AC 2012-4617: USING INSTRUCTION TO IMPROVE MATHEMATICAL MODELING IN CAPSTONE DESIGN

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Using instruction to improve mathematical modeling in capstone design

Abstract

Engineering students in capstone design have the opportunity to develop unique solutions to open-ended and analytically complex problems, allowing them to use their knowledge from previous coursework. One element of design is mathematical modeling, but students often struggle in recognizing when and how to apply the mathematical analysis encountered in prior coursework to their particular design solutions. Specifically, they struggle with creating, manipulating, and critiquing mathematical models to assist in the design of a product or process. The ultimate aim of our work is to improve students' ability to use models in capstone design after being exposed to instruction on mathematical modeling.

This study was a continuation of an earlier project in which we explored how students developed, used, and interpreted mathematical models. In the previous study, students were given instruction in the steps of mathematical modeling and a scenario in which they were asked to assist a hypothetical design team by creating a mathematical model that could be used in making decisions about the design. The instruction and the scenario broke down model creation and interpretation into six stages, from defining the model parameters to generating the mathematical equations and interpreting model outputs. In this study, we investigated how well students transferred these skills to their own open-ended design problems.

Specific instruction in mathematical modeling was required before we saw an impact on the use of modeling in students' own design projects. Eighty-two percent of teams were able to generate some type of mathematical relationship between model parameters after instruction, whereas in previous years, where no instruction occurred, only 25% of the teams developed a mathematical relationship to describe their system. However, the results also showed that even with instruction, students did not utilize models to their full potential. Only 41% of all teams adequately explored the parameter space of their model, and only 12% of teams discussed the limitations of their model. Seventy-one percent used the model results to inform their final design. This research has provided some insight into how to revise instruction in order to improve engineering students' abilities in mathematical modeling in the context of design.

Introduction

One of the traditional methods of teaching mathematical models in engineering courses has been to begin with analysis of a well established mathematical representation (or model) and then apply these equations to physical systems. Often these physical systems are special cases, where manipulation of the equations leads to a convenient analytical solution. In many cases, the assumptions, simplifications, and equation development have already occurred, and students do not get as much experience in these aspects of model setup. Students rarely start from a physical system and then develop appropriate mathematical relationships. Students, instead, try to fit the models they have learned to physical situations they encounter, which is a difficult and time-consuming way to approach an open-ended design problem.

The ability to create and manipulate a mathematical model for a physical system is a very important skill to possess, especially in engineering design. As future engineers, students should be prepared to approach physical systems with the appropriate analytical skills, including the ability to frame problems in terms of appropriate mathematical models and finding solutions to those models.

Modeling can be used in the design process in many ways: to avoid expensive and time-consuming tests of physical prototypes, to guide the range of physical models that should be tested, to rule out seemingly reasonable designs that are destined to fail, to avoid overdesign of components, to explore the likely range of performance of a device, and to estimate failure rates of a device composed of many elements. Thus, there are many reasons why students should possess the capabilities to do modeling.

To further understand how to prepare students to create and manipulate mathematical models we investigated students' abilities at modeling in the context of engineering design. The current study was a continuation of an earlier project [1,2] in which we explored how students developed, used, and interpreted mathematical models. In the previous study, students were given instruction in the steps of mathematical modeling and a scenario in which they were asked to assist a hypothetical design team by creating a mathematical model that could be used in making decisions about the design. The instruction and the scenario broke down model creation and interpretation into six stages [3], from defining the model parameters to generating the mathematical equations and interpreting model outputs. Rather than teaching particular modeling software, the purpose was to explain why modeling is done, what needs to be considered in moving from a physical system to a model, why one should explore a parameter space, and how to interpret models. We found that most students had difficulty in deciding what parameters were relevant, representing a physical situation in equations, and stating and justifying simplifications and assumptions. Students performed better at interpreting the model, and relating graphical results from the mathematical model to experimental data obtained from a physical model. They were also able to use the model outputs to make design decisions, or explain why the existing model was inadequate for this purpose. In this current phase of the study we investigated how well students transferred these skills to their own open-ended design problems.

Background Literature

Several researchers [4-6] have investigated how to instruct students in open-ended, design-type tasks, called Model Eliciting Activities (MEA). MEAs are designed to lead students through the process of open-ended problem solving from problem definition, through model use, to prototype design. A series of questions can be used to get students thinking about how to interpret physical problems and convert them into mathematical terms. An instructional framework [5] for developing an MEA includes the following six principles:

1. Model Construction
2. Reality

3. Self-assessment
4. Model Documentation
5. Model Shareability and Reusability
6. Effective Prototype

Most client-driven, open-ended design tasks lend themselves well to modeling activities. They are realistic engineering problems with physical phenomena that can be translated into mathematical representations. Models can be used to predict physical behavior based on patterns in real situations (or hypotheses of physical behavior when too little information is available).

However, the model construction step is not trivial. Lesh [5] breaks down the components of a model as "elements, relationships among elements, operations that describe how the elements interact, and patterns or rules that apply to the relationships and operations." Understanding these steps (identifying the phenomena and their relationships, "mathematizing" these phenomena and relationships, and manipulating these mathematical equations) is critical in creating a mathematical model. Once the model has been created, the user must know how to understand, evaluate, and use the model results to inform design decisions and build a successful prototype. The steps outlined below (creating and using a model) have been the focus of our research.

Our study explored students' abilities at creating mathematical models, interpreting model output, and using that output to inform design decisions. Mathematical modeling, and analysis of the resulting model output, can inform decisions by generating predictions for situations that are too difficult, too time consuming, or too expensive to test physically. Our previous work [1,2] used a framework for model creation and use based on Gainsburg's [3] six steps for what mathematical modeling should include:

1. Identify the real-world phenomenon
2. Simplify or idealize the phenomenon
3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)
5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

Our studies used Gainsburg's framework for the creation and use of mathematical models to evaluate students' abilities in different steps of modeling of an open-ended design problem. As implied by Gainsburg's framework, we considered a model to be more than a simple calculation. It needed to be a conception of a problem in a mathematical format that can yield different outputs depending on the values chosen for inputs, initial conditions, and/or model parameters.

