Excel.

Application Examples of Excel

One-Dimensional Heat Conduction in Fins

The coverage analytical solution of conduction in fins in undergraduate heat transfer textbooks is usually limited to fins of uniform cross-sectional area. For more complex fin configurations, only efficiency charts are provided in most heat transfer textbooks¹⁻¹⁵. Analysis for fins of variable cross-sectional areas or annular fins results in more complex differential equations. The solutions for temperature distribution involve complex functions such as Bessel functions. The analyses for these types of fins are not typically fully covered in an introductory heat transfer course. Instead the results are shown in the form of fin efficiency charts.

The fin efficiency is defined as

$$\eta_f = \frac{q_{act}}{q_{max}} = \frac{q_{act}}{hA(T_o - T_\infty)} \quad (1)$$

where T_o and T_∞ are the temperature at the base ambient temperatures, respectively, h is the heat transfer coefficient, A is the fin surface area, q_{act} denotes the actual heat transfer, q_{max} represents the maximum theoretical heat transfer by assuming that the entire fin is at the base temperature.

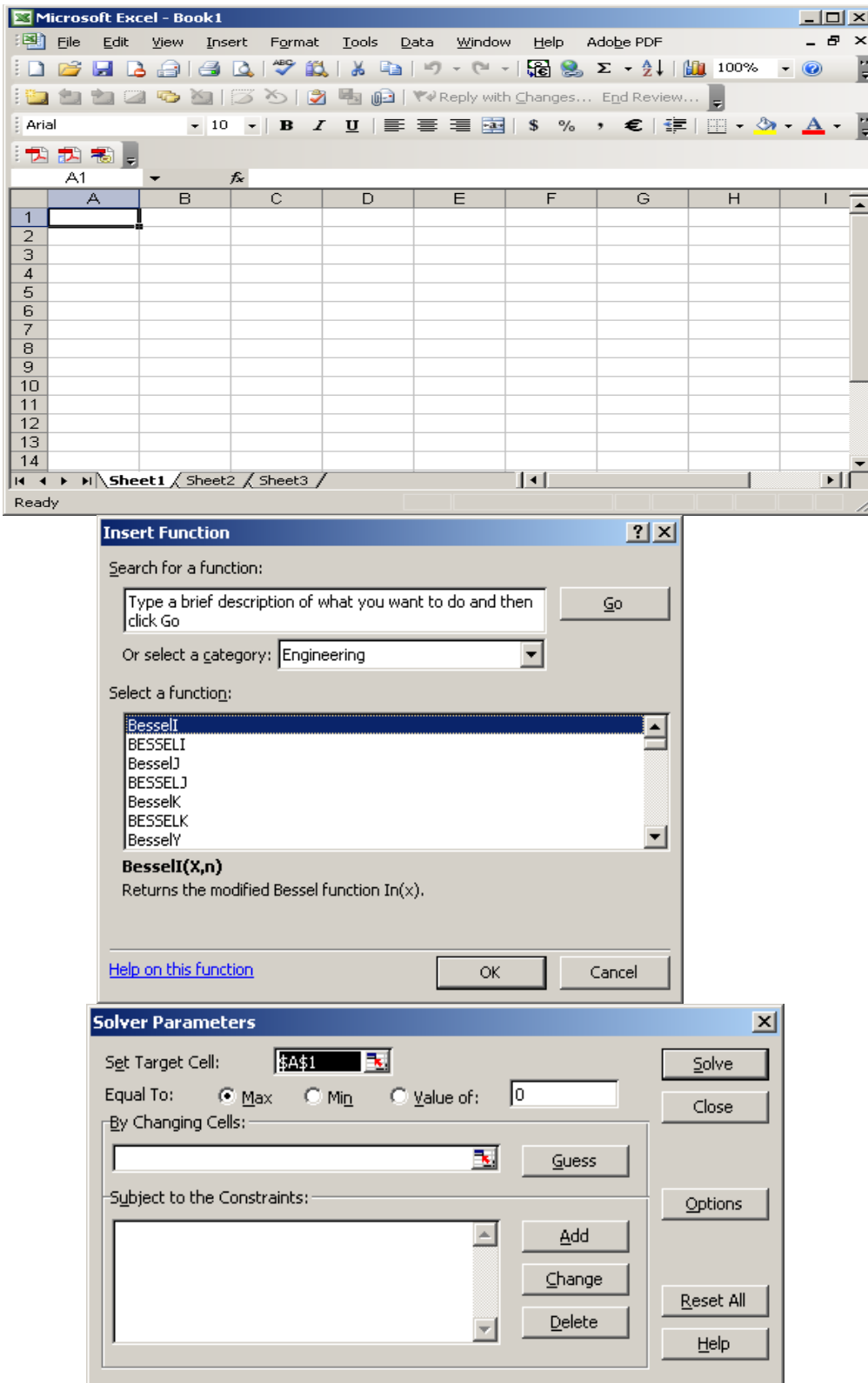


Fig. 1. Excel worksheet, function selection menu, and solver window

Fin efficiency charts approximate the rate of heat transfer, but do not provide any information on the temperature distribution in fins. Microsoft Excel can be a useful tool in solving heat conduction problems for a variety of fin configurations. Several modern textbooks⁹⁻¹² provide expressions for the efficiency of most common fin shapes. Somerton, *et.al.*¹⁸ and Karimi¹⁹ have demonstrated the use of Excel spreadsheet in solving one dimensional heat conduction problems in fins. The followings are two examples¹⁹.

Example 1

A straight fin of triangular profile (axial section) 0.1 m in length, 0.02 m thick at the base, and 0.2 m in depth is used to extend the surface of a wall at 200°C. The wall and the fin are made of mild steel ($k = 54 \text{ W/m}\cdot\text{°C}$). Air at 10 °C ($h = 200 \text{ W/m}^2\cdot\text{°C}$) flows over the surface of the fin. Evaluate the temperature at 0.05 m from the base and at the tip of the fin. Determine the rate of heat removal from the fin and the fin efficiency.

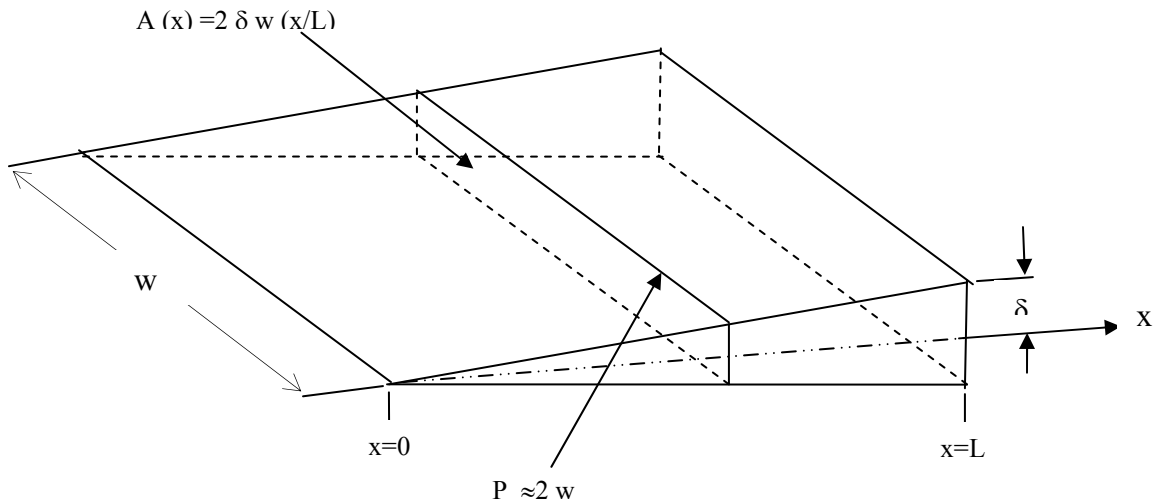


Fig. 2. Sketch of triangular fin in Example 1

Solution

An analytical solution to this problem⁷ gives the following expression for the dimensionless temperature distribution

$$\theta = \frac{T - T_{\infty}}{T_L - T_{\infty}} = \frac{I_0\left(2\sqrt{hLx/k\delta}\right)}{I_0\left(2\sqrt{hL^2/k\delta}\right)} = \frac{I_0\left(m\sqrt{Lx}\right)}{I_0\left(mL\right)} \quad (2)$$

where L is the length of the fin, x is the distance from the tip of the fin, δ is one half of the thickness at the base, $m = 2\sqrt{h/k\delta}$, and I_0 is the modified Bessel function of the first kind of order zero.

The rate of heat removal can be calculated by evaluating heat transfer at the base of the fin, where $x=L$.

$$q = -kA \left. \frac{dT}{dx} \right|_{x=L} \quad (3)$$

Thus the rate of heat transfer at the base can be expressed by

$$q = -2w\sqrt{hk\delta} (T_o - T_\infty) \frac{I_1\left(2\sqrt{hL^2/k\delta}\right)}{I_o\left(2\sqrt{hL^2/k\delta}\right)} \quad (4)$$

where w represents the width of the fin. The rate of heat removal from the base is equal to $-q$. Therefore, the fin efficiency can be determined by the following relation

$$\eta_f = \frac{-q}{2h\sqrt{L^2 + \delta^2} [w(T_o - T_\infty)]} \quad (5)$$

The formulation of solution in Excel for this problem is shown in Fig. 3. The data given in the problem statement are first entered into the cells of the worksheet. Using these data, the formulas for evaluating m , mL , $m\sqrt{xL}$, $I_o(mL)$, $I_1(mL)$, $I_o(mL)$, $I_o(m\sqrt{xL})$, θ , T , q , and η are entered into appropriate cells of the worksheet. To enter formulas an “=” sign is first entered into the cell followed with the terms needed for the evaluation of the formula. The basic mathematic operators used are +, -, * (multiplication), /, and ^ (power). The calculated results are presented in Fig. 4. By pressing CTRL + ` (grave accent) one can switch between the worksheet displaying formulas and their resulting values.

The worksheet shown in Fig. 4 can be expanded to evaluate the temperature profile in the fin and plot the results. To achieve this, the values for x ranging between 0 and 0.1 are entered in column A (cells A16 through A-26), as shown in Fig. 5. Then the cells B16 through E16 are highlighted and copied into lower rows by clicking on the bottom boundary corner of cell E16 and dragging it all the way to cell E26. By this copying action the values of $m\sqrt{xL}$, $I_o(m\sqrt{xL})$, θ , and T are automatically calculated for each value of x listed in column A. To plot T as a function of x , the cells A15 through A26 and E15 through E26 were first highlighted by pressing the Ctrl key. Then by clicking the chart wizard icon on the menu bar of the worksheet, a menu appears offering several standard options for plotting data. The x-y (scatter) option was selected and the four steps of chart wizard were preformed by providing the necessary information in each step and pressing the next button. Finally the Finish button was pressed to show the results in the worksheet.