Research Method

We analyzed final design reports from three sets of students taking the same Biomedical Engineering (BME) capstone design course. The first set of students took the course in 2009 (n=12 team projects). The second and third sets of students took the course the following two years (2010, n=17 team projects and 2011, n=13 team projects). In this paper we shall refer to these three sets of students as BME09, BME10, and BME11.

Data were collected from each student team's final design report and presentation for the capstone course. Each final project in this BME capstone design course was the culminating work of a partnership between a team of students and a real client with a real need. All student teams had different clients and unique open-ended projects, yet each team was required to design a working prototype of its final product design. Each project was completed in teams of 3-4 students in an 11-week academic quarter.

In addition to regular coursework, both sets of students were exposed to a collection of classroom activities on mathematical modeling. Students were presented with an open-ended design scenario to model and asked to respond to questions designed to evaluate their abilities to perform the steps presented by Gainsburg. There were four stages to the scenario activity, each one focused on one or more of the steps, presented during the course of the academic quarter. Students worked on this scenario in all three years. In BME10 and BME11 however, had additional lectures/discussions on mathematical modeling to supplement the scenario activities. These lectures provided instruction on what mathematical modeling is, why it is important, and identified the major mistakes that had been made in the completed stages of the scenario activities. The intent of these lectures was to bring all students back to the same starting place for the next stage of the activity and get them to reflect on how their work differed from what an experienced modeler might have done. The results of student performance on the in-class activities can be found in our earlier works [1,2].

The intent of the current portion of the study was to evaluate student performance at incorporating mathematical modeling on their team projects. A rubric was developed to assess student performance on the steps proposed by Gainsburg. The rubric consisted of a set of questions and decision trees that matched up to the modeling steps. This had to be flexible enough to apply to different projects, so it was different than the rubric that we had used to analyze student performance during the scenario study. It contains many branch points allowing for different possible outcomes. An example of a section of the rubric (about 20% of the overall rubric "tree") is shown in Figure 1. Three independent evaluators scoring three randomly selected reports evaluated the rubrics. The agreement rate between the evaluators was 85%. The remaining reports were scored primarily by one of the investigators.

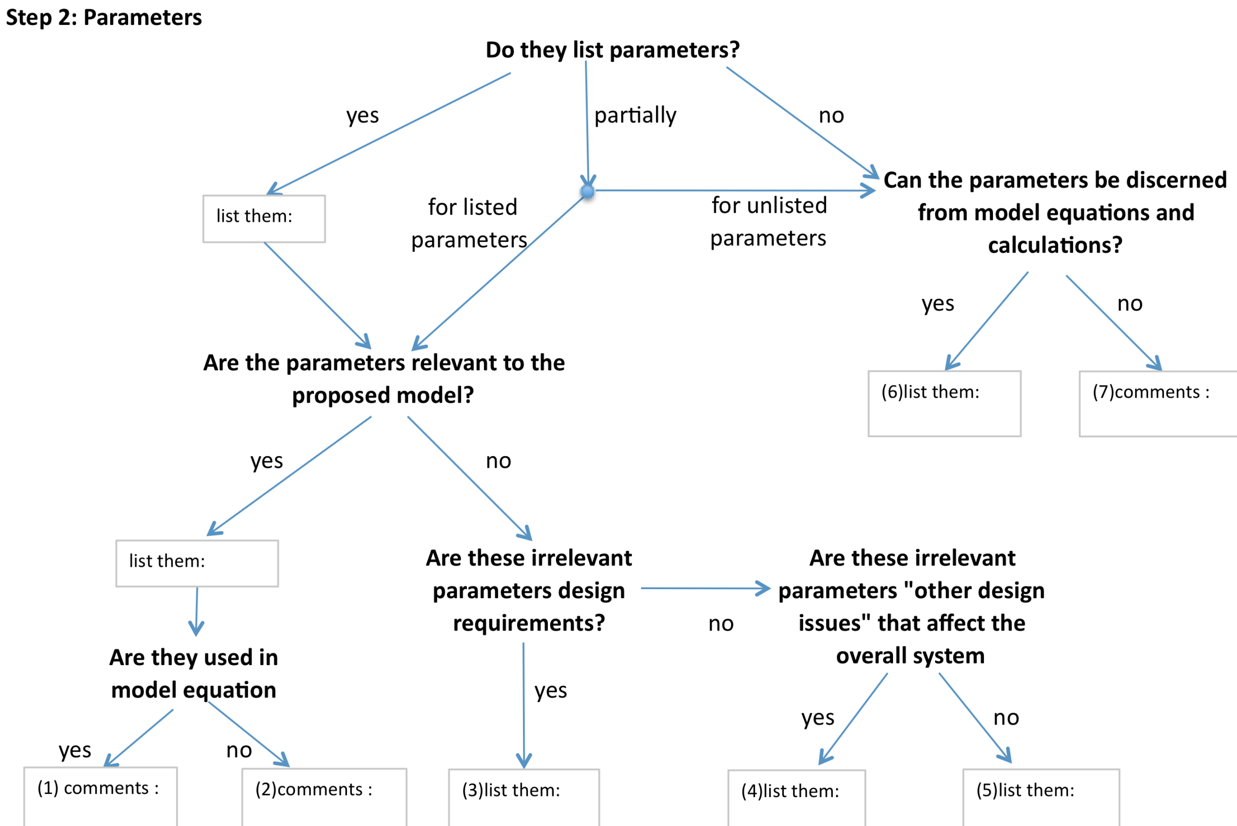


Figure 1. This decision tree was a portion of the rubric for assessing student performance in identifying and simplifying real world phenomena. Evaluators looked for instances when the student teams identified the parameters relevant to the model proposed in their project design.

Each team was assigned a unique project and client. The following list is a small sample of the types of projects the BME design teams attempted:

- Urinary incontinence: Design a device to detect and alert active adults with urinary incontinence to a failure in fluid retention by an absorbent product.
- Placenta simulator: Design a simulator device to train midwives in developing countries on third stage labor, the delivery of the placenta.
- CPAP: Design a device that will optimize Continuous Positive Airway Pressure (CPAP) in premature infants that is used in conjunction with Kangaroo Mother Care (KMC) in developing countries.
- Ultraviolet Germicidal Irradiation: Design an inexpensive, low-maintenance, and efficient system that utilizes Ultraviolet Germicidal Irradiation to reduce the number of new tuberculosis infections in a health-care setting in the developing world.

The nature of the projects was consistent with BME curriculum. No prediction was made when projects were selected for the course as to how well projects would lend themselves to modeling, but our experience has shown that all projects can benefit from some type of mathematical model. However, the diversity of projects requires that students be taught a framework for modeling, rather than some particular type of modeling software.

Results

Each team's project report was scored with the rubric. The results for each question and decision tree in the rubrics were normalized to a scale of 0-100%, where the percentage is the fraction of teams at a certain performance level.

1. Identify the real-world phenomenon

The first step of the framework proposed by Gainsburg was to identify the real-world phenomenon. We interpreted this step as outlining the problem that the proposed model was supposed to address. In the rubric, this step was translated into the rubric as a stand-alone question asking whether teams directly stated the modeling problem they were trying to solve, i.e., the project space that would be evaluated with a mathematical model.

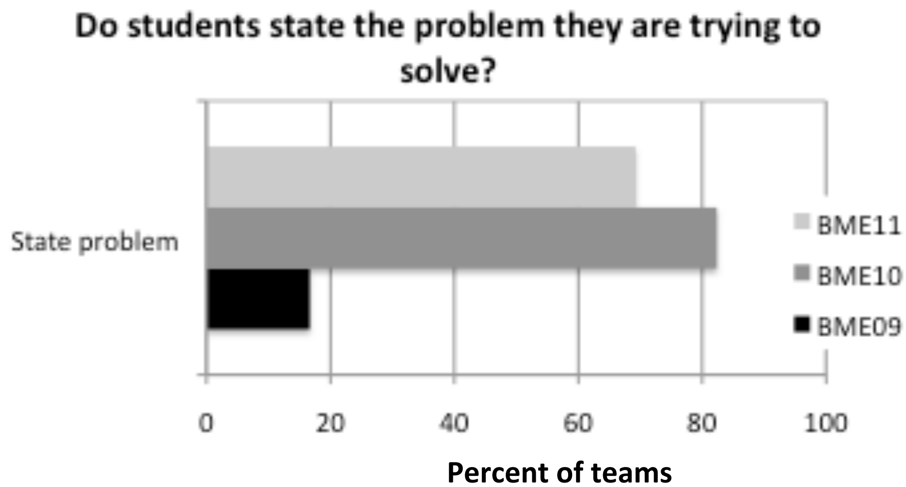


Figure 2. Rubric question one evaluated teams' ability to state the problem they are attempting to model (Framework step 1 – identify the real world phenomena).

The majority of teams in BME10 and BME11 identified the portion of the project (i.e. the phenomena) they were trying to model. Figure 2 shows that fourteen (82%) teams in BME10 and 9 (69%) teams in BME 11 made a problem statement. In contrast, 10 teams from BME09 (83%) failed to make a problem statement. Failure to identify the phenomena to be modeled did not prevent teams from creating a mathematical model; this portion of the rubric simply recognized teams that were cognizant of the need to identify the segment of the overall problem that would be approached with mathematical modeling.

2. Simplify or idealize the phenomenon

The second step of Gainsburg's framework is to simplify or idealize the phenomenon. This step serves to make the problem easier to understand and model. We translated this step into the rubric as three decision trees. These trees asked:

1) Do students list parameters? (*As seen in Figure 1*)
2) Do students state assumptions?

3) Do students provide sketches?

Each question addressed a different aspect of simplification. Parameters are the elements of a system. Equations describe how parameters interact and behave in a system. Listing parameters allows modelers to identify the important elements in a system, so that relationships between parameters (or equations) can easily be built. Assumptions allow modelers to neglect certain factors or simplify relationships between parameters and variables. Assumptions lead to simpler models that are easier to develop and understand. Sketches are simplified drawings. They help modelers to visualize the assumptions being made and the interactions of the parameters.

Because it was possible to state some assumptions and parameters and neglect to mention others, the decision tree in the rubric contained an option for partially stating the assumptions and parameters. The evaluator reading a report with partially stated assumptions or parameters would be able to assess both the stated and the unstated parameters or assumptions.

The results of the first question, "do students list parameters?" are shown in Figure 3. Seventy-five percent of teams from BME09 (9 teams) did not state any parameters. The remaining three teams (25%) only partially stated parameters. The BME10 and BME11 teams showed marked improvement, with seven teams (41%) in BME10 and six teams (46%) in BME11 fully identifying the parameters they planned to model. However, there were still ten teams (59%) in BME10 and seven teams (54%) in BME11 that failed to state any parameters or only partially stated them.

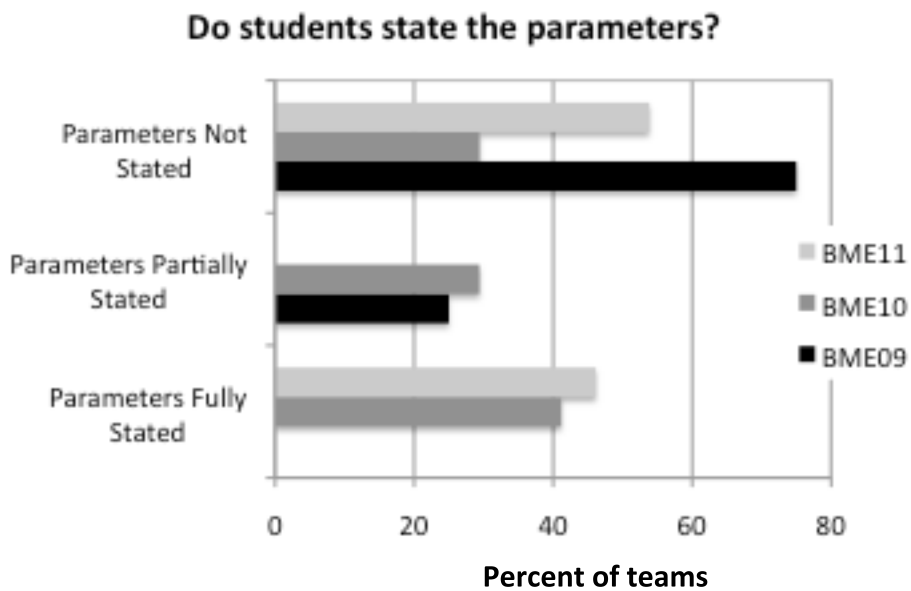


Figure 3. The rubric evaluated teams' ability to identify parameters they proposed to model (Framework step 2 – simplify or idealize the real world phenomena).