It should be noted that the derivation of equations derived for temperature profile and heat transfer are based on the assumption of one-dimensional heat conduction in the axial direction of the fin. For this assumption to be valid, the Biot number, Bi , must satisfy the following condition

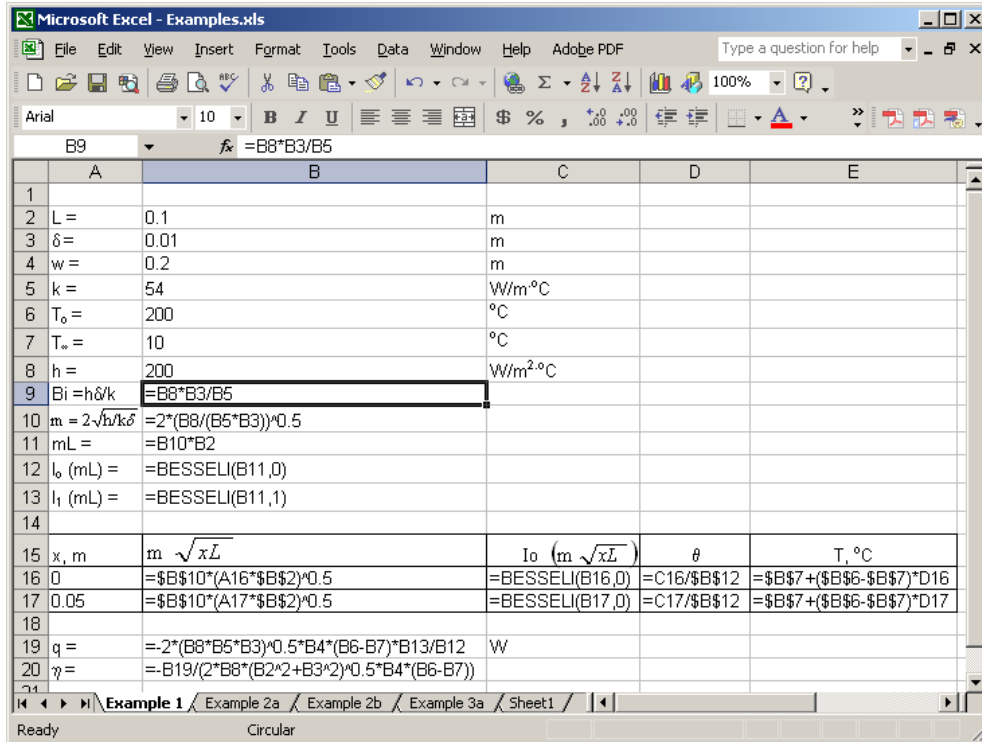


Fig. 3. Excel formulation of the solution for problem in Example 1.

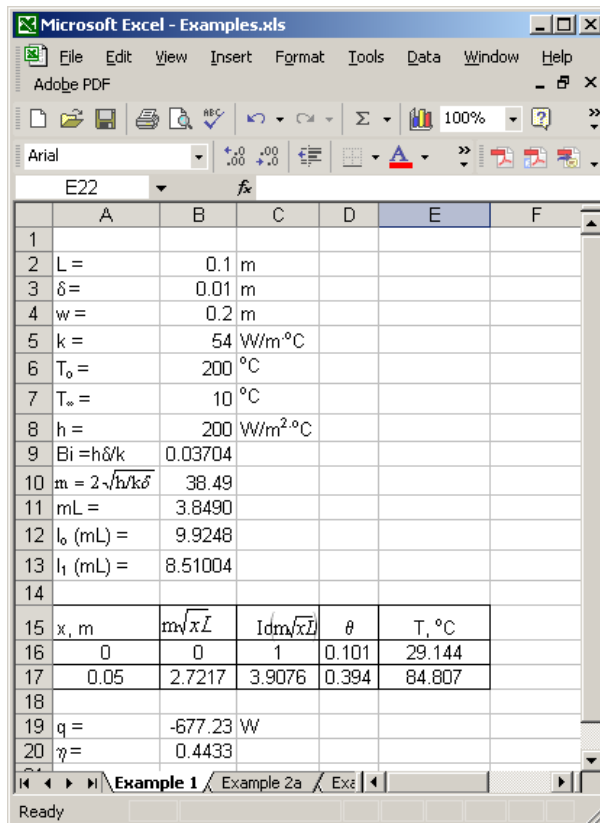


Fig. 4. Solution to Example problem 1

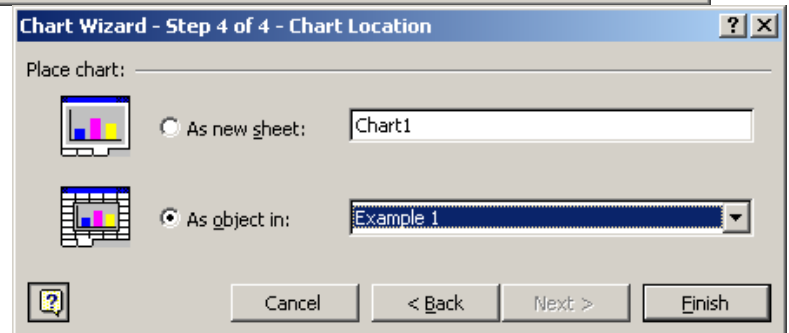
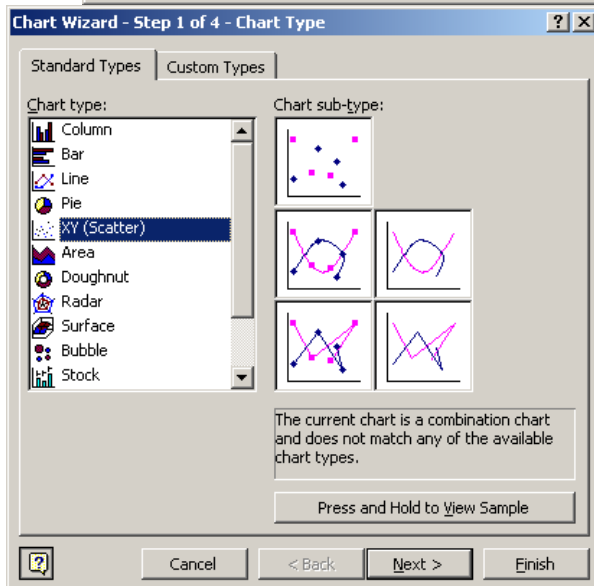
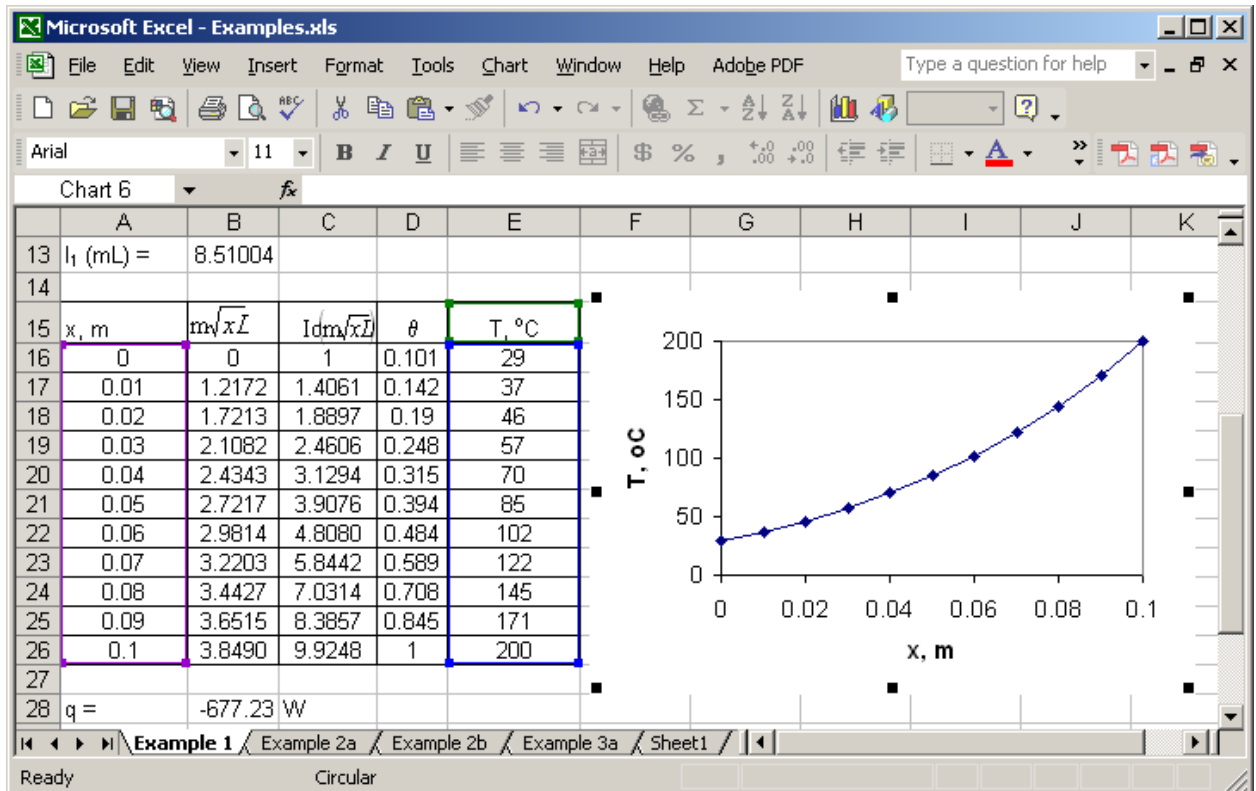


Fig. 5. Procedure for the evaluation and plotting of the temperature profile in Example 1

$$Bi = \frac{hL_{ch}}{k} = \frac{h(A/P)}{k} < 0.1 \quad (6)$$

where L_{ch} is a characteristic length, A is the cross sectional area, and P is the perimeter of the fin. For fins of circular cross sectional area, L_{ch} can be represented by the radius, R .

Example 2

A fin of triangular profile (axial section) 0.1 m in length, 0.02 m thick at the base, 0.2 m in depth is used to extend the surface of a wall at 200°C. The wall and the fin are made of mild steel ($k = 54 \text{ W/m}\cdot\text{°C}$). Air at 10°C ($h = 200 \text{ W/m}^2\cdot\text{°C}$) flows over the surface of the fin. Evaluate the distance from the base where the temperature is 175°C.

Solution:

The solution to this problem is based on the same equations used in the previous example. However, in this case the distance, x , cannot explicitly be determined, since it is a part of the argument for Bessel function in Eq. (2). A trial and error procedure is required to solve this problem.

An Excel spreadsheet can be used to solve this problem. One method is to use the same solution used in example 1, but in this case the values of x in the spreadsheet can be changed to achieve the desired temperature. The result of this procedure is shown in Fig. 6.

	A	B	C	D	E	F	G
2	L =	0.1 m					
3	$\delta =$	0.01 m					
4	w =	0.2 m					
5	k =	54 W/m \cdot °C					
6	$T_o =$	200 °C					
7	$T_\infty =$	10 °C					
8	h =	200 W/m $^2\cdot$ °C					
9	$Bi = h\delta/k$	0.03704					
10	$m = 2\sqrt{hk/\delta}$	38.49					
11	mL =	3.8490					
12	l_0 (mL) =	9.9248					
13	l_1 (mL) =	8.51004					
14	x, m	$m\sqrt{xL}$	l_0 (m \sqrt{xL})	θ	T, C		
15	0	0	1	0.1008	29.14		
16	0.05	2.722	3.908	0.3937	84.81		
17	0.06	2.981	4.808	0.4844	102.04		
18	0.07	3.220	5.844	0.5889	121.88		
19	0.08	3.443	7.031	0.7085	144.61		
20	0.09	3.651	8.386	0.8449	170.54		
21	0.091	3.672	8.531	0.8596	173.32		
22	0.092	3.692	8.678	0.8744	176.13		
23	0.0915	3.682	8.604	0.8670	174.72		
24	0.09155	3.683	8.612	0.8677	174.86		
25	0.09156	3.683	8.613	0.8678	174.89		
26	0.09157	3.683	8.615	0.8680	174.92		
27	0.09158	3.683	8.616	0.8681	174.95		
28	0.091598	3.684	8.619	0.8684	175.00		

Fig. 6 Solution of Example 2 by a trial and error procedure

A simpler way to solve the problem in Example 2, is to take the advantage of “Goal Seek” tool in Excel. The procedure and the final solution are shown in Fig.7. Figure 7-a shows the value of the temperature at an arbitrary position in the fin. By using the tool menu, and selecting the Goal Seek option a dialog box appears, as shown in Fig 7. The target cell (temperature in this case, cell E16) then is selected and its value is set to a desired value for that cell (175). The cell that its value must be changed is identified (cell A16). After clicking on the Solve button, the value in the selected cell A16 (x) is automatically changes to a value that yields the desired temperature of 175 °C in the target cell (E16). The solution is presented in Fig. 7-b.