Looking deeper into the parameters that the teams were identifying, (or not identifying), the rubric's decision tree attempted to divide the parameters into relevant and irrelevant categories. We found that six teams (35%) in BME10 and six teams (46%) in BME11 provided parameters that were both relevant to the model and used in the model equations, whereas BME09 had no teams providing relevant parameters. One team in both BME09 and BME10 listed design requirements (constraints given to certain variables in the problem statement) confusing them with parameters that could be adjusted in the model. While this demonstrated an intention to have parameters, there was a misunderstanding of what constituted a proper parameter. This is a great improvement over student performance on the in-class activities, where students frequently listed the design requirements as parameters [1,2]. These results are summarized in Table 1. Because more than one outcome was possible, the percentages may add up to more than 100 in the table.

Table 1. Main results of the decision tree for the question "Do students state the parameters?" (Framework step 2 – simplify or idealize the real world phenomena).

	BME09	BME10	BME11
Parameters relevant and used in equation	0 (0%)	6 (35%)	6 (46%)
Parameters relevant and not used in equation	0 (0%)	1 (6%)	0 (0%)
Parameters are actually design requirements	1 (8%)	1 (6%)	0 (0%)
Parameters are not listed but can be discerned from equation	3 (25%)	7 (41%)	2 (15%)
Parameters are not listed and cannot be discerned from model equation	9 (75%)	3 (18%)	5 (38%)

The results of the second question, "do students state assumptions?" are shown in Figure 4. Ten teams (83%) in BME09 did not state any assumptions of their models. The remaining two teams (17%) of BME09 fully stated assumptions required to build the equations relevant to their model. The reviewers made a judgment call when evaluating whether assumptions were "fully stated." A team's assumptions were considered fully stated when the assumptions necessary to use the given equations, simplifications, or boundary conditions specified by the team were provided.

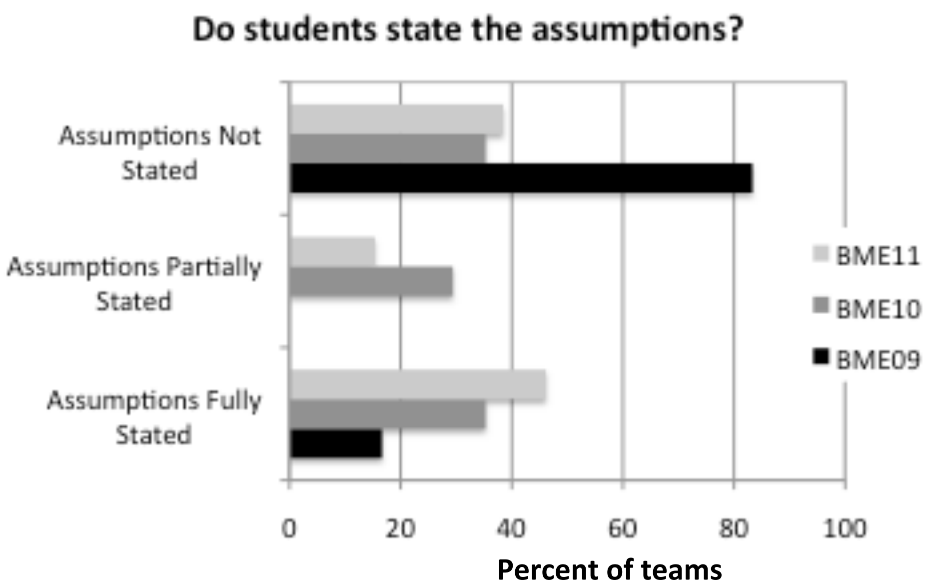


Figure 4. The rubric evaluated teams' ability to identify assumptions that would simplify the system they are attempting to model (Framework step 2 – simplify or idealize the real world phenomena).

In BME10, there was a relatively even split among all options. There were six teams (35%) that fully stated assumptions, five teams (29%) that partially stated assumptions, but still six teams that failed to state any assumptions. BME11 had six teams (46%) that fully stated assumptions, two teams (15%) partially stated assumptions, and five teams (38%) that failed to state assumptions.

The rubric further distinguished the stated assumptions into "flawed" and "not flawed." A "not flawed," or flawless, assumption simplifies or describes the system in a way that is consistent with reality (for example, treating an LED as a point source of light when it is some distance away from a surface of interest). A "flawed" assumption simplifies or describes the system in a way that is inconsistent with reality (for example. treating an LED as a laser with coherent light projecting in only one direction).

Table 2. Results of the decision tree for the question "Do students state assumptions?" (Framework step 2 – simplify or idealize the real world phenomena.)

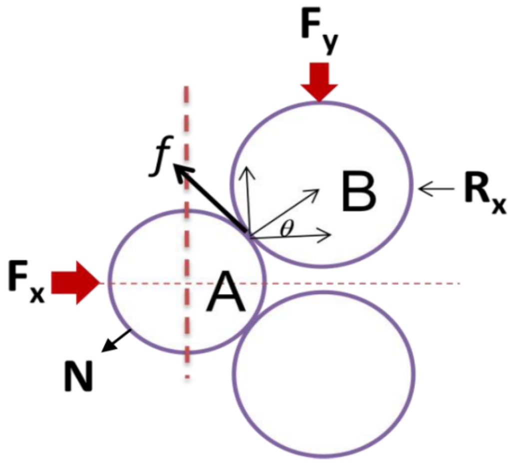
	2009	**2010**	**2011**
Assumptions are not flawed and used in proposed model	1 (8%)	11 (65%)	7 (47%)
Assumptions are not flawed but not used in proposed model	1 (8%)	0 (0%)	1 (7%)
Assumptions are flawed and used in the proposed model	0 (0%)	1 (6%)	1 (7%)
Assumptions are not stated (no model proposed)	8 (67%)	1 (6%)	2 (13%)
Assumptions are not stated and are needed to develop the proposed model	2 (17%)	10 (59%)	4 (27%)

The unstated (or missing) assumptions were also further distinguished into assumptions that were "needed" or "not needed" to develop the proposed model. Because of this design in the rubric, a team could have both stated and unstated assumptions, leading to a total tally in Table 2 greater that the total number of teams.