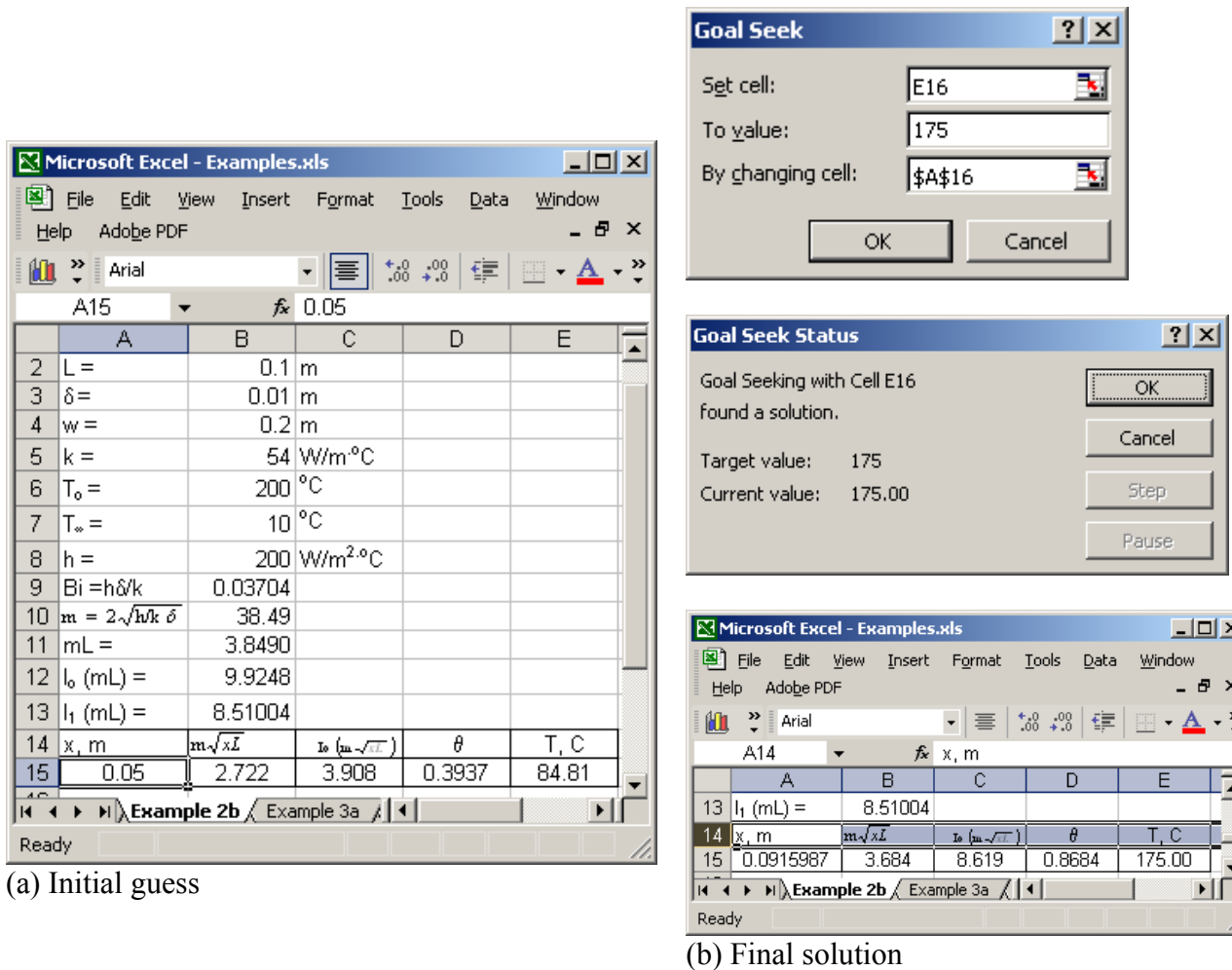


Fig 7 Procedure of using the Goal Seek tool to find x where T =175°C.

Transient and Multi-Dimensional Conduction Problems

Recently, Sarker and Ketkar²⁰ described the use of Excel in solving one-dimensional transient heat conduction problems. In this work the general heat diffusion equation in a cylindrical coordinate system was simplified by assuming no internal heat generation and ignoring heat transfer in the axial and angular directions. The resulting equation was transformed into finite difference equations and the resulting matrix for the system of equations was solved using Excel. In another attempt, Dent, *et.al*²¹ described a procedure for using Excel to find the temperature

profiles from infinite series equations given for one-dimensional transient heat conduction problems. Baughn²² has developed a unified numerical technique for solving multi-dimensional steady state and transient conduction problems using Excel spreadsheet. The numerical scheme uses Gauss-Seidel iteration process for steady state problems and explicit method for transient problems. Karimi, *et.al*²³ provided an example for the application of Excel in solving transient heat conduction problems in a semi-infinite slab. The following is an example of a problem that the solution requires a trial and error procedure.

Example 3

A semi-infinite concrete slab ($k=0.8 \text{ W/m}\cdot\text{°C}$) having a uniform temperature of 55°C is suddenly exposed to an air stream of 10°C . The average heat transfer coefficient is $15 \text{ W/m}^2\cdot\text{°C}$. Determine the distance below the surface of the slab where the temperature reaches 45 °C after 20 minutes. Thermal diffusivity of concrete is $\alpha = 5.31 \times 10^{-7} \text{ m/s}^2$.

Solution

An analytical solution for transient temperature distribution in a semi-infinite slab is expressed as⁷

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = \text{erf} \frac{\zeta}{2} + \exp(\beta\zeta + \beta^2) \left[\text{erfc} \left(\frac{\zeta}{2} + \beta \right) \right] \quad (7)$$

where

$$\beta\zeta = \frac{hx}{k} \quad (8)$$

$$\beta^2 = \frac{h^2\alpha t}{k^2} \quad (9)$$

where erf denotes error function and erfc is the complimentary error function. Since the location x is a part of the arguments for erf and erfc, it can not be found explicitly from Eq. (7). Therefore, the solution requires a trail and error process. Figure 8 shows the result of the trial and error process, using an Excel spreadsheet. The formulas for the parameters of Eq. (7) were entered into cells B9 through F9. An initial value is assumed for x (0.002 m) and entered into cell A-9. The initial guess for x resulted in a value of 39.4 °C for the temperature (cell F9). Cells B9 through F9 were copied into the following rows and the value of x was changed in each row until column F produced a temperature close to 45 °C . Figure 8 shows that when $x = 0.014 \text{ m}$, the corresponding temperature is 44.8 °C and when $x = 0.016 \text{ m}$, the corresponding temperature is 45.6 . Therefore, Fig. 8 suggests that x should be slightly larger than 0.014 m where temperature is 45 °C .

Solver tool of Excel can be used to speed up the iteration process. Again the formulas for the parameters in Eq. (7) were entered into cells B-9 through F-9. The solution process is presented in Fig. 9. An initial value was assumed for x (0.001 m) and entered in cell A-9, which resulted in a corresponding temperature of 38.9 °C (Fig 9-a). Using the tool menu on the spreadsheet, Solver was selected. A dialog box appeared (Fig. 9-b) for setting the Solver parameters. In this box the target cell was set to F9 and its value was set to 45. Cell A9 was entered for the

parameter that its value to be changed by the Solver. Then the “solve” button is clicked which produced the final result as shown in Fig. 9-c. The solution shows that at $x = 0.0144$ m below the surface, the temperature is 45°C .

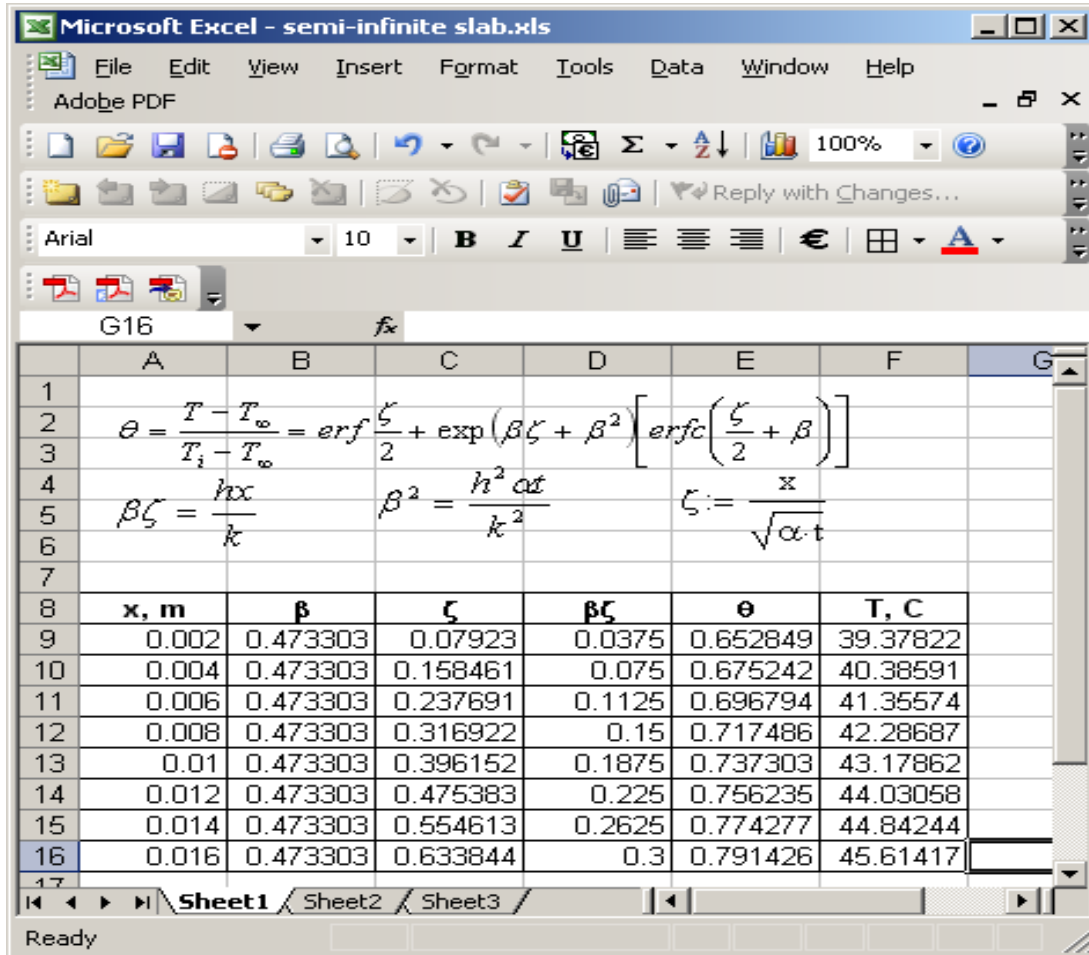
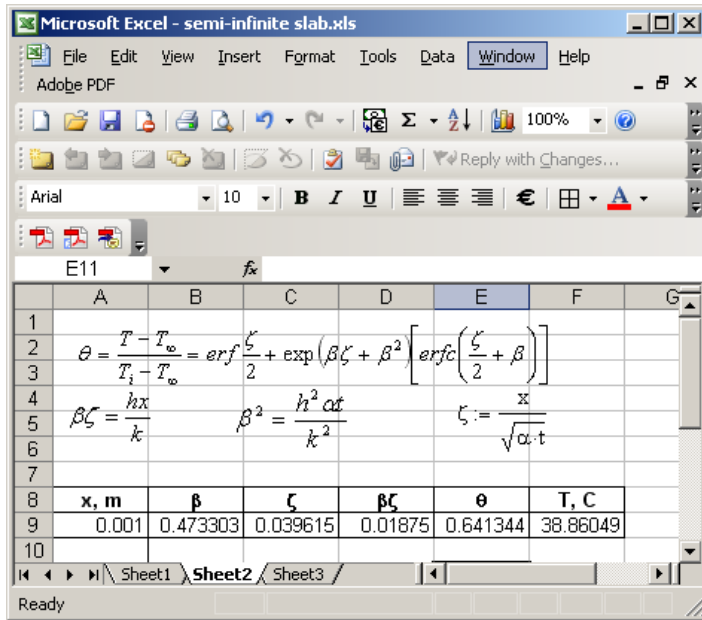
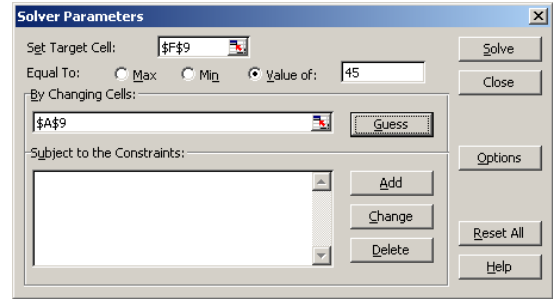