Of the seventeen BME10 teams, eleven (65%) used flawless assumptions to develop their model. In contrast, ten teams (59%) failed to use assumptions that were necessary to develop their model. This suggested that the teams would either identify, and use, relevant assumptions or they would completely fail to state the assumption. Of the twelve BME09 teams, ten (83%) failed to state assumptions. However, eight of these twelve teams were scored as "no model proposed." This was because these teams failed to develop a model. If there was no model, there was no need to state any assumptions. If we were to omit those eight teams without models, there would be a roughly 50:50 split between stating and not stating assumptions in BME09. Of the thirteen BME11 teams, seven (47%) used flawless assumptions to develop their model, while four teams (27%) did not identify assumptions that were needed to develop their model. Two teams (13%) in BME11 did not provide any mathematical model.

The last question in the rubrics for step two was "do students provide sketches?" In particular, we categorized the quality of the sketches, looking for simplifications, meaningful interactions between parameters, and proper labels. The sketching rubric identified two major factors:

meaningful interactions and helpful details. If a sketch had meaningful interactions, it outlined how the parameters in the final equation interacted with each other. If a sketch had helpful details, it showed physical dimensions and characteristics. Examples of these factors can be seen in Figure 5.



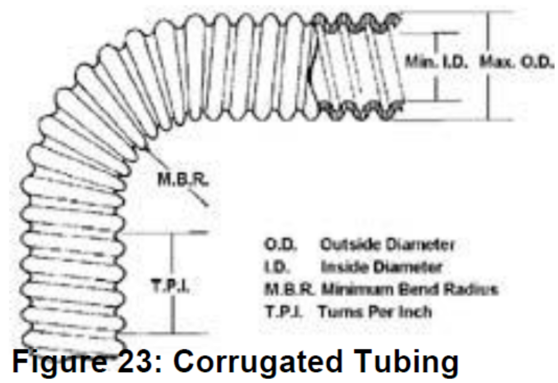


Figure 5. (Left) Sketch containing meaningful interactions – note the decomposition of forces on the spheres. (Right) Sketch containing helpful details – note the physical dimensions and features. (Images taken from BME10 final reports for Placenta Simulator and CPAP projects).

As shown in Figure 6, only one of twelve teams (8%) in BME09 provided any form of sketch. This sketch had helpful details but no meaningful interactions. In BME10, a total of nine teams (53%) had a sketch that contained meaningful interactions and four teams (24%) provided no sketch. In BME11, three teams (23%) provided a sketch with meaningful interactions, five teams (38%) had sketches with some details but no meaningful interactions, and five teams (38%) had no sketches. This indicated that more students in BME10 and BME11 were aware of the importance of this step in the simplification of the problem.

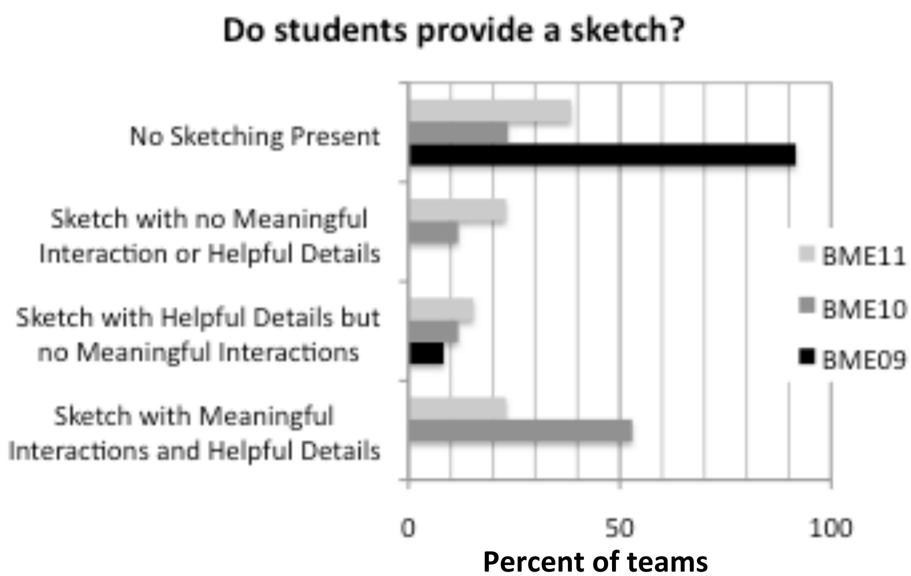


Figure 6. The rubric evaluated sketches of the system teams attempted to model (Framework step 2 – simplify or idealize the real world phenomena).

3. Express the idealized phenomenon mathematically, and
4. Perform the mathematical manipulations

Steps three and four of Gainsburg's framework were to express the idealized phenomenon mathematically and perform the mathematical manipulations. These steps served to develop the relationships between the parameters that were identified in step two of the framework. The relationships could be manipulated using the stated assumptions, simplifications, or other equations.

In the rubric, these two steps were translated into one decision tree. This tree identified when students provided a relationship between parameters, and how students utilized those relationships. A distinction was made between equations and other forms of relationships in the rubric. This was done to accommodate teams with models that were not conducive to traditional equations. For example, a team used statistical relationships from similar systems to gather insight into their proposed system. A distinction was also made between developed and undeveloped equations. Developed equations started from some general theory or equation, and were tailored to the specific model in question. Undeveloped equations had a tendency to be generic and selected from the literature, without an attempt to relate it specifically to the design problem.

Figure 7 shows that teams in BME10 and BME11 were more likely than teams in BME09 to complete steps three and four, producing and manipulating mathematical relationships in their model. Nine teams (75%) from BME09 had no equation or relationship, and hence no model. Only three teams (18%) from BME10 and four teams (31%) from BME11 had no relationship.