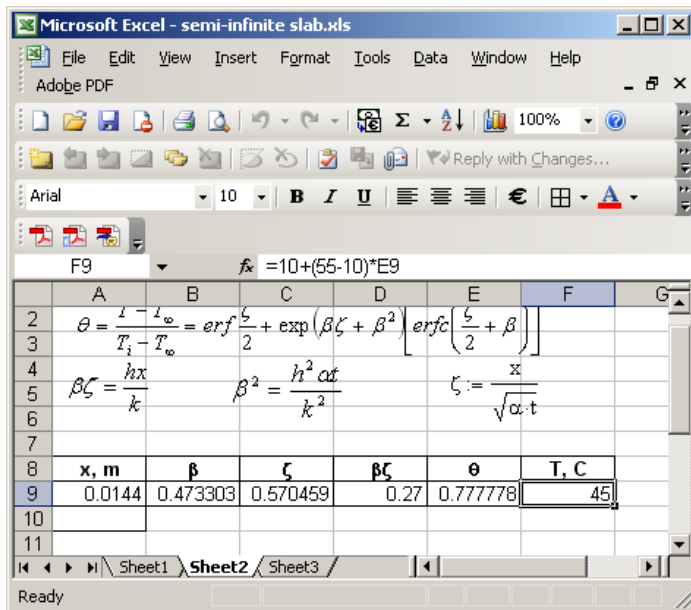
Fig. 8 Solution of example 3 using a trial and error process



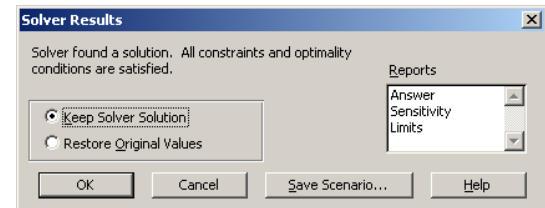
(a)



(b)



(c)



(d)

Similarity Solution for Laminar Flow over Isothermal Flat Plate

Assuming steady incompressible laminar flow with constant fluid properties, the continuity, momentum, and energy equations, respectively, are given by the following relations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (10)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (11)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (12)$$

Using the Blasius method, the continuity and momentum equations reduce to a single ordinary differential equation. The velocity components in the boundary layer are defined as

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x} \quad (13)$$

where $\psi(x, y)$ is the stream function. Equation (11) is satisfied by substituting velocity components defined in Eq. (13). Substituting the following similarity variables, Eq. (12) reduces to an ordinary differential equation.

$$\eta \equiv y \sqrt{u_\infty / \nu x} \quad (14)$$

$$f(\eta) \equiv \frac{\psi}{u_\infty \sqrt{\nu x / u_\infty}} \quad (15)$$

The resulting ordinary differential equation is expressed as

$$2 \frac{d^3 f}{d\eta^3} + f \frac{d^2 f}{d\eta^2} = 2f''' + f \cdot f'' = 0 \quad (16)$$

The standard boundary conditions are

$$f|_{\eta=0} = \frac{df}{d\eta}|_{\eta=0} = f'|_{\eta=0} = 0 \quad \text{and} \quad \frac{df}{d\eta}|_{\eta \rightarrow \infty} = f'|_{\eta \rightarrow \infty} = 1 \quad (17)$$

It can be shown¹¹ that the velocity components u and v can be express as

$$u = u_\infty \frac{df}{d\eta} \quad (18)$$

$$v = \frac{1}{2} \sqrt{\frac{\nu u_\infty}{x}} \left(\eta \frac{df}{d\eta} - f \right) = \frac{1}{2} \sqrt{\frac{\nu u_\infty}{x}} (\eta \cdot f' - f) \quad (19)$$

Therefore, the velocity components in the hydrodynamic boundary layer can be obtained from the solution of Eq. (16).

Defining the following expression for the dimensionless temperature

$$\theta = \frac{T - T_w}{T_\infty - T_w} \quad (20)$$

and substituting Eqs. (18) and (19), Eq. (12) reduces to

$$\frac{d^2\theta}{d\eta^2} + \frac{\text{Pr}}{2} f \frac{d\theta}{d\eta} = \theta' + \frac{\text{Pr}}{2} f \theta' = 0 \quad (21)$$

where Pr is the Prandtl number. The applicable boundary conditions for Eq. (21) are

$$\theta|_{\eta=0} = 0 \quad \text{and} \quad \theta|_{\eta \rightarrow \infty} = 1 \quad (22)$$

The standard procedure to find the velocity profile in the hydrodynamic boundary layer is to obtain a solution to Eq. (16) using a numerical scheme. The results of the solution of Eq. (16) are used in Eq. (21) to determine the temperature profile in the thermal boundary layer. The solution of equation (16) is necessary for the evaluation of shear stress and skin friction coefficient for the plate. The solution of Eq. (21) is needed to derive an expression for the heat transfer coefficient and Nusselt number.

The results from the solution of Eq. (16) are presented in forms of table or graphs in many undergraduate heat transfer textbooks. Very few textbooks show the temperature profile resulting from the solution of Eq. (21).

Example 4

Use a numerical scheme to solve Eqs. (16) and (21) to evaluate velocity profile in the hydrodynamic boundary layer and temperature profile in the thermal boundary layer, respectively. Use the results to develop relationships for local friction factor and Nusselt number.

Solution

Excel can be employed as a tool for solving Eqs. (16) and (21). In two separate works Fakheri²⁴ and Naraghi²⁴ demonstrated the use of Excel in solving the boundary layer problem. The following is a summary of procedure used by Naraghi²⁵.

Using a forward finite difference method, the first, second, and third derivatives of function f are expressed as

$$f'_i = \frac{f_{i+1} - f_i}{\Delta\eta} \quad (23)$$

$$f''_i = \frac{f'_{i+1} - f'_i}{\Delta\eta} \quad (24)$$

$$f'''_i = \frac{f''_{i+1} - f''_i}{\Delta\eta} \quad (25)$$

Equations (23) through (25) could be rearranged and presented in the following forms

$$f_{i+1} = f_i + f'_i(\Delta\eta) \quad (26)$$

$$f'_{i+1} = f'_i + f''_i(\Delta\eta) \quad (27)$$

$$f''_{i+1} = f''_i + f'''_i(\Delta\eta) \quad (28)$$

From Eq. (16)

$$f'''_i = -\frac{f_i \cdot f''_i}{2} \quad (29)$$

A procedure was developed to solve Eq. (16) in an Excel worksheet using Eqs. (26) through (29). Fig. 10 shows the process and the results of the procedure. As shown in Fig. 10-a, η was set to zero (0) in cell A3 and the boundary conditions $f = f' = 0$, Eq. (17), were entered into cells B3 and C3. Since $f(0)$ is set to zero, then f''' in Eq. (29) is also zero at $\eta = 0$. Therefore, cell E3 was also set to zero. The value of f'' at $\eta = 0$ is unknown and it must be determined by trial and error. Therefore, an arbitrary value of 0.8 was selected and entered into cell D3. To obtain accurate results, the increment for $\Delta\eta$ was set to 0.01 and entered into cell G1. The value of η was increased by an increment of $\Delta\eta$ in cells A4 through A803 where $\eta = 8$. Equations (26) through (29) were entered in cells B4 through E4, respectively. These cells were highlighted and their contents (formulas) were copied into the following rows, through row 803. The third boundary condition requires that f' should approach a value of 1 as η becomes very large. This condition is not satisfied by the cell C803 in Fig. 10-a. Therefore, the value of f' in cell D3 must be changed until the value of f' in cell C803 approaches 1.

To speed up this trial and error process the Solver tool of Excel was employed. The tool menu was used and Solver tool was selected. A dialog box appeared for entering the parameters for the Solver tool. As shown in Fig 10-b, the target cell was set to C803, the target value was set to 1, and D3 was selected for the cell which its value to be changed. After clicking on the solver button, the values of the variables in the worksheet changed to the final results. The results are presented in Fig. 10-c.

A comparison of data in Fig 10-c with the accepted value in the literature indicates that the results are highly accurate. The boundary layer thickness is defined as a location away from the surface of the plate where $u = 0.99 u_\infty$. Therefore, Eq. (18) indicates that at the edge of boundary layer thickness, f' must be equal to 0.99. Table 10-c shows that $f' = 0.99$ where $\eta = 4.9$. This compares very well with the established value of $\eta = 4.92$ (a relative error of 0.4%). The resulting value for f' at $\eta = 0$ was used in the evaluation of shear stress at the wall $\tau_{w,x}$ and the local friction coefficient, $C_{f,x}$. Fig. 10-c shows a value of 0.3298 for f' at $\eta = 0$ which compares well with the published value of 0.332 (0.7% relative error). Therefore based on this results the following relations can be expressed for $\tau_{w,x}$ and $C_{f,x}$.

$$\tau_{w,x} = 0.3298 \mu u_\infty \sqrt{u_\infty / \nu x} \quad (30)$$

$$C_{f,x} = \frac{\tau_{w,x}}{\rho u_\infty^2 / 2} = \frac{0.6596}{\text{Re}_x^{1/2}} \quad (31)$$

A similar procedure was used to solve Eq. (21) for the thermal boundary layer for various Pr numbers. This procedure was adopted by Naraghi²⁵ to solve Eq. (21) for values of Pr number ranging between 0.6 and 100, using Excel. Again a forward finite difference method was used to express the first and second derivatives of function θ and resulting equations were rearranged into the following forms