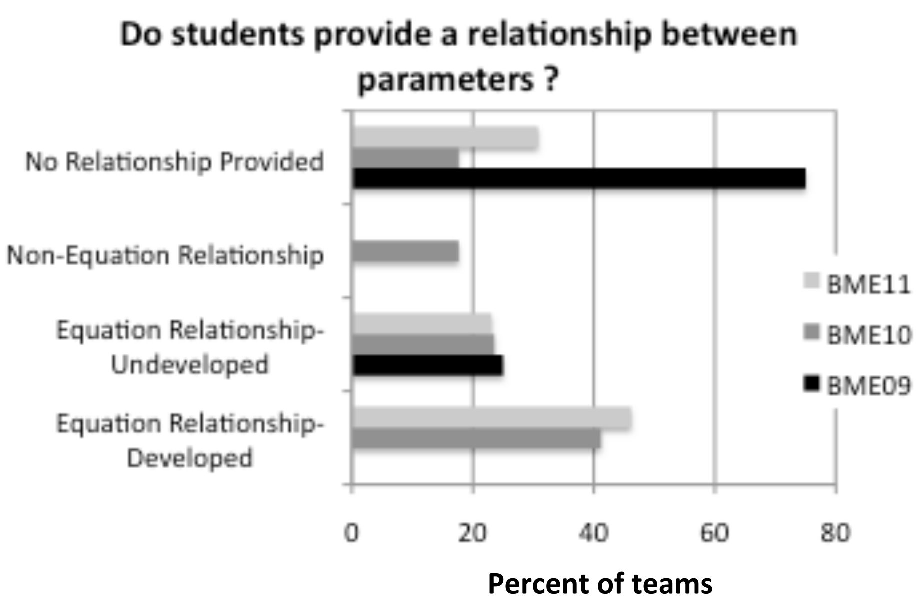


Figure 7. The rubric examined teams' use of mathematical relationships and manipulations. (Framework steps 3 and 4 – express the idealized phenomenon mathematically, and perform the mathematical manipulations).

Further, seven teams (41%) in BME10 and six teams (46%) in BME11 had an equation that was tailored to the proposed model. This was a significant improvement over BME09, in which many teams were unable to provide a mathematical relationship between parameters. Of those four teams (25%) in BME09 that were able to provide a relationship, those relationships were not manipulated, or tailored to the proposed model, leaving step four of the framework incomplete.

5. Interpret the mathematical solution in real-world terms

The fifth step of Gainsburg's framework was to interpret the mathematical solution in real-world terms. This was translated into the rubric as a decision tree and two stand-alone questions. The decision tree addressed whether students had model outputs and whether those outputs were interpreted correctly. The stand-alone questions identified whether the parameter space was adequately explored and whether the teams used model outputs to guide their design.

As shown in Figure 8, three teams (25%) in BME09 gave correctly interpreted model outputs. The remaining nine teams in BME09 were unable to provide a model, and therefore unable to provide model outputs. In BME10, however, seven teams (41%) provided correctly interpreted model outputs. Another three teams (18%) provided model outputs, but did not interpret them or interpreted them incorrectly. The remaining seven teams (41%) in BME10 failed to provide the model output, or discussion of the model output, in their final reports. In BME11, six teams (46%) provided correctly interpreted model outputs and six teams (46%) failed to provide model outputs. These results implied that, if students were able to obtain model output, they were likely to interpret it correctly. However, teams were just as likely to stop after generating a formal description of a model, without using it to generate any outputs that contained useful information.

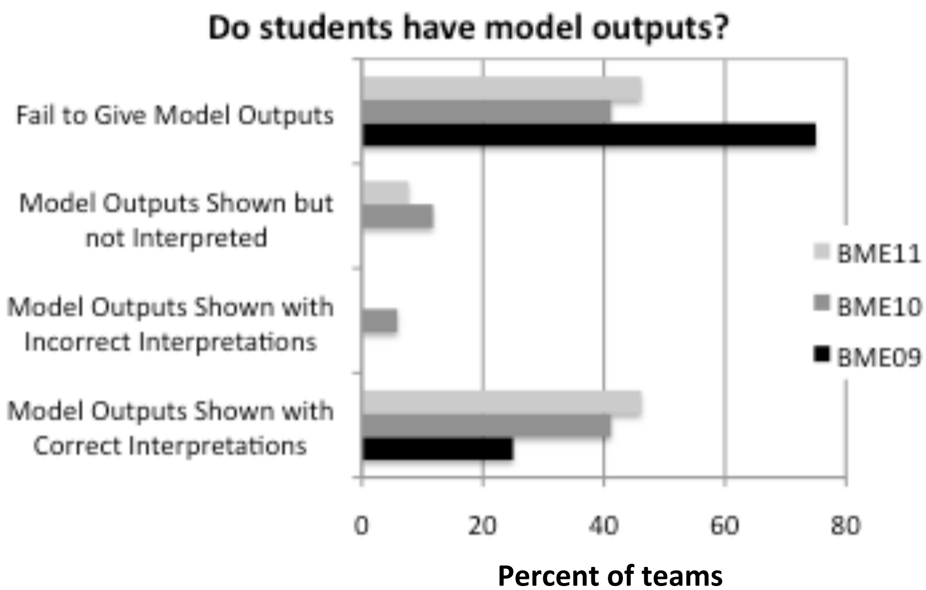


Figure 8. The rubric evaluated teams' ability to understand and use model output. (Framework step 5 – interpret the mathematical solution in real-world terms).

The rubric further identified when teams adequately explored the parameter space. This portion of the rubric helped us understand what range of values students would use as inputs for their models and whether their models were going to be utilized to their full potential. If a team only plugged in one set of values that they felt was the best option, this was considered as a partial exploration of the parameter space. In our judgment, adequate exploration required varying parameter values to investigate how the behavior of the system changed with these changing values. Inadequate exploration occurred when students used a single set of inputs to the model (single set of parameter values).

The nine teams (75%) in BME09 without models had no model exploration, and were categorized as "inadequate exploration." As shown in Figure 9, the remaining three teams (25%) only had partial exploration of the parameter space. The teams in BME10 showed marked improvement. Seven teams (41%) had adequate exploration and five teams (29%) had partial exploration for a total of twelve teams (70%) with some form of exploration of the parameter space. The teams in BME11 did not fare as well as teams in BME10, with only two teams (15%) providing adequate exploration of the parameter space and five teams (38%) partially exploring the parameter space. Four of the six teams in BME11 that failed to explore the parameter space also failed to develop a mathematical model.