$$\theta_{i+1} = \theta_i + \theta'_i(\Delta\eta) \tag{32}$$

$$\theta'_{i+1} = \theta'_i + \theta''_i(\Delta\eta) \tag{33}$$

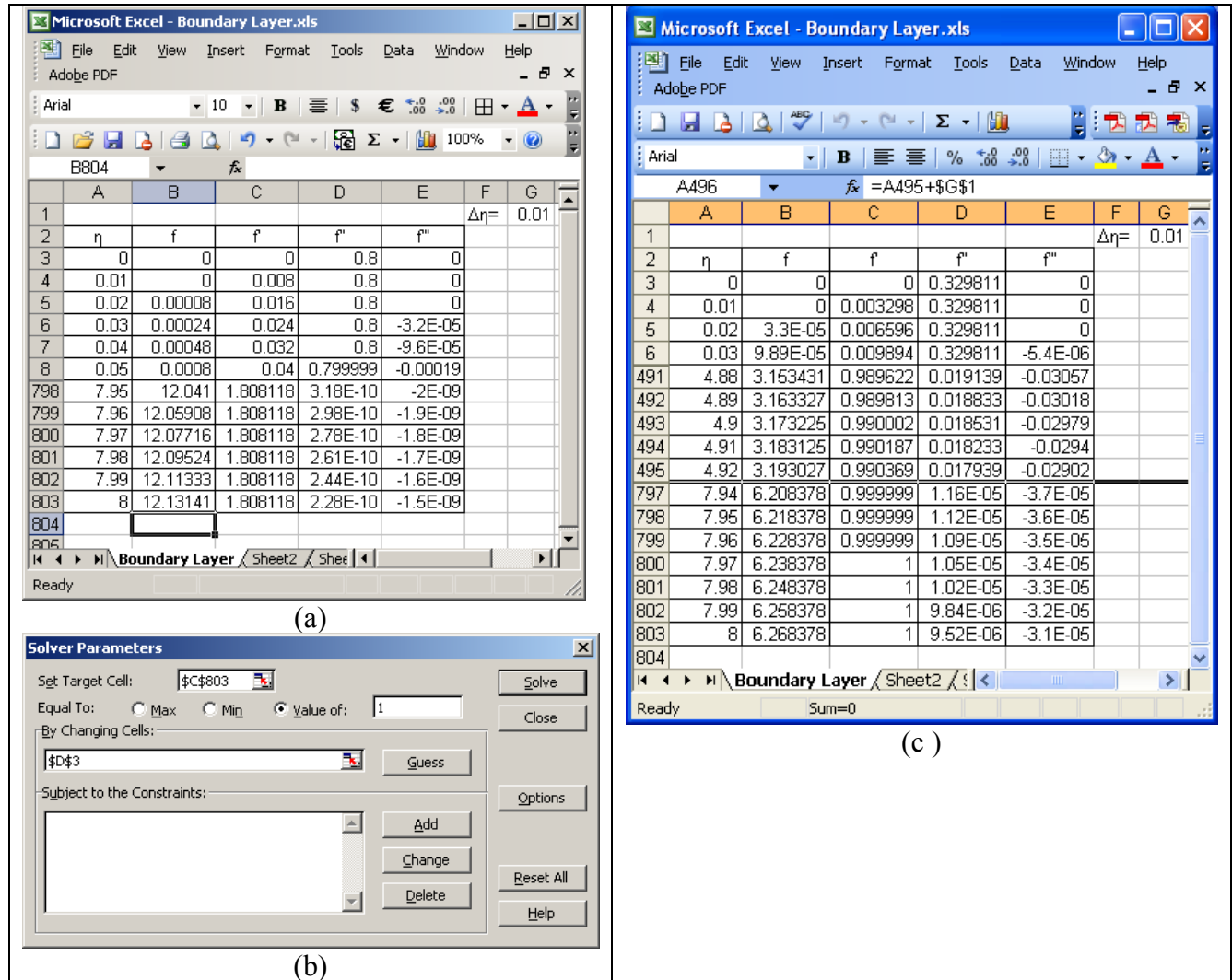


Fig. 10 Solution of hydrodynamic boundary layer equation, Eq. (16)

From Eq. (21)

$$\theta''_i = -\frac{\text{Pr} f_i \theta'_i}{2} \tag{34}$$

The Excel worksheet that resulted in the solution of hydrodynamic boundary layer equation was expanded to include columns for θ , θ' , and θ'' , as shown in Fig. 11. The boundary condition θ

$(\eta = 0) = 0$ was entered into cell F3. From Eq. (29), $\theta' = 0$ at $\eta = 0$. Therefore, cell H3 was also set to zero. The value of θ' at $\eta = 0$ is unknown and must be determined by trial and error. Again, an arbitrary value of 0.8 was selected and entered into cell G3. A specific value for Pr number was entered into cell I1 (2.0) in this case. Equations (32) through (34) were entered in cells F4 through H4, respectively. These cells were highlighted and their contents (formulas) were copied into the following rows, through row 803. The second boundary condition in Eq. (22) requires that the value of θ should approach 1.0 as η becomes very large. This condition is not satisfied by the cell C803 in Fig. 11. Therefore, the value of θ in cell G3 must be changed until the value of θ' in cell C803 approaches 1. Again the Solver tool of Excel was employed to force the solution to satisfy the second boundary condition. The target cell was set to F803, the target value was set to 1, and G3 was selected for the cell which its value to be changed. After clicking on the solver button, the values of the variables in the worksheet change to the final results. The results are presented in Fig. 12.

The thermal boundary layer thickness is defined as a location away from the surface of the plate where $\theta = 0.99$. Table 12 shows that this condition is met where $\eta = 3.79$ for $Pr = 2.0$.

The local Nusselt number can be expressed as a function of $\theta'(\eta = 0)$

$$Nu_x = \frac{hx}{k} = Re_x^{1/2} \theta'(0) \quad (35)$$

The procedure described above was repeated for pr numbers ranging between 0.6 and 100. For each value of Pr, the corresponding value of $\theta'(0)$ was entered into an Excel worksheet and the results were plotted on a graph. The equation for trend-line resulted in the following correlation ($R^2 = 0.9999$)

$$\theta'(0) = 0.3313 Pr^{0.3357} \quad (36)$$

Combining Eqs (35) and (36) yields the following relationship

$$Nu_x = \frac{hx}{k} = 0.3313 Re_x^{1/2} Pr^{0.3357} \quad (37)$$

Equation (37) compares well with the established relationship for the local Nu_x

$$Nu_x = \frac{hx}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad (38)$$

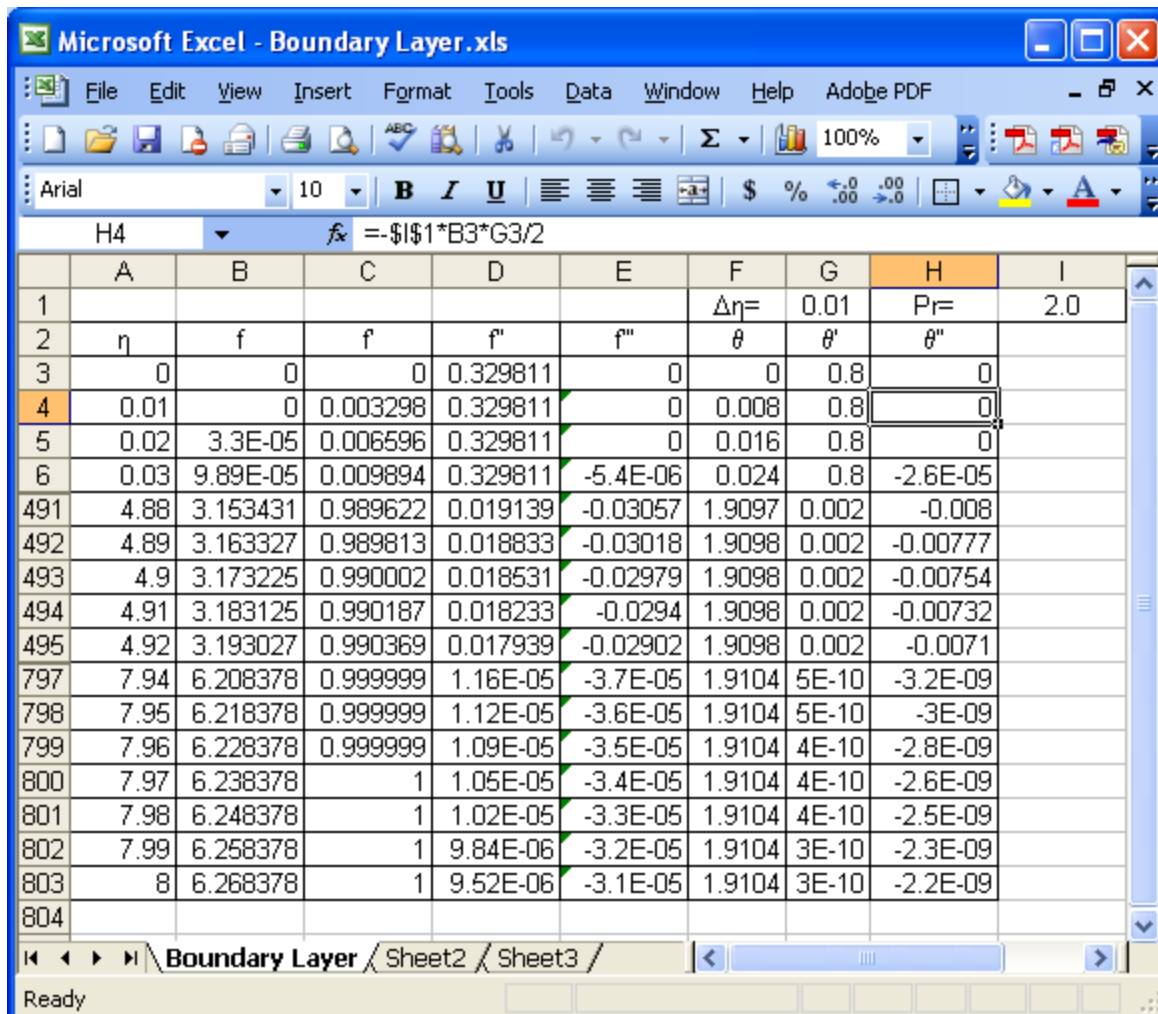


Fig. 11 Initial attempt to solve the thermal boundary layer equation, Pr = 0.2

Heat Exchanger Analysis

In an undergraduate heat transfer course students are introduced to two different methods used in the analysis and design of heat exchangers. They are Logarithmic Mean Temperature Difference (LMTD) method and Effectiveness-NTU methods. In a heat exchanger the flow heat capacity rate, in general, is defined as

$$C = \dot{m}c_p \quad (39)$$

where \dot{m} denotes the mass flow rate and c_p represents the specific heat of a given fluid stream. The rate of heat transfer from or to each fluid stream can be calculated from the following relation

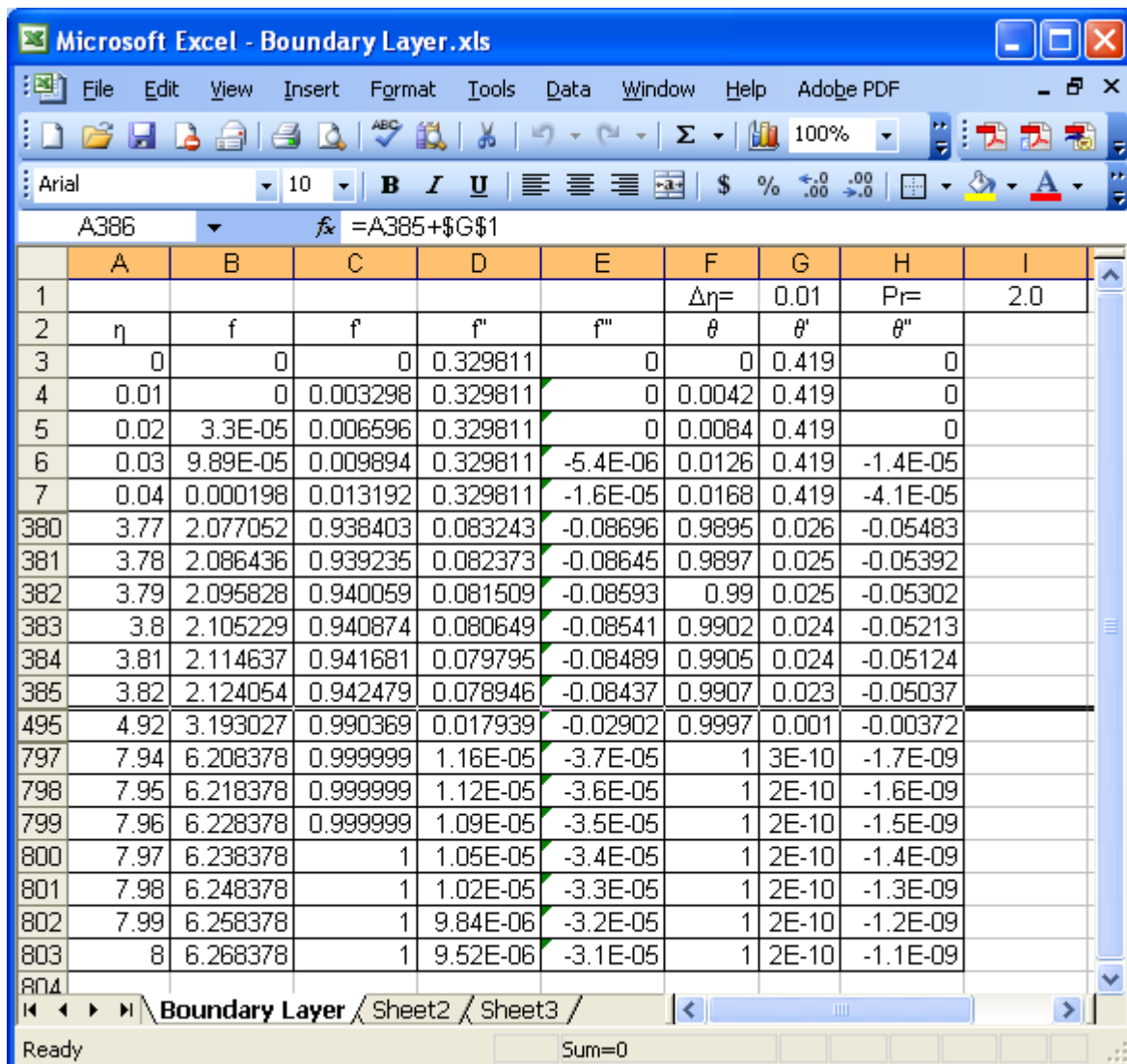


Fig. 12 Results of the solution for the thermal boundary layer equation, Pr =2.0

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}) \quad (40)$$

where q is the rate of heat transfer, T denotes temperature; subscripts c and h identify the cold and the fluids, respectively; subscripts i and o represent inlet and outlet conditions respectively. Equation (40) is the result of an energy balance on each fluid stream.