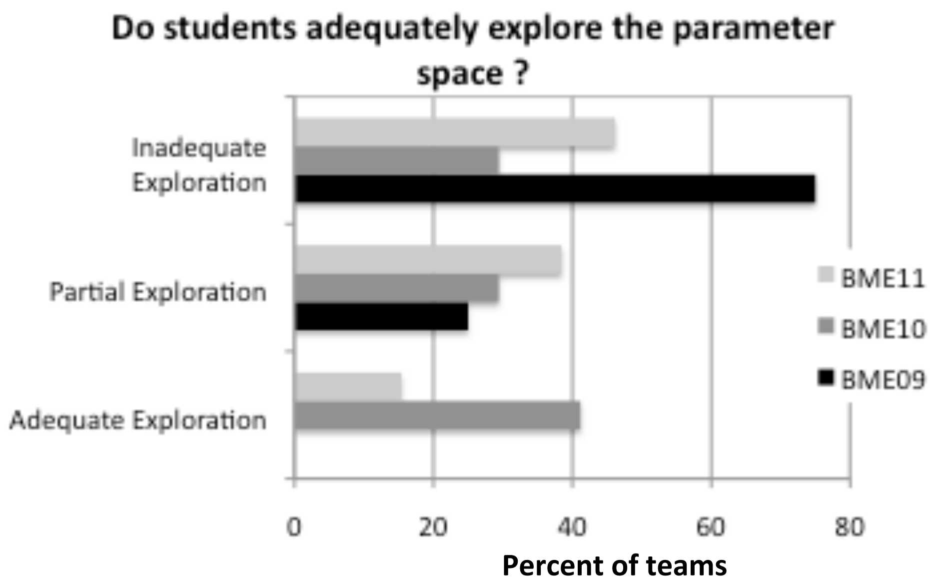


Figure 9. The rubric evaluated teams' ability to explore the parameter space of their model. (Framework step 5 – interpret the mathematical solution in real-world terms).

A major purpose for mathematical modeling in design is to use model outputs to inform design decisions. This portion of the rubric used a stand-alone question to determine whether teams utilized model outputs to guide their design. Figure 10 shows that seven teams (54%) in BME11, twelve teams (71%) in BME10, and three teams (25%) in BME09 used model outputs to guide design. Given the importance of using models to guide designs, the results from all years appeared disappointing. However, a prerequisite to using model outputs is to have a model. In BME10, fourteen teams provided some relationship between parameters. Of these fourteen, twelve teams used their model to guide design. This was a success rate of 86%. In BME11, nine teams had a relationship between parameters and seven used the outputs to guide their design; a success rate of 78%. In BME09, three teams had a relationship between parameters, and all three of these teams used their model to guide design. This suggested that, once a model was developed, using it to inform design was a natural progression.

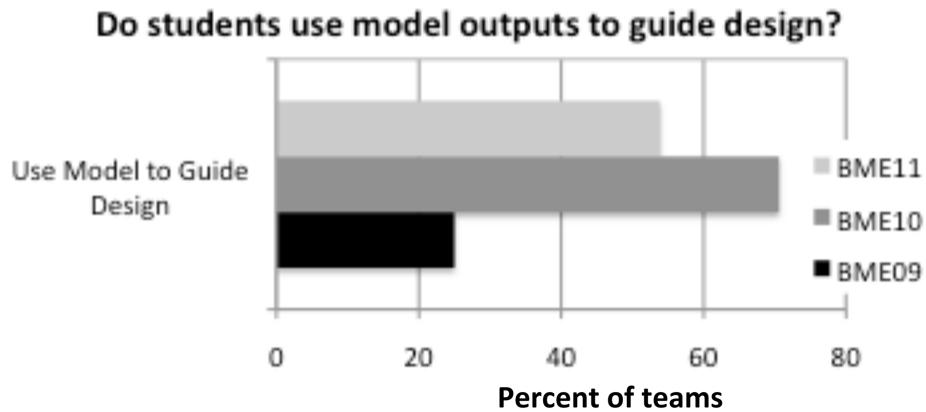


Figure 10. The rubric distinguished whether teams used model outputs to guide their design decisions. (Framework step 5 – interpret the mathematical solution in real-world terms).

6. Test the interpretation against reality

The last step in the modeling framework was to test the interpretation against reality. An assessment of the accuracy or believability of the model outputs can inform the user to what degree the model outputs can be trusted. This step was translated into the rubric as two stand-alone questions. The first question determined whether teams verified the model with a real test. The second question evaluated students' abilities to discuss the limitations of the model.

As shown in Figure 11, no teams in BME09 verified their model using a real test. Only four teams (24%) from BME10, and three teams (23%) in BME11 had some form of verification. Additionally, one team from BME09 (8%) and two teams (12%) from BME10 stated an intention for model verification, indicating that teams understood the importance of testing the model outputs, however these teams did not have a plan for proceeding with that verification.

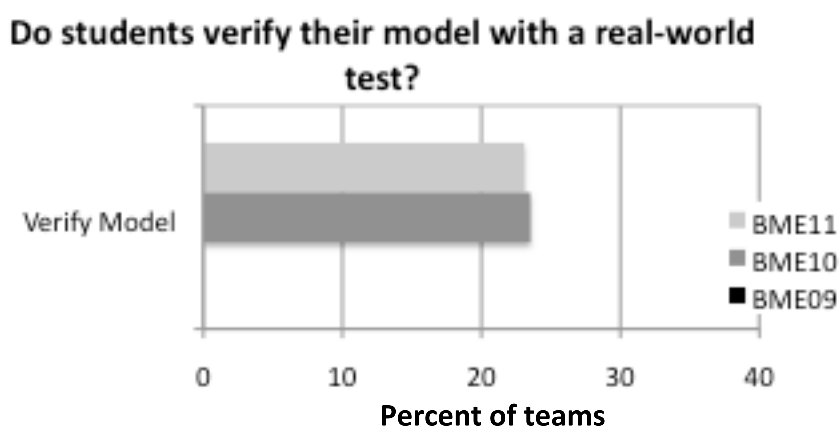


Figure 11. The rubric evaluated whether teams verified their models. (Framework step 6 – test the interpretation against reality).