The heat transfer rate, based on heat transfer concepts, is expressed as

$$q = UA_s (F) LMTD \quad (41)$$

where U is the overall heat transfer coefficient, A_s is the surface area separating the two fluid streams, $LMTD$ is the logarithmic mean temperature difference between the two fluid streams, and F is an appropriate correction factor which value depends on the type of heat exchanger and flow conditions.

In general, LMTD can be expressed as

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a / \Delta T_b)} \quad (42)$$

where ΔT_a and ΔT_b are the temperature differences between the two fluid streams at the terminal points of the heat exchangers, as shown in Fig. 13.

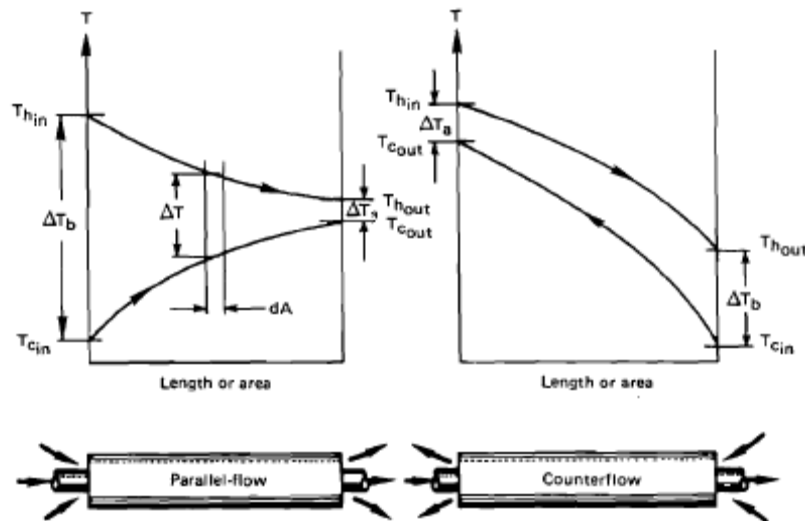


Fig. 13 Temperature variation through parallel-flow and counter flow heat exchangers⁷

The terminal point temperature differences shown for the counter-flow heat exchangers in Fig. 13 are used in Eq. (42) for the evaluation of LMTD of fluid streams in any other types of heat exchangers such as shell-and-tube or cross flow heat exchangers.

For parallel-flow and counter-flow heat exchangers, the correction factor in Eq. (41) has a value of $F=1$. For other types of heat exchangers, specific charts or equations are used for the correction factor, F . For example, Fig 14 is a correction factor chart for a one shell-pass, even number tube-pass heat exchanger.

As shown in Fig. 14, the correction factor, F , is a function of two parameters P and R . The first parameter, P , is defined as

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} \quad (43)$$

Since the denominator in Eq. (43) represents the maximum temperature difference between the two fluid streams, the value of P is always less than one. The second parameter, R , is defined as

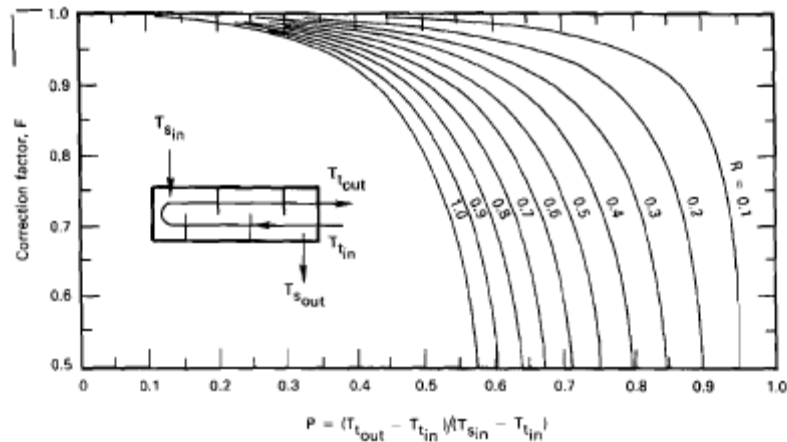


Fig. 14. LMTD correction factor, F, for a one shell-pass, even number of tube-passes heat exchanger.⁷

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{C_c}{C_h} \quad (44)$$

Depending on the flow heat capacity ratios (or temperature changes for the hot and cold fluids), the value of R could be less than one or greater than one. If the value of R in Eq. (44) ends up to be greater than one, then R should be replaced by 1/R and P replaced by PR, since Fig. 1 displays curves only for R values that are less than or equal to one. In other words,

$$F = F(P, R) = F(PR, 1/R) \quad (45)$$

LMTD method is useful for sizing heat exchangers. That is when the inlet and outlet temperatures of the fluid streams are known or could be calculated directly from Eq. (40), LMTD and correction factor, F, can be easily evaluated and used in Eq. (41) for calculating the surface area or the overall heat transfer coefficient of heat exchangers. However, when both outlet temperatures of a heat exchanger are unknown and must be evaluated, LMTD can not be evaluated explicitly from Eq. (42). Hence, in these situations an iterative procedure is required for the evaluation of LMTD and the correction factor. In these cases the effectiveness-NTU method is a more efficient procedure for heat exchanger analysis.

The effectiveness of a heat exchanger is defined as

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

or

$$\varepsilon = \frac{C_h (T_{h,i} - T_{h,o})}{C_{\min} (T_{h,i} - T_{c,i})} = \frac{C_c (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} \quad (46)$$

where the C_{\min} is the smaller of the C_h and C_c . From Eq. (40) it follows that

$$q = \varepsilon C_{\min} (T_{h,i} - T_{c,i}) \quad (47)$$

The number of transfer units is defined as

$$NTU = \frac{UA}{C_{\min}} \quad (48)$$

The capacitance ratio is defined as

$$C_R = \frac{C_{\min}}{C_{\max}} \quad (49)$$

The derivation of heat exchanger effectiveness equation for parallel-flow is given in most heat transfer textbooks¹⁻¹⁵. For a parallel flow heat exchanger, the effectiveness is expressed as

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_R)]}{1 + C_R} \quad (50)$$

In this form the effectiveness is explicitly expressed as a function of NTU and C_R . Alternatively, NTU could be expressed as a function of ε and C_R . For a parallel-flow heat exchanger, NTU is given as

$$NTU = \frac{\ln[1 - \varepsilon(1 + C_R)]}{1 + C_R} \quad (52)$$

Heat transfer textbooks provide effectiveness charts for several types of heat exchangers. For example, Fig. 15 displays the effectiveness chart for a single pass cross-flow heat exchanger, one fluid unmixed.

More recent heat transfer textbooks also provide equations for $\varepsilon = \varepsilon(NTU, C_R)$ or $NTU = NTU(\varepsilon, C_R)$ for several types of heat exchanger.

There are two inherent problems with using charts in thermal analysis of heat exchanger systems. First, the accuracy of solutions is highly dependent on how precise one can read the charts, but also in problems requiring several stages of iteration, the process could become extremely tedious. Use of equations for correction factors or the effectiveness in a numerical scheme increases the accuracy and eases the task of solving problems involving repeated calculations. Microsoft Excel is one of the tools that can be used in solving heat exchanger problems. Recently, Karimi²⁶ described methods of application of Excel in heat exchanger analysis and provided a few examples. Two examples are included

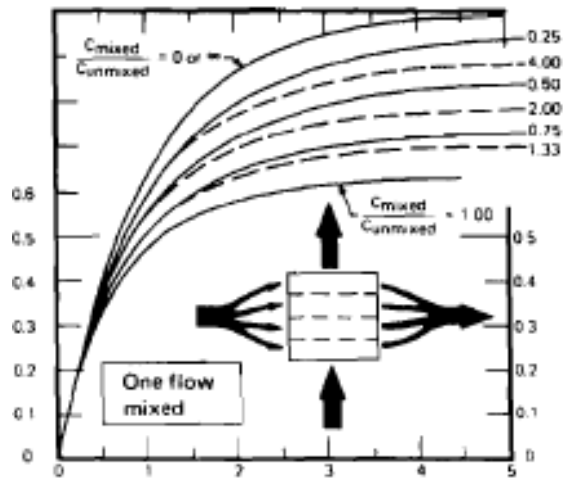


Fig. 15 Effectiveness chart for a one-pass cross-flow heat exchanger, one fluid unmixed⁷

Example 5

Water at 15 °C with a mass flow rate of 8 kg/s is available to cool hot oil from 90 °C to 30 °C. The oil mass flow rate is 4 kg/s. A shell-and-tube heat exchanger with one-shell pass and four-tube-passes is proposed for this process. Using uniform c_p values of 2.5 kJ/(kg °C) and 4.2 kJ/(kg °C) for oil and water, respectively, and assuming an overall heat transfer coefficient of 250 W/(m²·°C) for the heat exchanger

- determine the surface area of the heat exchanger
- plot the heat exchanger surface area as a function of water mass flow rate, when the mass flow rates vary between 6 and 30 kg/s.

Solution

This problem can be easily solved, using the LMTD method and the correction chart in Fig. 14. Using the specific heat and mass flow rate data given in the problem statement, Eq. (38) yields the following results:

$$C_c = 33.6 \text{ kW/}^\circ\text{C}$$

$$C_h = 10 \text{ kW/}^\circ\text{C}$$

The results are substituted into Eq. (40) to obtain $T_{c,o}$

$$q = C_h \Delta T_h = 10 \text{ kW/}^\circ\text{C} (90 - 30) ^\circ\text{C} = 600 \text{ kW}$$

$$q = 600 \text{ kW} = C_c (T_{c,o} - T_{c,i}) = 33.6 \text{ kW/}^\circ\text{C} (T_{c,o} - 15) ^\circ\text{C}. \text{ This gives}$$

$$T_{c,o} = 32.86 ^\circ\text{C}$$

Then,

$$\Delta T_a = 90 - 32.86 = 57.14 ^\circ\text{C}$$

$$\Delta T_b = 30 - 15 = 15 ^\circ\text{C}$$

Substituting ΔT_a and ΔT_b into Eq. (42), yields

$$LMTD = 31.51 \text{ }^\circ\text{C}$$

In order to evaluate the correction factor, the terminal temperatures of the heat exchanger are substituted into Eqs. (43) and (44) to find the values of P and R.