Lastly, student performance was measured on their discussion of the limitations of their models. "Failure to discuss limitations" indicated that teams had little understanding or acknowledgement of how the model differs from reality. In "partial discussion," there was admission by students that the model differs from reality in some ways but no elaboration on how or why it differs. A team with "full discussion" explained to at least some extent how the model differs from reality and included details such as whether the model outputs over- or underestimates the behavior of the physical system.

No team from either BME09 or BME10 provided a full discussion of the model limitations and only one team from BME11 (8%) provided full discussion of model limitations, as shown in Figure 12. Only one team (8%) from BME09, two teams (12%) from BME10, and two teams (15%) from BME11 gave a partial discussion of limitations. This was seen as weakness in students' modeling performance.

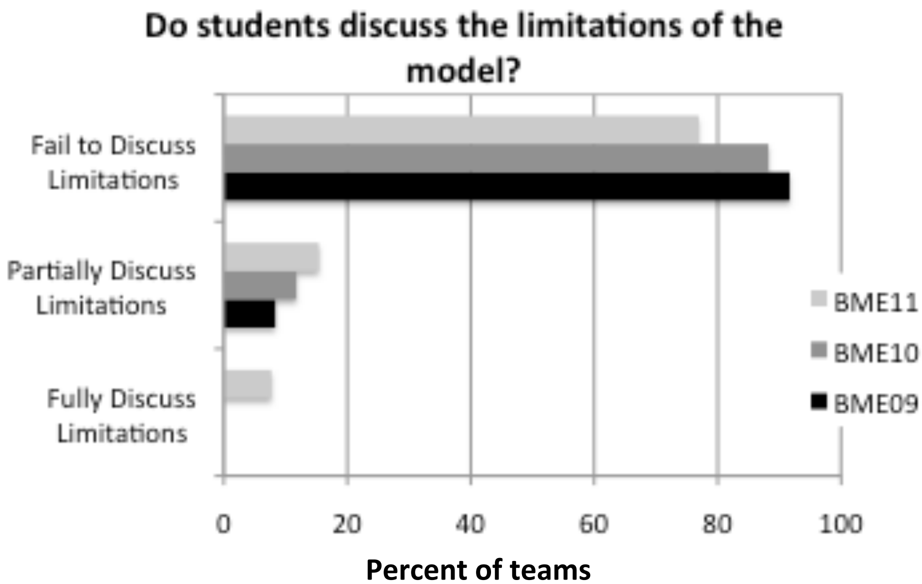


Figure 12. The rubric evaluated teams' discussion of model limitations. (Framework step 6 – test the interpretation against reality).

Discussion

Mathematical modeling in the service of design is one of the most advanced skills in the undergraduate engineering curriculum, and our studies have reinforced how difficult it is. It calls upon a great deal of students' prior knowledge, used differently than they have used it in previous courses. Our experience has been that students disconnect design and mathematical modeling, and a concerted effort is required to bring these together. In BME09 we assessed students' abilities in using modeling in a design scenario with a combination of in-class and homework assignments, but we did not use their work to teach modeling. In contrast, in BME10 and BME11, students worked through the same design scenario but between phases we constructed classroom lectures and discussions about modeling. Here we have shown that this intervention and explicit teaching about modeling had considerable impact, as seen in the shift in modeling used in projects between BME09 and BME10 and BME11.

It is important to point out that the scenario problem concerned modeling the interaction of light with a baby's skin. None of the students' actual design projects involved anything like this. The projects were a range of topics that required completely different models, as is characteristic of BME design projects, yet their performance in these other modeling domains showed improvement. Thus we feel that our discussion of modeling in Gainsburg's general framework was very beneficial, probably more beneficial than having students work with a specific modeling language or package in terms of their use of future models.

This improvement in modeling can be seen in student performance on the six steps of the framework proposed by Gainsburg. The BME09 teams had lower levels of performance on all six steps when compared to BME10 and BME11. Figure 7 showed that teams in BME10 and BME11 were more likely than teams in BME09 to complete steps three and four of the framework, producing and manipulating mathematical relationships in their model. Three teams (25%) from BME09, eleven teams (65%) from BME10, and nine teams (69%) from BME11 had some sort of mathematical relationship between parameters. This outcome was due mainly to the lack of mathematical models produced by teams in BME09. Because a large number of BME09 teams failed to create a model, they were unable to perform well on the later modeling steps that required the existence of a model.

We can attribute improvement in creating models to improvement in several of the early steps of the framework from teams in BME10 and BME11. Notably, an improvement in identifying parameters, assumptions, and sketching of the relationships between parameters led to an improved success rate in creating mathematical models in BME10 and BME11. The most important result is that specific instruction in modeling, which we did in the context of taking the student step by step through modeling in a particular scenario, can translate to improvements in at least some of the abilities critical to using models in situations where teams are on their own, and it is not possible to use that step by step approach.

However, in the later steps of the framework, there was no discernable improvement in the ability to use model output to guide design decisions. This may be partially due to the difficulty of completing all the elements of the design in a short time frame, leaving inadequate time for some teams to generate outputs from the modeling. Of the teams that were able to complete a mathematical model in BME09, BME10, and BME11, there was a high success rate in utilizing the model output to inform design. This suggested student familiarity with interpreting and utilizing completed models, and may be a consequence of how we teach equations and models in other courses – where instructors teach well-established mathematical models of physical systems.

It is not reasonable to expect undergraduates to fully deal with all the aspects of the different steps of mathematical modeling in their first experience. One confounding issue was time limitations. The academic term is only ten weeks long and testing models may have been too resource intensive to complete within the term. This may have been one reason for the low numbers of teams that verified their model with a real test or discussed the limitations of their proposed model. However, having model outputs isn't necessarily a requirement for discussing limitations of a model. For a simple model, students could evaluate the equations, parameters, and assumptions to determine the direction of the error. Still, the time limitation of completing a design project within one academic quarter is the likeliest reason that students may not have completed the final steps of the modeling framework.

Conclusion

The data from this study suggest that the lectures given in BME10 and BME11 influenced students' performance on several of the steps in mathematical modeling, and allowed for marked improvement over their peers from BME09. While there is more room for improvement, it has been shown that instruction can have an impact on students' modeling abilities. Additionally, the weaknesses that students still possess have been identified. Using these results, instruction can be improved to further develop students' mathematical modeling skills.

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