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 0.2381$$

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = 3.36$$

However, since $R > 1$, in order to use Fig. 14 to evaluate the correction factor, P needs to be replaced with PR, and R replaced with 1/R

$$PR = (0.2381)(3.36) = 0.8$$

$$1/R = 1/3.36 = 0.2978$$

Then from Fig.14, the correction factor is approximated as

$$F = 0.74$$

Substituting the known values into Eq. (41) the heat exchanger area is calculated

$$q = UA_s (F) LMTD$$

$$600 \text{ kW} = 0.250 \text{ kW}/(\text{m}^2 \cdot ^\circ\text{C}) A_s (0.74) (31.51 \text{ }^\circ\text{C})$$

$$A_s = 102.9 \text{ m}^2$$

The same procedure can be used to solve part (b) of this example, by varying the mass flow rate of water. However, it is clear that the manual solution of part (b) will consume a great deal of time without adding much to the learning process. Employing Excel will ease and speed up the calculation process.

In order to use Excel to solve part (b) of this example, the correction factor chart, Fig. 14, must be replaced by an appropriate equation. For a one-shell-pass and even number of tube passes, the equation for the correction factor is given as⁸

$$F = \frac{\sqrt{1+R^2}}{1-R} \ln\left(\frac{1-RP}{1-P}\right) \left[\ln\left(\frac{2-P(1+R-\sqrt{1+R^2})}{2-P(1+R+\sqrt{1+R^2})}\right) \right]^{-1} \quad (51)$$

where R and P are defined by Eqs. (43) and (44).

Equation (51), along with equations for C_c , LMTD, P, R, were used in an Excel spreadsheet to determine the surface area of the heat exchanger by varying the cooling water mass flow rates. Table 1 represents the results of the heat exchanger area calculations. Excel was used to plot the heat exchanger area as a function of water mass flow rate, as shown Fig. 16.

Table 1. Excel spreadsheet calculation of heat exchanger area for example 5.

m_w , kg/s	C_c kW/°C	C_h , kW/°C	ΔT_b , °C	$T_{c,o}$, °C	ΔT_a , °C	LMTD °C	P	R	F	A_s , m ²
6	25.2	10	15	38.81	51.19	29.48	0.3175	2.52	0.500	162.81
7	29.4	10	15	35.41	54.59	30.65	0.2721	2.94	0.675	115.99
8	33.6	10	15	32.86	57.14	31.51	0.2381	3.36	0.750	101.51
9	37.8	10	15	30.87	59.13	32.17	0.2116	3.78	0.796	93.73
10	42.0	10	15	29.29	60.71	32.70	0.1905	4.2	0.827	88.76
11	46.2	10	15	27.99	62.01	33.12	0.1732	4.62	0.850	85.28
12	50.4	10	15	26.90	63.10	33.48	0.1587	5.04	0.867	82.69
13	54.6	10	15	25.99	64.01	33.78	0.1465	5.46	0.881	80.68
14	58.8	10	15	25.20	64.80	34.03	0.1361	5.88	0.892	79.08
15	63.0	10	15	24.52	65.48	34.25	0.1270	6.3	0.901	77.77
16	67.2	10	15	23.93	66.07	34.45	0.1190	6.72	0.909	76.67
17	71.4	10	15	23.40	66.60	34.61	0.1120	7.14	0.915	75.74
18	75.6	10	15	22.94	67.06	34.76	0.1058	7.56	0.921	74.95
19	79.8	10	15	22.52	67.48	34.90	0.1003	7.98	0.926	74.26
20	84.0	10	15	22.14	67.86	35.02	0.0952	8.4	0.931	73.65
21	88.2	10	15	21.80	68.20	35.13	0.0907	8.82	0.934	73.11
22	92.4	10	15	21.49	68.51	35.23	0.0866	9.24	0.938	72.64
23	96.6	10	15	21.21	68.79	35.32	0.0828	9.66	0.941	72.21
24	100.8	10	15	20.95	69.05	35.40	0.0794	10.08	0.944	71.83
25	105.0	10	15	20.71	69.29	35.48	0.0762	10.5	0.946	71.48
26	109.2	10	15	20.49	69.51	35.55	0.0733	10.92	0.949	71.16
27	113.4	10	15	20.29	69.71	35.61	0.0705	11.34	0.951	70.87
28	117.6	10	15	20.10	69.90	35.67	0.0680	11.76	0.953	70.60
29	121.8	10	15	19.93	70.07	35.73	0.0657	12.18	0.955	70.36
30	126.0	10	15	19.76	70.24	35.78	0.0635	12.6	0.956	70.13

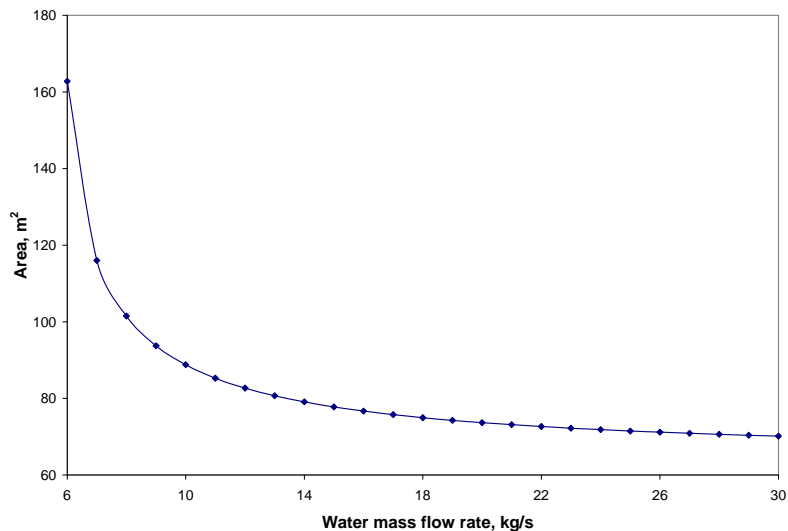


Fig. 16 Example 5, Variation of heat exchanger area with the mass flow rate of cooling water

Example 6

Consider a cross-flow heat exchanger containing a tube bank that consists of a square array of 100 thin-walled tubes (10x10), each 2.5 cm in diameter and 5 meter long. The tubes are aligned with a transverse pitch of 5 cm. Water is used in this heat exchanger to cool hot air from 800 K, to 500 K. Water makes a single pass through each tube entering at 12 °C. Hot air enters the heat exchanger with a velocity of 5.0 m/s in a cross flow over tubes with a mass flow rate of 2.25 kg/s. Determine the water mass flow rate and the exit temperature.

Solution

The following property values are given for air at an average temperature of 650 K: $c_{p,a} = 1063$ J/kg.K, $\mu_a = 322.5 \times 10^{-7}$ N.s/m, $\nu_a = 60.21 \times 10^{-6}$ m²/s, $k_a = 0.0497$ W/m.K, and $Pr_a = 0.69$. Assuming an average temperature of 340 K for water, the following property values are obtained $c_{p,w} = 4188$ J/kg.K, $\mu_w = 420 \times 10^{-6}$ N.s/m, $\nu_w = 5.35 \times 10^{-7}$ m²/s, $k_w = 0.660$ W/m.K, and $Pr_w = 2.66$

For the external flow over an aligned tube bundle, as shown in Fig 17, the maximum velocity is given by

$$u_{\max} = \frac{S_T}{S_T - D} u_{\infty} \quad (52)$$

where u_{∞} is the free stream velocity, D denotes the tube diameter, S_T represents the transverse pitch. Using the data given in the problem statement

$$u_{\max} = 10 \text{ m/s}$$

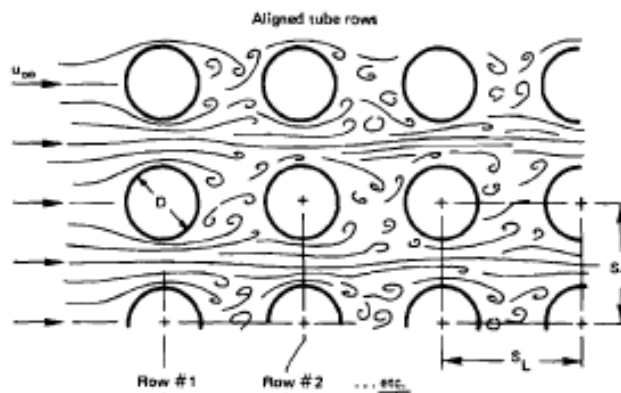


Fig. 17. External flow over an aligned tube bundle⁷

The Reynolds number based on the maximum flow velocity is expressed as

$$Re_{D,\max} = \frac{u_{\max} D}{\nu} \quad (53)$$

where ν is the kinematic viscosity. Using the kinematic viscosity value given for air

$$Re_{D,\max} = 4152.1$$

For air flow across a tube bundles consisting of 10 or more rows, Grimson²⁸ gave the following correlation for the average Nusselt number

$$Nu_{D,\max} = \frac{h_o D}{k} = C Re_{D,\max}^m \left[\begin{array}{l} n_L \geq 10 \\ 2000 < Re_{D,\max} < 40,000 \\ Pr \approx 0.7 \end{array} \right] \quad (54)$$

where n_L represents the number of rows, and h_o is the external heat transfer coefficient. The values of C and m depend on the ratios of S_T/D and S_L/D . For the case when $S_T/D = S_L/D = 2$ (this example), the values of C and m are given as 0.229 and 0.632, respectively²⁸. Based on these values, Eq. (54) reduces to

$$Nu_{D,\max} = 0.229 Re_{D,\max}^{0.632} \quad (55)$$

Substituting the value of $Re_{D,\max} = 4152.1$, Eq. (55) yields

$$Nu_{D,\max} = 44.318$$

Then,

$$h_o = k Nu_{D,\max}/D = 88.11 \text{ W/m}^2 \cdot \text{K}$$

For the internal flow, the Reynolds number is defined as

$$Re_D = \frac{u D}{\nu} = \frac{4(\dot{m}_w)_i}{\pi D \mu_w} \quad (56)$$

where $(\dot{m}_w)_i$ represents the mass flow rate in each tube. If Re_D indicates a fully developed laminar flow, the Nusselt number, assuming constant surface temperature, is given as

$$Nu_D = \frac{h_i D}{k} = 3.66 \quad (57)$$

where h_i represents the heat transfer coefficient inside the tubes. When Re_D indicates a fully developed turbulent flow, the Nusselt number, can be approximated by Dittus-Boelter equation²⁹

$$Nu_D = \frac{h_i D}{k} = 0.023 Re_D^{0.8} Pr^{0.4} \quad (58)$$

Ignoring the thermal resistance of the tube wall, the overall heat transfer coefficient is expressed as

$$U = \left[\frac{1}{h_i} + \frac{1}{h_o} \right]^{-1} \quad (59)$$

The surface area of the heat exchanger is evaluated, using the following relationship

$$A = N \pi DL = 100 \pi (0.025 \text{ m}) (4 \text{ m}) = 31.42 \text{ m}^2$$

NTU was defined in Eq. (48) as

$$NTU = \frac{UA}{C_{\min}}$$

Using the specific heat and mass flow rate data, C_h is calculated from Eq. (39)

$$C_h = C_a = 2364.8 \text{ W/K}$$

The rate of heat transfer is calculated from Eq. (40)

$$q = C_h \Delta T_h = 2364.8 \text{ W/K} (800-500) \text{ K} = 709,425 \text{ W}$$

At this point there exist too many unknowns to solve the heat exchanger problem directly either by the LMTD method or the effectiveness method. For the LMTD, the exit temperature of water is unknown and cannot be calculated directly. For the effectiveness method NTU, C_R , and ε cannot be calculated directly without the knowledge of the mass flow rate of water. Therefore, an iterative procedure is required to solve this heat exchanger problem.

We will employ the effectiveness-NTU method in the iterative procedure described below. Some steps in the procedure depend on which fluid is assumed to represent the C_{\min} . When $C_c = C_w$ is chosen as the C_{\min} , the steps operation is presented in [*brackets and italic*]. If the one assumption does not converge to an answer, then the other assumption can be implemented in iteration process).

1. Assume $C_h = C_a$ represents C_{\min} [*Assume $C_c = C_w$ represents C_{\min}*]
2. Assume a value for $C_R = C_{\min}/C_{\max}$
3. Evaluate $C_c = C_w = C_{\max} = C_{\min}/C_R = C_h/C_R$ [*evaluate $C_c = C_w = C_{\min} = C_{\max} C_R = C_h C_R$*]
4. Calculate the total mass flow rate of water, $\dot{m}_w = C_w/c_{p,w}$; $(\dot{m}_w)_i = \dot{m}_w/100$
5. Use the calculated value of $(\dot{m}_w)_i$ in Eq. (56) to evaluate Re_D
6. If the flow is laminar use Eq. (57) to evaluate Nu_D . Otherwise use Eq. (58)
7. Calculate the internal heat transfer coefficient from the results in step 6
8. Evaluate the overall heat transfer coefficient from Eq. (59)
9. Evaluate NTU from Eq. (48)
10. Substitute the values of NTU and C_R in an appropriate effectiveness equation. For a cross-flow heat exchanger when fluid representing C_{\min} is mixed and fluid representing C_{\max} is unmixed, the effectiveness, ε , is expressed as

$$\varepsilon = 1 - \exp\left\{-C_R^{-1} \left\{1 - \exp(-C_R NTU)\right\}\right\} \quad (60)$$

[or a cross-flow heat exchanger when fluid representing C_{max} is mixed and fluid representing C_{min} is unmixed, the effectiveness, ε , is expressed as

$$\varepsilon = \left(\frac{1}{C_R} \right) \left(1 - \exp\{-C_R [1 - \exp(-NTU)]\} \right) \quad] \quad (61)$$

11. Use the value of C_c , evaluated at step 3, in Eq. (46), to calculate $(T_{c,o} - T_{c,i})$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{C_R (T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon C_R (T_{h,i} - T_{c,i})$$

[Use the value of C_c , evaluated at step 3, in Eq. (46), to calculate $(T_{c,o} - T_{c,i})$

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon (T_{h,i} - T_{c,i})]$$

12. Calculate $(T_{h,i} - T_{h,o})$ from Eq. (40)

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c / C_h) (T_{c,o} - T_{c,i}) = (T_{h,i} - T_{h,o}) / C_R$$

[Calculate $(T_{h,i} - T_{h,o})$ from Eq. (40)

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c / C_h) (T_{c,o} - T_{c,i}) = C_R (T_{h,i} - T_{h,o})]$$

13. If the value of $(T_{h,i} - T_{h,o})_{cal}$ evaluated in step 12 is the same (or approximately the same) as the actual value of $(T_{h,i} - T_{h,o})_{act}$ (determined from the values given in the problem statement), stop the process and use the last values of C_c and $(T_{c,o} - T_{c,i})$ to evaluate the mass flow rate and exit temperature for water. Otherwise, assume a new value for C_R , go to step 3 and repeat the iteration process.

Excel was employed to implement the procedure described above. In this process it was assumed that $C_h = C_a$ represents C_{min} . Table 2 shows the results of the iteration process. It shows that when $C_R = 0.26105$, the calculated value of ΔT_h converges to the actual value of $\Delta T_h = 300$. Then water exit temperature can be calculated from

$$T_{w,o} = T_{w,i} + \Delta T_c = 12 + 78.31 = 90.31 \text{ } ^\circ\text{C}$$

Table 2 shows that the water mass flow rate in each tube is 0.0216 kg/s or the total mass flow rate of water is 2.16 kg/s.

The Goal Seek or the Solver tools of Excel can be employed to speed up the iteration process for solving the problem in example 6. Fig. 18 shows the assumed (C_R) and calculated values for each step of the procedure described earlier for the iteration process. It shows that for an

Table 2 Iteration process results from Excel spreadsheet for example 6

step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
C_R	$C_c = C_{max}$	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ϵ	ΔT_c	$(\Delta T_h)_{calc}$
0.1	23648	0.0565	6847.0	39.82	1051.2	81.29	1.0800	0.6407	33.00	329.98
0.2	11824	0.0282	3423.5	22.87	603.8	76.89	1.0214	0.6030	62.11	310.55
0.3	7883	0.0188	2282.3	16.53	436.5	73.31	0.9739	0.5702	88.10	293.67
0.21	11261	0.0269	3260.5	21.99	580.6	76.50	1.0163	0.5995	64.84	308.76
0.22	10749	0.0257	3112.3	21.19	559.4	76.12	1.0112	0.5961	67.54	307.00
0.23	10282	0.0245	2977.0	20.45	539.9	75.74	1.0063	0.5927	70.21	305.26
0.24	9853	0.0235	2852.9	19.77	521.8	75.38	1.0014	0.5894	72.85	303.54
0.25	9459	0.0226	2738.8	19.13	505.0	75.02	0.9966	0.5861	75.46	301.85
0.26	9095	0.0217	2633.5	18.54	489.4	74.66	0.9919	0.5829	78.05	300.17
0.27	8758	0.0209	2535.9	17.99	474.9	74.32	0.9873	0.5796	80.60	298.52
0.261	9060	0.0216	2623.4	18.48	487.9	74.63	0.9915	0.5825	78.30	300.01
0.262	9026	0.0216	2613.4	18.43	486.5	74.59	0.9910	0.5822	78.56	299.84
0.2611	9057	0.0216	2622.4	18.48	487.8	74.63	0.9914	0.5825	78.33	299.99
0.26105	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00

assumed value of $C_R = 0.4$ the procedure calculates $(\Delta T_h)_{calc} = 278.66$ °C, which is different from the actual value of $\Delta T_h = 300$ °C given in the problem statement.

By using the tool menu on the Excel worksheet and selecting the Solver option a menu appears as shown in Figure 19. In this menu we can set the target cell (K22) equal to a value of 300 [$(\Delta T_h)_{calc} = 300$ °C]. We also identify the cell (A22, C_R) as the value that needs to be changed during the iteration process. By clicking on the Solve button, the Solver will search for a value of C_R that results in a value of $(\Delta T_h)_{calc} = 300$ °C. The values of all other cells will be changed to correspond to the final value of C_R . Figure 20 displays the final results obtained from Excel's Solver tool for example 6.

	A	B	C	D	E	F	G	H	I	J	K	L
1		Air				Water						
2	$m_a =$	2.25	kg/s		$m_w =$?	kg/s					
3	$c_{p,a} =$	1051	J/kg.K		$c_{p,w} =$	4188	J/kg.K		L=	4	m	
4	$\mu_a =$	3.23E-05	N.s/m		$\mu_w =$	4.20E-04	N.s/m		A=	31.42	m ²	
5	$\nu_a =$	6.02E-05	m ² /s,		$\nu_w =$	5.35E-07	m ² /s,					
6	$k_a =$	0.0497	W/m.K		$k_w =$	0.66	W/m.K					
7	$Pr_a =$	0.69			$Pr_w =$	2.66						
8	$T_{h,i} =$	800	K		$T_{c,i} =$	285	K					
9	$T_{h,o} =$	500	K		$*T_{c,o} =$?	K					
10	$u_{\infty} =$	5	m/s									
11	$C_h =$	2364.75	W/K		$*C_e =$?	W/K					
12	q =	709425	W				W					
13	D =	0.025	m		D =	0.025	m					
14	$S_T =$	0.05	m				m					
15	$u_{max} =$	10.0	m/s				m/s					
16	$Re_{D,max} =$	4152.1										
17	$Nu_{D,max} =$	44.318										
18	$h_o =$	88.105	W/m ² .K									
19												
20	step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step 10	step 11	step 12	
21	C_R	$C_e = C_{max}$	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ϵ	ΔT_c	$(\Delta T_h)_{calc}$	
22	0.4	5912	0.0141	1711.7	13.14	346.8	70.25	0.9333	0.5411	111.47	278.66	
23												

Fig. 18. First step in using Solver for the iteration process in Example 6

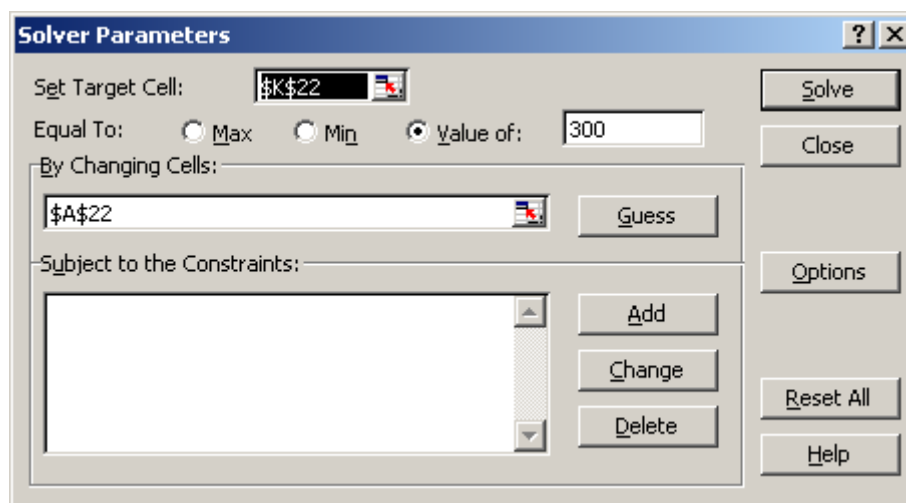


Fig. 19. Solver Parameter menu

	A	B	C	D	E	F	G	H	I	J	K
17	Nu _{D,max} =	44.318									
18	h _o	88.105	W/m ² .K								
19											
20	step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
21	C _R	C _c = C _{max}	(m _w) _i	Re _D	Nu _D	h _i	U	NTU	ε	ΔT _c	(ΔT _h) _{calc}
22	0.261045	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00
23											
24											

Fig. 20. The iteration results obtained by Excel's Solver tool for example 6

Summary

The application of Excel spreadsheet in solving a variety of heat transfer problems was demonstrated through several examples. It was shown that Excel is a useful computational tool when the solution to problems requires (a) varying one of the parameters, (b) plotting the results of calculations, and (c) an iteration process. Excel is accessible to all students and, typically is available on most desktop and laptop computers.

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Nomenclature

- A = surface area or cross-sectional area, m²
 C= Fluid capacitance, W/K
 C_R= capacitance ratio, C_{min}/C_{max}
 c_p specific heat, J/kg.K
 D diameter, m
 F= correction factor
 h = heat transfer coefficient, W/m²·K
 I₀(x), I₁(x) = modified Bessel function of the first kind of order zero, order one
 J₀(x) or J₁(x) = Bessel function of the first kind of order zero or order one
 K₀(x), K₁(x) = modified Bessel function of the first kind of order zero, order one
 k = thermal conductivity, W/m·K
 L= length, m
 \dot{m} = mass flow rate, kg/s
 LMTD= logarithmic mean temperature difference, K
 NTU= number of transfer units, Eq. 10
 Nu Nusselt number
 P= correction factor parameter, Eq. 5
 q = heat transfer rate, W
 R = correction factor parameter, Eq. 6
 Re = Reynolds number
 S pitch
 T = temperature, °C or K
 U = overall heat transfer coefficient, W/m²·K

Greek letters

- α = thermal diffusivity, m²/s

Δ	difference
ε	heat exchanger effectiveness
$\theta =$	dimensionless temperature parameter, a ratio of temperature differences
$\eta_f =$	fin efficiency
$\mu =$	viscosity, N.s/m
$\nu =$	kinematic viscosity, m ² /s

Subscripts

c=	cold fluid stream
h =	hot fluid stream
i =	inlet condition
L	linear
min=	minimum value
max	maximum value
o =	outlet condition or location at x=0
T	transverse
$\infty =$	ambient condition
f =	fin

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